Computer algebra independent integration tests

Summer 2022 edition

5-Inverse-trig-functions/5.3-Inverse-tangent/148-5.3.2-d-x-^m-a+b-arctan-c-x^n-^p

Nasser M. Abbasi

September 27, 2022 Compiled on September 27, 2022 at 4:36am

Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [166]. This is test number [148].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (166)	0.00 (0)
Mathematica	98.19 (163)	1.81 (3)
Maple	87.35 (145)	12.65 (21)
Mupad	65.06 (108)	34.94 (58)
Sympy	57.83 (96)	42.17 (70)
Maxima	56.02 (93)	43.98 (73)
Fricas	55.42 (92)	44.58 (74)
Giac	51.20 (85)	48.80 (81)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

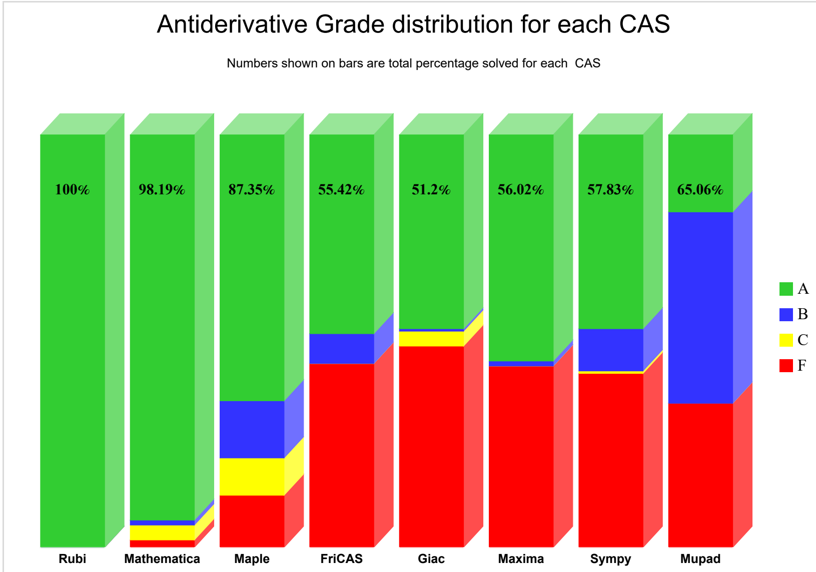
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

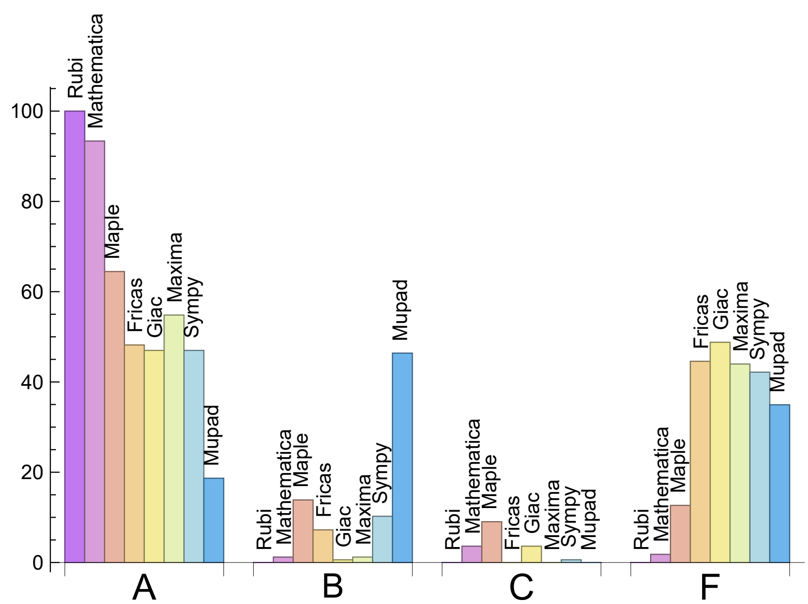
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	93.37	1.20	3.61	1.81
Maple	64.46	13.86	9.04	12.65
Maxima	54.82	1.20	0.00	43.98
Fricas	48.19	7.23	0.00	44.58
Giac	46.99	0.60	3.61	48.80
Sympy	46.99	10.24	0.60	42.17
Mupad	N/A	46.39	0.00	34.94

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	3	100.00 %	0.00 %	0.00 %
Maple	21	100.00 %	0.00 %	0.00 %
Fricas	74	83.78 %	0.00 %	16.22 %
Giac	81	97.53 %	2.47 %	0.00 %
Maxima	73	82.19 %	1.37 %	16.44 %
Sympy	70	88.57 %	11.43 %	0.00 %
Mupad	58	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

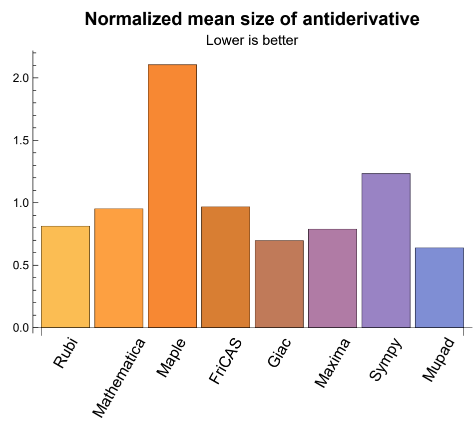
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

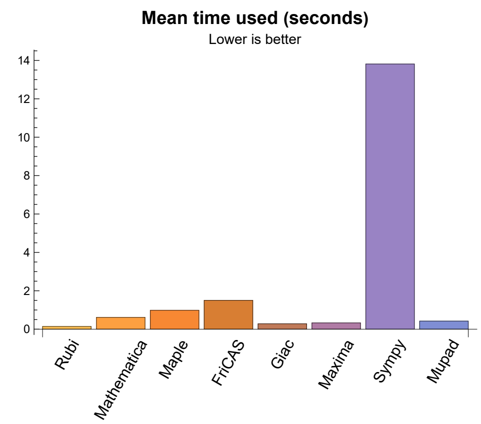
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.14	120.70	0.81	75.50	1.00
Mathematica	0.61	161.23	0.95	65.00	1.07
Maple	0.98	304.32	2.11	61.00	1.02
Maxima	0.33	59.97	0.79	43.00	0.91
Fricas	1.50	91.00	0.97	42.00	0.96
Sympy	13.81	98.90	1.23	58.00	1.07
Giac	0.28	51.49	0.70	39.00	0.83
Mupad	0.42	46.49	0.64	38.00	0.81

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0.0
0.5
1.0
1.5
2.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
2
4
6
8
10
12
14
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 57, 58, 59, 91, 92, 94, 95, 127, 128, 130, 131\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{82, 83\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

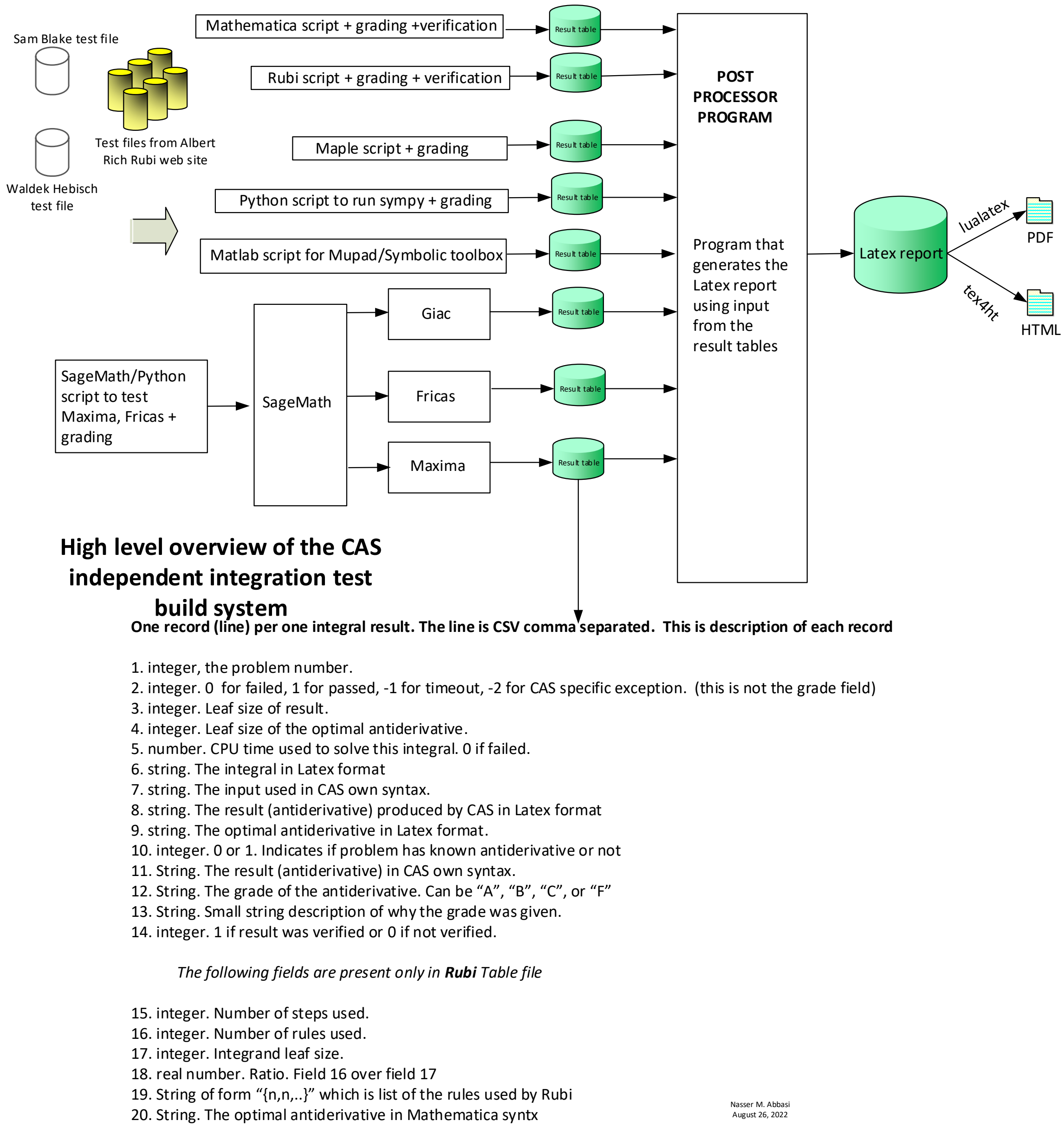
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 160, 161, 162, 163, 164, 165, 166 }

B grade: { 82, 83 }

C grade: { 9, 11, 66, 102, 158, 159 }

F grade: { 81, 84, 85 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 18, 21, 23, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 80, 91, 92, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 119, 127, 128, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 142, 145, 146, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

B grade: { 7, 14, 16, 20, 22, 24, 26, 28, 29, 32, 34, 75, 87, 123, 136, 141, 143, 147, 149, 152, 153, 157, 166 }

C grade: { 19, 25, 27, 30, 31, 33, 64, 86, 100, 122, 144, 148, 150, 151, 165 }

F grade: { 56, 78, 79, 81, 82, 83, 84, 85, 88, 89, 90, 93, 114, 117, 118, 120, 121, 124, 125, 126, 129 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 35, 36, 37, 38, 39, 40, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 80, 91, 92, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 119, 127, 128, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 142, 146, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

B grade: { 157, 165 }

C grade: { }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 64, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 129, 136, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 166 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 35, 36, 37, 38, 39, 40, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 74, 76, 80, 91, 92, 94, 95, 96, 97, 98, 99, 101, 102, 103, 105, 107, 109, 111, 113, 115, 119, 127, 128, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 142, 146, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

B grade: { 68, 69, 70, 71, 72, 73, 104, 106, 108, 110, 112, 166 }

C grade: { }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 56, 64, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 129, 136, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 157, 165 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 62, 65, 66, 67, 68, 69, 70, 71, 73, 74, 76, 92, 96, 98, 102, 104, 105, 106, 108, 110, 112, 132, 133, 134, 135, 137, 138, 139, 140, 142, 146, 154, 155, 156, 161, 162, 163 }

B grade: { 61, 63, 80, 97, 99, 101, 103, 107, 109, 111, 113, 115, 119, 158, 159, 160, 164 }

C grade: { 72 }

F grade: { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 56, 64, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 100, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 136, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 157, 165, 166 }

2.1.7 Giac

A grade: { 6, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 65, 67, 68, 69, 70, 71, 72, 73, 74, 76, 91, 92, 94, 95, 96, 97, 98, 99, 101, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 127, 128, 130, 131, 133, 135, 137, 139, 154, 155, 156, 158, 159, 160, 161, 162, 163, 164 }

B grade: { 136 }

C grade: { 66, 102, 132, 134, 138, 146 }

F grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 56, 64, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 100, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 129, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 157, 165, 166 }

2.1.8 Mupad

A grade: { 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 57, 58, 59, 91, 92, 94, 95, 127, 128, 130, 131 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 53, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 80, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 119, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 146, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165 }

C grade: { }

F grade: { 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 56, 75, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 114, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 129, 141, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 161, 166 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	64	58	57	54	63	0	52
N.S.	1	1.00	1.08	0.98	0.97	0.92	1.07	0.00	0.88
time (sec)	N/A	0.023	0.004	0.205	0.464	3.602	0.334	0.000	0.442

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	61	59	56	59	60	0	54
N.S.	1	1.00	1.09	1.05	1.00	1.05	1.07	0.00	0.96
time (sec)	N/A	0.030	0.012	0.052	0.265	2.026	0.265	0.000	0.417

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	53	49	48	47	53	0	44
N.S.	1	1.00	1.10	1.02	1.00	0.98	1.10	0.00	0.92
time (sec)	N/A	0.022	0.004	0.105	0.469	6.432	0.208	0.000	0.210

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	50	50	46	49	49	0	42
N.S.	1	1.00	1.11	1.11	1.02	1.09	1.09	0.00	0.93
time (sec)	N/A	0.023	0.008	0.053	0.265	4.570	0.194	0.000	0.370

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	42	40	37	34	42	0	34
N.S.	1	1.00	1.14	1.08	1.00	0.92	1.14	0.00	0.92
time (sec)	N/A	0.011	0.004	0.112	0.463	3.426	0.138	0.000	0.123

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	31	33	26	31	27
N.S.	1	1.00	1.00	0.97	1.07	1.14	0.90	1.07	0.93
time (sec)	N/A	0.009	0.004	0.083	0.256	3.293	0.091	0.415	0.098

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	74	0	0	0	0	28
N.S.	1	1.00	1.00	2.11	0.00	0.00	0.00	0.00	0.80
time (sec)	N/A	0.022	0.004	0.095	0.000	0.000	0.000	0.000	0.287

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	38	45	39	37	37	0	36
N.S.	1	1.00	1.09	1.29	1.11	1.06	1.06	0.00	1.03
time (sec)	N/A	0.018	0.004	0.078	0.270	0.920	0.267	0.000	0.319

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	46	44	31	26	37	0	42
N.S.	1	1.00	1.24	1.19	0.84	0.70	1.00	0.00	1.14
time (sec)	N/A	0.015	0.005	0.122	0.470	0.712	0.218	0.000	0.332

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	57	51	50	61	0	46
N.S.	1	1.00	1.02	1.08	0.96	0.94	1.15	0.00	0.87
time (sec)	N/A	0.025	0.013	0.082	0.266	1.259	0.377	0.000	0.119

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	46	53	46	41	46	0	42
N.S.	1	1.00	0.96	1.10	0.96	0.85	0.96	0.00	0.88
time (sec)	N/A	0.018	0.004	0.131	0.472	0.629	0.296	0.000	0.363

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	66	62	59	71	0	56
N.S.	1	1.00	1.00	1.03	0.97	0.92	1.11	0.00	0.88
time (sec)	N/A	0.027	0.015	0.092	0.260	1.296	0.536	0.000	0.371

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	138	171	163	152	199	0	171
N.S.	1	1.00	0.96	1.19	1.13	1.06	1.38	0.00	1.19
time (sec)	N/A	0.217	0.087	0.326	0.483	0.832	0.461	0.000	0.667

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	169	312	0	0	0	0	-1
N.S.	1	1.00	0.99	1.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.206	0.358	0.471	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	111	135	136	121	155	0	134
N.S.	1	1.00	0.99	1.21	1.21	1.08	1.38	0.00	1.20
time (sec)	N/A	0.153	0.046	0.158	0.474	0.958	0.322	0.000	0.316

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	131	276	0	0	0	0	-1
N.S.	1	1.00	0.95	2.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.196	0.345	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	75	97	104	83	107	0	88
N.S.	1	1.00	0.99	1.28	1.37	1.09	1.41	0.00	1.16
time (sec)	N/A	0.076	0.035	0.193	0.474	0.768	0.194	0.000	0.410

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	90	123	0	0	0	0	-1
N.S.	1	1.00	1.08	1.48	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.050	0.759	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	144	1128	0	0	0	0	-1
N.S.	1	1.00	1.09	8.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.181	0.052	8.300	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	102	318	0	0	0	0	-1
N.S.	1	1.00	1.24	3.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.097	0.799	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	90	115	98	94	119	0	140
N.S.	1	1.00	1.14	1.46	1.24	1.19	1.51	0.00	1.77
time (sec)	N/A	0.091	0.036	0.239	0.474	0.782	0.341	0.000	2.310

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	153	365	0	0	0	0	-1
N.S.	1	1.00	1.09	2.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.268	0.840	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	128	152	152	135	170	0	171
N.S.	1	1.00	1.10	1.31	1.31	1.16	1.47	0.00	1.47
time (sec)	N/A	0.159	0.052	0.250	0.475	1.154	0.507	0.000	2.257

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	291	494	0	0	0	0	-1
N.S.	1	1.00	1.14	1.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.655	0.554	0.474	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	396	2951	0	0	0	0	-1
N.S.	1	1.00	1.46	10.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.513	0.636	18.189	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	225	411	0	0	0	0	-1
N.S.	1	1.00	1.16	2.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.383	0.414	0.535	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	269	1921	0	0	0	0	-1
N.S.	1	1.00	1.31	9.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.313	0.420	5.489	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	152	323	0	0	0	0	-1
N.S.	1	1.00	1.16	2.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.195	0.374	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	192	254	0	0	0	0	-1
N.S.	1	1.00	1.61	2.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.058	1.516	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	212	2309	0	0	0	0	-1
N.S.	1	1.00	1.03	11.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.289	0.099	4.770	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	214	2126	0	0	0	0	-1
N.S.	1	1.00	1.84	18.33	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.256	3.302	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	176	422	0	0	0	0	-1
N.S.	1	1.00	1.32	3.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.218	1.055	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	321	5807	0	0	0	0	-1
N.S.	1	1.00	1.51	27.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.331	0.618	19.816	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	265	512	0	0	0	0	-1
N.S.	1	1.00	1.34	2.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.412	0.489	1.583	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.006	0.483	1.810	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	9	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.11
time (sec)	N/A	0.002	0.011	1.324	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.010	0.314	1.341	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.006	0.442	2.015	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	9	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.11
time (sec)	N/A	0.002	0.650	0.460	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.010	0.765	1.384	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.005	1.508	1.301	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.002	1.331	0.694	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.009	0.927	0.840	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.005	0.708	1.239	0.000	0.000	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.003	1.606	0.685	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.009	0.822	0.802	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.005	0.937	1.360	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.002	0.018	0.705	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.009	0.854	0.836	0.000	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.005	1.096	1.335	0.000	0.000	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.09
time (sec)	N/A	0.002	1.911	0.708	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.009	2.022	0.799	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	108	69	86	118	104	86	49
N.S.	1	1.00	0.92	0.59	0.74	1.01	0.89	0.74	0.42
time (sec)	N/A	0.048	0.019	0.093	0.492	5.000	1.315	0.465	0.321

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	2.712	1.527	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.015	1.779	1.783	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.024	1.711	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	0.194	0.975	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.004	0.337	0.487	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.015	0.246	1.220	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	59	50	54	51	58	60	49
N.S.	1	1.00	1.09	0.93	1.00	0.94	1.07	1.11	0.91
time (sec)	N/A	0.028	0.008	0.211	0.468	2.638	28.518	0.428	0.357

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	52	45	48	51	80	47	44
N.S.	1	1.00	1.11	0.96	1.02	1.09	1.70	1.00	0.94
time (sec)	N/A	0.024	0.011	0.198	0.268	2.858	21.110	0.431	0.349

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	48	41	43	38	48	43	40
N.S.	1	1.00	1.12	0.95	1.00	0.88	1.12	1.00	0.93
time (sec)	N/A	0.020	0.006	0.187	0.464	3.464	10.193	0.430	0.327

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	41	38	38	39	66	40	35
N.S.	1	1.00	1.14	1.06	1.06	1.08	1.83	1.11	0.97
time (sec)	N/A	0.010	0.008	0.147	0.256	3.686	5.671	0.443	0.314

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	63	0	0	0	0	32
N.S.	1	1.00	1.00	1.62	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.036	0.005	0.235	0.000	0.000	0.000	0.000	0.327

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	44	39	41	43	75	60	38
N.S.	1	1.00	1.13	1.00	1.05	1.10	1.92	1.54	0.97
time (sec)	N/A	0.018	0.007	0.089	0.263	2.432	12.450	0.436	0.343

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	48	39	35	30	42	72	41
N.S.	1	1.00	1.17	0.95	0.85	0.73	1.02	1.76	1.00
time (sec)	N/A	0.018	0.007	0.125	0.465	2.265	10.297	0.460	0.356

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	60	51	53	54	92	69	50
N.S.	1	1.00	1.09	0.93	0.96	0.98	1.67	1.25	0.91
time (sec)	N/A	0.024	0.011	0.105	0.259	2.203	35.832	0.457	0.365

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	179	140	147	372	153	169	64
N.S.	1	1.00	1.11	0.87	0.91	2.31	0.95	1.05	0.40
time (sec)	N/A	0.081	0.038	0.170	0.484	1.370	14.355	0.487	0.363

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	177	138	145	337	143	165	62
N.S.	1	1.00	1.11	0.87	0.91	2.12	0.90	1.04	0.39
time (sec)	N/A	0.072	0.028	0.085	0.487	2.398	7.254	0.461	0.392

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	107	125	127	319	617	149	49
N.S.	1	1.00	0.76	0.89	0.91	2.28	4.41	1.06	0.35
time (sec)	N/A	0.077	0.030	0.082	0.466	1.387	4.139	0.410	0.390

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	158	125	132	322	121	138	55
N.S.	1	1.00	1.10	0.87	0.92	2.25	0.85	0.97	0.38
time (sec)	N/A	0.065	0.031	0.102	0.485	2.090	8.217	0.452	0.201

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	177	132	142	389	529	159	63
N.S.	1	1.00	1.11	0.83	0.89	2.45	3.33	1.00	0.40
time (sec)	N/A	0.072	0.037	0.109	0.477	1.283	16.586	0.477	0.428

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	177	138	138	350	155	150	63
N.S.	1	1.00	1.11	0.87	0.87	2.20	0.97	0.94	0.40
time (sec)	N/A	0.074	0.040	0.117	0.481	1.948	27.329	0.495	0.452

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	121	151	169	137	199	145	150
N.S.	1	1.00	0.98	1.22	1.36	1.10	1.60	1.17	1.21
time (sec)	N/A	0.183	0.055	0.224	0.516	1.184	41.041	0.453	1.016

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	141	367	0	0	0	0	-1
N.S.	1	1.00	0.92	2.38	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.234	0.432	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	85	113	126	100	155	100	112
N.S.	1	1.00	0.94	1.26	1.40	1.11	1.72	1.11	1.24
time (sec)	N/A	0.103	0.043	0.133	0.522	1.802	14.779	0.427	0.683

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	107	142	0	0	0	0	-1
N.S.	1	1.00	1.06	1.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.061	0.181	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	165	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.072	0.013	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	127	0	0	0	0	0	-1
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.104	0.015	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	98	118	110	115	167	0	152
N.S.	1	1.00	1.13	1.36	1.26	1.32	1.92	0.00	1.75
time (sec)	N/A	0.117	0.046	0.134	0.537	2.367	21.552	0.000	0.613

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1393	1393	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.845	4.143	0.027	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1191	1191	5293	0	0	0	0	0	-1
N.S.	1	1.00	4.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.232	30.757	0.027	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1164	1164	5293	0	0	0	0	0	-1
N.S.	1	1.00	4.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.151	30.545	0.033	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1360	1360	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.549	2.000	0.079	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1444	1444	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.690	2.033	0.048	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	170	758	0	0	0	0	-1
N.S.	1	1.00	1.14	5.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.214	0.295	0.471	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	224	289	0	0	0	0	-1
N.S.	1	1.00	1.56	2.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.181	0.044	0.407	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	245	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.342	0.132	0.014	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	239	0	0	0	0	0	-1
N.S.	1	1.00	1.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.218	0.288	0.015	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	196	0	0	0	0	0	-1
N.S.	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.226	0.219	0.015	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.015	1.271	0.014	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.825	0.014	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	65	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.042	0.014	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.018	0.221	0.013	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.018	0.233	0.013	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	59	50	54	51	58	60	49
N.S.	1	1.00	1.09	0.93	1.00	0.94	1.07	1.11	0.91
time (sec)	N/A	0.025	0.007	0.047	0.487	1.468	125.815	0.420	0.454

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	52	45	48	51	117	47	44
N.S.	1	1.00	1.11	0.96	1.02	1.09	2.49	1.00	0.94
time (sec)	N/A	0.023	0.011	0.041	0.262	1.082	69.177	0.438	0.379

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	48	41	43	38	48	43	40
N.S.	1	1.00	1.12	0.95	1.00	0.88	1.12	1.00	0.93
time (sec)	N/A	0.019	0.006	0.063	0.470	1.406	35.579	0.441	0.405

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	41	38	38	39	102	40	35
N.S.	1	1.00	1.14	1.06	1.06	1.08	2.83	1.11	0.97
time (sec)	N/A	0.014	0.008	0.029	0.260	1.294	19.447	0.404	0.100

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	63	0	0	0	0	32
N.S.	1	1.00	1.00	1.62	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.033	0.005	0.044	0.000	0.000	0.000	0.000	0.351

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	44	39	41	43	110	60	38
N.S.	1	1.00	1.13	1.00	1.05	1.10	2.82	1.54	0.97
time (sec)	N/A	0.017	0.006	0.041	0.267	1.072	38.418	0.404	0.377

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	48	39	35	30	42	72	41
N.S.	1	1.00	1.17	0.95	0.85	0.73	1.02	1.76	1.00
time (sec)	N/A	0.018	0.006	0.059	0.465	1.511	35.408	0.427	0.386

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	60	51	53	54	129	69	50
N.S.	1	1.00	1.09	0.93	0.96	0.98	2.35	1.25	0.91
time (sec)	N/A	0.023	0.010	0.052	0.260	1.463	131.390	0.434	0.401

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	179	165	148	399	255	167	114
N.S.	1	1.00	1.03	0.95	0.85	2.29	1.47	0.96	0.66
time (sec)	N/A	0.231	0.035	0.092	0.471	1.673	23.275	0.477	1.164

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	131	98	92	234	755	95	91
N.S.	1	1.00	1.30	0.97	0.91	2.32	7.48	0.94	0.90
time (sec)	N/A	0.069	0.030	0.028	0.466	1.330	12.936	0.432	2.295

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	170	148	137	505	245	137	107
N.S.	1	1.00	1.03	0.90	0.83	3.06	1.48	0.83	0.65
time (sec)	N/A	0.209	0.033	0.085	0.474	1.532	29.053	0.466	0.909

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	183	105	102	121	286	108	118
N.S.	1	1.00	1.59	0.91	0.89	1.05	2.49	0.94	1.03
time (sec)	N/A	0.064	0.031	0.066	0.466	1.157	58.144	0.495	2.567

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	181	167	152	439	264	171	122
N.S.	1	1.00	1.03	0.95	0.86	2.49	1.50	0.97	0.69
time (sec)	N/A	0.310	0.056	0.096	0.467	1.744	61.127	0.488	1.002

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	185	113	106	137	292	119	106
N.S.	1	1.00	1.58	0.97	0.91	1.17	2.50	1.02	0.91
time (sec)	N/A	0.069	0.020	0.047	0.484	1.604	29.536	0.432	1.936

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	170	154	137	408	246	157	113
N.S.	1	1.00	1.03	0.93	0.83	2.47	1.49	0.95	0.68
time (sec)	N/A	0.290	0.019	0.058	0.464	1.679	17.088	0.481	0.689

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	170	104	98	90	262	91	99
N.S.	1	1.00	1.63	1.00	0.94	0.87	2.52	0.88	0.95
time (sec)	N/A	0.057	0.026	0.046	0.479	1.293	24.348	0.430	1.828

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	179	159	147	595	264	161	120
N.S.	1	1.00	1.03	0.91	0.84	3.42	1.52	0.93	0.69
time (sec)	N/A	0.297	0.036	0.072	0.485	1.622	45.930	0.590	0.706

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	121	151	169	129	243	145	150
N.S.	1	1.00	0.98	1.22	1.36	1.04	1.96	1.17	1.21
time (sec)	N/A	0.177	0.055	0.101	0.552	1.348	151.091	0.468	1.140

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	141	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.173	0.195	0.018	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	85	113	126	91	194	100	112
N.S.	1	1.00	0.94	1.26	1.40	1.01	2.16	1.11	1.24
time (sec)	N/A	0.101	0.036	0.141	0.543	1.030	50.500	0.422	0.724

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	107	142	0	0	0	0	-1
N.S.	1	1.00	1.03	1.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.044	0.174	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	167	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.076	0.015	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	125	0	0	0	0	0	-1
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.102	0.014	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	98	118	110	102	207	0	152
N.S.	1	1.00	1.13	1.36	1.26	1.17	2.38	0.00	1.75
time (sec)	N/A	0.113	0.043	0.128	0.541	1.238	72.195	0.000	0.691

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	167	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.189	0.261	0.016	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	346	0	0	0	0	0	-1
N.S.	1	1.00	1.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.331	0.348	0.015	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	170	935	0	0	0	0	-1
N.S.	1	1.00	1.16	6.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.089	0.654	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	224	289	0	0	0	0	-1
N.S.	1	1.00	1.61	2.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.177	0.045	0.165	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	248	0	0	0	0	0	-1
N.S.	1	1.00	1.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.334	0.125	0.016	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	240	0	0	0	0	0	-1
N.S.	1	1.00	1.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.268	0.016	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	196	0	0	0	0	0	-1
N.S.	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.226	0.221	0.015	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	1.343	0.015	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.015	0.867	0.015	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	65	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.044	0.016	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	0.227	0.015	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.016	0.234	0.014	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	55	56	45	41	46	81	45
N.S.	1	1.00	1.10	1.12	0.90	0.82	0.92	1.62	0.90
time (sec)	N/A	0.024	0.009	0.110	0.467	1.868	0.150	0.448	0.411

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	48	62	43	40	41	69	40
N.S.	1	1.00	1.12	1.44	1.00	0.93	0.95	1.60	0.93
time (sec)	N/A	0.021	0.007	0.051	0.263	0.800	0.131	0.405	0.352

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	44	47	36	31	36	72	36
N.S.	1	1.00	1.13	1.21	0.92	0.79	0.92	1.85	0.92
time (sec)	N/A	0.012	0.007	0.063	0.470	0.660	0.105	0.433	0.344

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	38	27	25	22	46	25
N.S.	1	1.00	1.00	1.41	1.00	0.93	0.81	1.70	0.93
time (sec)	N/A	0.009	0.003	0.066	0.266	0.758	0.084	0.453	0.303

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	94	0	0	0	72	32
N.S.	1	1.00	1.00	2.41	0.00	0.00	0.00	1.85	0.82
time (sec)	N/A	0.031	0.006	0.091	0.000	0.000	0.000	0.432	0.338

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	37	38	38	41	36	39	43
N.S.	1	1.00	1.09	1.12	1.12	1.21	1.06	1.15	1.26
time (sec)	N/A	0.014	0.008	0.053	0.274	0.657	0.302	0.448	0.344

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	48	47	42	37	44	60	50
N.S.	1	1.00	1.12	1.09	0.98	0.86	1.02	1.40	1.16
time (sec)	N/A	0.017	0.009	0.071	0.470	0.719	0.335	0.445	0.384

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	60	53	54	55	60	51	56
N.S.	1	1.00	1.09	0.96	0.98	1.00	1.09	0.93	1.02
time (sec)	N/A	0.026	0.008	0.076	0.266	0.696	0.409	0.443	0.369

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	111	163	134	125	144	0	140
N.S.	1	1.00	0.91	1.34	1.10	1.02	1.18	0.00	1.15
time (sec)	N/A	0.176	0.055	0.420	0.482	1.882	0.214	0.000	0.473

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	152	412	0	0	0	0	-1
N.S.	1	1.00	1.00	2.71	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.238	0.379	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	124	104	88	97	0	98
N.S.	1	1.00	0.89	1.51	1.27	1.07	1.18	0.00	1.20
time (sec)	N/A	0.099	0.032	0.359	0.474	0.938	0.143	0.000	0.404

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	105	357	0	0	0	0	-1
N.S.	1	1.00	1.27	4.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.072	0.213	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	148	1249	0	0	0	0	-1
N.S.	1	1.00	1.00	8.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.196	0.060	3.372	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	107	142	0	0	0	0	-1
N.S.	1	1.00	1.11	1.48	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.085	0.507	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	99	112	120	109	117	137	143
N.S.	1	1.00	1.18	1.33	1.43	1.30	1.39	1.63	1.70
time (sec)	N/A	0.090	0.043	0.944	0.498	0.757	0.370	0.432	2.749

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	253	569	0	0	0	0	-1
N.S.	1	1.00	1.18	2.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.420	0.443	6.785	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	330	6282	0	0	0	0	-1
N.S.	1	1.00	1.44	27.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.348	0.493	13.416	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	174	473	0	0	0	0	-1
N.S.	1	1.00	1.20	3.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	0.192	2.487	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	215	2333	0	0	0	0	-1
N.S.	1	1.00	1.81	19.61	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.175	3.202	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	219	2542	0	0	0	0	-1
N.S.	1	1.00	0.95	11.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.311	0.136	0.866	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	222	289	0	0	0	0	-1
N.S.	1	1.00	1.63	2.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	0.068	1.468	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	178	368	0	0	0	0	-1
N.S.	1	1.00	1.21	2.50	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.191	4.392	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	34	32	31	27	39	31	31
N.S.	1	1.00	0.67	0.63	0.61	0.53	0.76	0.61	0.61
time (sec)	N/A	0.010	0.010	0.024	0.471	1.090	1.320	0.415	0.349

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	28	27	26	20	32	26	26
N.S.	1	1.00	0.67	0.64	0.62	0.48	0.76	0.62	0.62
time (sec)	N/A	0.007	0.008	0.012	0.469	0.667	0.783	0.407	0.365

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	18	17	16	14	19	16	16
N.S.	1	1.00	0.82	0.77	0.73	0.64	0.86	0.73	0.73
time (sec)	N/A	0.004	0.013	0.011	0.471	1.214	0.693	0.429	0.074

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	61	35	0	0	0	24
N.S.	1	1.00	1.00	1.97	1.13	0.00	0.00	0.00	0.77
time (sec)	N/A	0.024	0.005	0.020	0.474	0.000	0.000	0.000	0.302

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	30	22	21	17	94	21	21
N.S.	1	1.00	1.11	0.81	0.78	0.63	3.48	0.78	0.78
time (sec)	N/A	0.008	0.008	0.010	0.477	2.374	0.566	0.440	0.348

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	34	27	26	26	160	26	24
N.S.	1	1.00	0.81	0.64	0.62	0.62	3.81	0.62	0.57
time (sec)	N/A	0.009	0.009	0.013	0.478	3.210	1.193	0.443	0.353

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	24	85	24	24
N.S.	1	1.00	0.83	0.69	0.67	0.67	2.36	0.67	0.67
time (sec)	N/A	0.010	0.011	0.012	0.267	2.531	0.978	0.449	0.351

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	20	19	19	24	19	-1
N.S.	1	1.00	0.86	0.69	0.66	0.66	0.83	0.66	-0.03
time (sec)	N/A	0.008	0.008	0.011	0.257	3.450	0.782	0.447	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	16	17	16	16
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.85	0.80	0.80
time (sec)	N/A	0.005	0.007	0.020	0.263	3.861	0.118	0.412	0.346

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	18	26	20	18	22
N.S.	1	1.00	1.00	0.86	0.82	1.18	0.91	0.82	1.00
time (sec)	N/A	0.006	0.009	0.024	0.256	5.143	0.383	0.451	0.358

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	31	26	25	33	143	28	27
N.S.	1	1.00	0.84	0.70	0.68	0.89	3.86	0.76	0.73
time (sec)	N/A	0.010	0.014	0.027	0.261	3.127	1.180	0.415	0.354

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	57	50	0	0	0	25
N.S.	1	1.00	1.00	1.73	1.52	0.00	0.00	0.00	0.76
time (sec)	N/A	0.025	0.006	0.064	0.480	0.000	0.000	0.000	0.336

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	32	83	0	63	0	0	-1
N.S.	1	1.00	0.82	2.13	0.00	1.62	0.00	0.00	-0.03
time (sec)	N/A	0.027	0.010	0.080	0.000	2.295	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [74] had the largest ratio of [16]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	12	0.250
2	A	4	3	1.00	12	0.250
3	A	4	3	1.00	12	0.250
4	A	4	3	1.00	12	0.250
5	A	3	3	1.00	10	0.300
6	A	3	2	1.00	8	0.250
7	A	3	2	1.00	12	0.167
8	A	5	5	1.00	12	0.417
9	A	3	3	1.00	12	0.250
10	A	4	3	1.00	12	0.250
11	A	4	3	1.00	12	0.250
12	A	4	3	1.00	12	0.250
13	A	16	7	1.00	14	0.500
14	A	14	9	1.00	14	0.643
15	A	11	7	1.00	14	0.500
16	A	9	8	1.00	14	0.571
17	A	6	5	1.00	12	0.417
18	A	5	5	1.00	10	0.500
19	A	6	5	1.00	14	0.357
20	A	4	4	1.00	14	0.286
21	A	8	7	1.00	14	0.500
22	A	8	7	1.00	14	0.500
23	A	13	8	1.00	14	0.571
24	A	33	11	1.00	14	0.786
25	A	24	11	1.00	14	0.786

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	18	10	1.00	14	0.714
27	A	12	9	1.00	14	0.643
28	A	8	8	1.00	12	0.667
29	A	5	6	1.00	10	0.600
30	A	8	6	1.00	14	0.429
31	A	5	6	1.00	14	0.429
32	A	7	6	1.00	14	0.429
33	A	14	11	1.00	14	0.786
34	A	16	8	1.00	14	0.571
35	A	0	0	0.00	0	0.000
36	A	0	0	0.00	0	0.000
37	A	0	0	0.00	0	0.000
38	A	0	0	0.00	0	0.000
39	A	0	0	0.00	0	0.000
40	A	0	0	0.00	0	0.000
41	A	0	0	0.00	0	0.000
42	A	0	0	0.00	0	0.000
43	A	0	0	0.00	0	0.000
44	A	0	0	0.00	0	0.000
45	A	0	0	0.00	0	0.000
46	A	0	0	0.00	0	0.000
47	A	0	0	0.00	0	0.000
48	A	0	0	0.00	0	0.000
49	A	0	0	0.00	0	0.000
50	A	0	0	0.00	0	0.000
51	A	0	0	0.00	0	0.000
52	A	0	0	0.00	0	0.000
53	A	12	9	1.00	8	1.125
54	A	0	0	0.00	0	0.000
55	A	0	0	0.00	0	0.000
56	A	2	2	1.00	14	0.143
57	A	0	0	0.00	0	0.000
58	A	0	0	0.00	0	0.000
59	A	0	0	0.00	0	0.000
60	A	5	4	1.00	14	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	4	3	1.00	14	0.214
62	A	4	4	1.00	14	0.286
63	A	2	2	1.00	12	0.167
64	A	4	3	1.00	14	0.214
65	A	5	5	1.00	14	0.357
66	A	4	4	1.00	14	0.286
67	A	4	3	1.00	14	0.214
68	A	11	8	1.00	14	0.571
69	A	11	8	1.00	14	0.571
70	A	11	7	1.00	10	0.700
71	A	10	7	1.00	14	0.500
72	A	11	8	1.00	14	0.571
73	A	11	8	1.00	14	0.571
74	A	12	8	1.00	16	0.500
75	A	10	9	1.00	16	0.562
76	A	7	6	1.00	16	0.375
77	A	6	6	1.00	14	0.429
78	A	7	6	1.00	16	0.375
79	A	5	5	1.00	16	0.312
80	A	9	8	1.00	16	0.500
81	A	86	27	1.00	16	1.687
82	A	69	23	1.00	12	1.917
83	A	47	23	1.00	16	1.438
84	A	64	25	1.00	16	1.562
85	A	77	25	1.00	16	1.562
86	A	9	9	1.00	16	0.562
87	A	6	7	1.00	14	0.500
88	A	9	7	1.00	16	0.438
89	A	6	7	1.00	16	0.438
90	A	8	7	1.00	16	0.438
91	A	0	0	0.00	0	0.000
92	A	0	0	0.00	0	0.000
93	A	2	2	1.00	16	0.125
94	A	0	0	0.00	0	0.000
95	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	5	4	1.00	14	0.286
97	A	4	3	1.00	14	0.214
98	A	4	4	1.00	14	0.286
99	A	2	2	1.00	14	0.143
100	A	4	3	1.00	14	0.214
101	A	5	5	1.00	14	0.357
102	A	4	4	1.00	14	0.286
103	A	4	3	1.00	14	0.214
104	A	12	8	1.00	14	0.571
105	A	9	8	1.00	10	0.800
106	A	11	7	1.00	14	0.500
107	A	9	9	1.00	14	0.643
108	A	12	8	1.00	14	0.571
109	A	9	9	1.00	14	0.643
110	A	11	7	1.00	12	0.583
111	A	8	8	1.00	14	0.571
112	A	12	8	1.00	14	0.571
113	A	12	8	1.00	16	0.500
114	A	10	9	1.00	16	0.562
115	A	7	6	1.00	16	0.375
116	A	6	6	1.00	16	0.375
117	A	7	6	1.00	16	0.375
118	A	5	5	1.00	16	0.312
119	A	9	8	1.00	16	0.500
120	A	9	8	1.00	16	0.500
121	A	13	10	1.00	16	0.625
122	A	9	9	1.00	16	0.562
123	A	6	7	1.00	16	0.438
124	A	9	7	1.00	16	0.438
125	A	6	7	1.00	16	0.438
126	A	8	7	1.00	16	0.438
127	A	0	0	0.00	0	0.000
128	A	0	0	0.00	0	0.000
129	A	2	2	1.00	16	0.125
130	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	0	0	0.00	0	0.000
132	A	5	4	1.00	14	0.286
133	A	5	4	1.00	14	0.286
134	A	4	4	1.00	12	0.333
135	A	4	3	1.00	10	0.300
136	A	4	3	1.00	14	0.214
137	A	2	2	1.00	14	0.143
138	A	4	4	1.00	14	0.286
139	A	5	4	1.00	14	0.286
140	A	14	9	1.00	16	0.562
141	A	9	8	1.00	16	0.500
142	A	9	8	1.00	14	0.571
143	A	6	6	1.00	12	0.500
144	A	7	6	1.00	16	0.375
145	A	6	6	1.00	16	0.375
146	A	7	6	1.00	16	0.375
147	A	17	9	1.00	16	0.562
148	A	15	12	1.00	16	0.750
149	A	8	7	1.00	14	0.500
150	A	6	7	1.00	12	0.583
151	A	9	7	1.00	16	0.438
152	A	6	7	1.00	16	0.438
153	A	9	9	1.00	16	0.562
154	A	6	4	1.00	10	0.400
155	A	5	4	1.00	8	0.500
156	A	4	4	1.00	6	0.667
157	A	4	3	1.00	10	0.300
158	A	4	4	1.00	10	0.400
159	A	5	4	1.00	10	0.400
160	A	3	2	1.00	12	0.167
161	A	3	2	1.00	12	0.167
162	A	2	2	1.00	12	0.167
163	A	4	4	1.00	12	0.333
164	A	3	2	1.00	12	0.167
165	A	4	3	1.00	10	0.300

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	4	3	1.00	10	0.300

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^5(a+b\text{ArcTan}(cx))\,dx$

Optimal. Leaf size=59

$$-\frac{bx}{6c^5}+\frac{bx^3}{18c^3}-\frac{bx^5}{30c}+\frac{b\text{ArcTan}(cx)}{6c^6}+\frac{1}{6}x^6(a+b\text{ArcTan}(cx))$$

```
[Out] -1/6*b*x/c^5+1/18*b*x^3/c^3-1/30*b*x^5/c+1/6*b*arctan(c*x)/c^6+1/6*x^6*(a+b
*arctan(c*x))
```

Rubi [A]
time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {4946, 308, 209}

$$\frac{1}{6}x^6(a+b\text{ArcTan}(cx))+\frac{b\text{ArcTan}(cx)}{6c^6}-\frac{bx}{6c^5}+\frac{bx^3}{18c^3}-\frac{bx^5}{30c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/6*(b*x)/c^5 + (b*x^3)/(18*c^3) - (b*x^5)/(30*c) + (b*ArcTan[c*x])/(6*c^6
) + (x^6*(a + b*ArcTan[c*x]))/6
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+b\tan^{-1}(cx)\right)\,dx &= \frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)-\frac{1}{6}(bc)\int\frac{x^6}{1+c^2x^2}\,dx\\ &= \frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)-\frac{1}{6}(bc)\int\left(\frac{1}{c^6}-\frac{x^2}{c^4}+\frac{x^4}{c^2}-\frac{1}{c^6\left(1+c^2x^2\right)}\right)\,dx\\ &= -\frac{bx}{6c^5}+\frac{bx^3}{18c^3}-\frac{bx^5}{30c}+\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)+\frac{b\int\frac{1}{1+c^2x^2}\,dx}{6c^5}\\ &= -\frac{bx}{6c^5}+\frac{bx^3}{18c^3}-\frac{bx^5}{30c}+\frac{b\tan^{-1}(cx)}{6c^6}+\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 64, normalized size = 1.08

$$-\frac{bx}{6c^5}+\frac{bx^3}{18c^3}-\frac{bx^5}{30c}+\frac{ax^6}{6}+\frac{b\mathrm{ArcTan}(cx)}{6c^6}+\frac{1}{6}bx^6\mathrm{ArcTan}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/6*(b*x)/c^5 + (b*x^3)/(18*c^3) - (b*x^5)/(30*c) + (a*x^6)/6 + (b*ArcTan[
c*x])/(6*c^6) + (b*x^6*ArcTan[c*x])/6
```

Maple [A]
time = 0.20, size = 58, normalized size = 0.98

method	result	size
derivativedivides	$\frac{\frac{c^6x^6a}{6}+\frac{b\,c^6x^6\arctan(cx)}{6}-\frac{b\,c^5x^5}{30}+\frac{b\,c^3x^3}{18}-\frac{xbc}{6}+\frac{b\arctan(cx)}{6}}{c^6}$	58
default	$\frac{\frac{c^6x^6a}{6}+\frac{b\,c^6x^6\arctan(cx)}{6}-\frac{b\,c^5x^5}{30}+\frac{b\,c^3x^3}{18}-\frac{xbc}{6}+\frac{b\arctan(cx)}{6}}{c^6}$	58
risch	$-\frac{ix^6b\ln(icx+1)}{12}+\frac{ix^6b\ln(-icx+1)}{12}+\frac{x^6a}{6}-\frac{b\,x^5}{30c}+\frac{b\,x^3}{18c^3}-\frac{bx}{6c^5}+\frac{b\arctan(cx)}{6c^6}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^6*(1/6*c^6*x^6*a+1/6*b*c^6*x^6*arctan(c*x)-1/30*b*c^5*x^5+1/18*b*c^3*x^
3-1/6*x*b*c+1/6*b*arctan(c*x))
```

Maxima [A]
time = 0.46, size = 57, normalized size = 0.97

$$\frac{1}{6}\,ax^6+\frac{1}{90}\left(15\,x^6\arctan\left(cx\right)-c\left(\frac{3\,c^4x^5-5\,c^2x^3+15\,x}{c^6}-\frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/90*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^
6 - 15*arctan(c*x)/c^7))*b
```

Fricas [A]
time = 3.60, size = 54, normalized size = 0.92

$$\frac{15\,ac^6x^6 - 3\,bc^5x^5 + 5\,bc^3x^3 - 15\,bcx + 15\,(bc^6x^6 + b)\arctan(cx)}{90\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*a*c^6*x^6 - 3*b*c^5*x^5 + 5*b*c^3*x^3 - 15*b*c*x + 15*(b*c^6*x^6 +
 b)*arctan(c*x))/c^6
```

Sympy [A]
time = 0.33, size = 63, normalized size = 1.07

$$\begin{cases} \frac{ax^6}{6} + \frac{bx^6\operatorname{atan}(cx)}{6} - \frac{bx^5}{30c} + \frac{bx^3}{18c^3} - \frac{bx}{6c^5} + \frac{b\operatorname{atan}(cx)}{6c^6} & \text{for}\ c \neq 0 \\ \frac{ax^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*x**6/6 + b*x**6*atan(c*x)/6 - b*x**5/(30*c) + b*x**3/(18*c**3)
 - b*x/(6*c**5) + b*atan(c*x)/(6*c**6), Ne(c, 0)), (a*x**6/6, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.44, size = 52, normalized size = 0.88

$$\frac{\frac{b\operatorname{atan}(c\,x)}{6} + \frac{b\,c^3\,x^3}{18} - \frac{b\,c^5\,x^5}{30} - \frac{b\,c\,x}{6}}{c^6} + \frac{a\,x^6}{6} + \frac{b\,x^6\operatorname{atan}(c\,x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atan(c*x)),x)
```

```
[Out] ((b*atan(c*x))/6 + (b*c^3*x^3)/18 - (b*c^5*x^5)/30 - (b*c*x)/6)/c^6 + (a*x^
6)/6 + (b*x^6*atan(c*x))/6
```

3.2 $\int x^4(a + b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=56

$$\frac{bx^2}{10c^3} - \frac{bx^4}{20c} + \frac{1}{5}x^5(a + b\text{ArcTan}(cx)) - \frac{b\log(1 + c^2x^2)}{10c^5}$$

```
[Out] 1/10*b*x^2/c^3-1/20*b*x^4/c+1/5*x^5*(a+b*arctan(c*x))-1/10*b*ln(c^2*x^2+1)/
c^5
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4946, 272, 45}

$$\frac{1}{5}x^5(a + b\text{ArcTan}(cx)) + \frac{bx^2}{10c^3} - \frac{b\log(c^2x^2 + 1)}{10c^5} - \frac{bx^4}{20c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*x^2)/(10*c^3) - (b*x^4)/(20*c) + (x^5*(a + b*ArcTan[c*x]))/5 - (b*Log[1
+ c^2*x^2])/(10*c^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^4\left(a+b\tan^{-1}(cx)\right)\,dx &= \frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{5}(bc)\int\frac{x^5}{1+c^2x^2}\,dx\\ &= \frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{10}(bc)\text{Subst}\left(\int\frac{x^2}{1+c^2x}\,dx,x,x^2\right)\\ &= \frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)-\frac{1}{10}(bc)\text{Subst}\left(\int\left(-\frac{1}{c^4}+\frac{x}{c^2}+\frac{1}{c^4(1+c^2x)}\right)\,dx,x,x\right.\\ &= \frac{bx^2}{10c^3}-\frac{bx^4}{20c}+\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)-\frac{b\log\left(1+c^2x^2\right)}{10c^5}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 61, normalized size = 1.09

$$\frac{bx^2}{10c^3}-\frac{bx^4}{20c}+\frac{ax^5}{5}+\frac{1}{5}bx^5\text{ArcTan}(cx)-\frac{b\log\left(1+c^2x^2\right)}{10c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*x^2)/(10*c^3) - (b*x^4)/(20*c) + (a*x^5)/5 + (b*x^5*ArcTan[c*x])/5 - (b*
Log[1 + c^2*x^2])/(10*c^5)
```

Maple [A]
time = 0.05, size = 59, normalized size = 1.05

method	result	size
derivativedivides	$\frac{\frac{c^5x^5a}{5}+\frac{c^5x^5b\arctan(cx)}{5}-\frac{c^4x^4b}{20}+\frac{bc^2x^2}{10}-\frac{b\ln\left(c^2x^2+1\right)}{10}}{c^5}$	59
default	$\frac{\frac{c^5x^5a}{5}+\frac{c^5x^5b\arctan(cx)}{5}-\frac{c^4x^4b}{20}+\frac{bc^2x^2}{10}-\frac{b\ln\left(c^2x^2+1\right)}{10}}{c^5}$	59
risch	$-\frac{ix^5b\ln(icx+1)}{10}+\frac{ix^5b\ln(-icx+1)}{10}+\frac{ax^5}{5}-\frac{bx^4}{20c}+\frac{bx^2}{10c^3}-\frac{b\ln\left(-c^2x^2-1\right)}{10c^5}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(1/5*c^5*x^5*a+1/5*c^5*x^5*b*arctan(c*x)-1/20*c^4*x^4*b+1/10*b*c^2*x^
2-1/10*b*ln(c^2*x^2+1))
```

Maxima [A]
time = 0.27, size = 56, normalized size = 1.00

$$\frac{1}{5}ax^5+\frac{1}{20}\left(4x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2x^2}{c^4}+\frac{2\log\left(c^2x^2+1\right)}{c^6}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*
x^2 + 1)/c^6))*b
```

Fricas [A]
time = 2.03, size = 59, normalized size = 1.05

$$\frac{4\,bc^5x^5\arctan\left(cx\right)+4\,ac^5x^5-bc^4x^4+2\,bc^2x^2-2\,b\log\left(c^2x^2+1\right)}{20\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*c^5*x^5*arctan(c*x) + 4*a*c^5*x^5 - b*c^4*x^4 + 2*b*c^2*x^2 - 2*b
*log(c^2*x^2 + 1))/c^5
```

Sympy [A]
time = 0.26, size = 60, normalized size = 1.07

$$\begin{cases} \frac{ax^5}{5}+\frac{bx^5\operatorname{atan}\left(cx\right)}{5}-\frac{bx^4}{20c}+\frac{bx^2}{10c^3}-\frac{b\log\left(x^2+\frac{1}{c^2}\right)}{10c^5} & \text{for}\; c\neq 0\\ \frac{ax^5}{5} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*x**5/5 + b*x**5*atan(c*x)/5 - b*x**4/(20*c) + b*x**2/(10*c**3)
 - b*log(x**2 + c**(-2))/(10*c**5), Ne(c, 0)), (a*x**5/5, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.42, size = 54, normalized size = 0.96

$$\frac{a\,x^5}{5}-\frac{\frac{b\,\ln\left(c^2\,x^2+1\right)}{10}-\frac{b\,c^2\,x^2}{10}+\frac{b\,c^4\,x^4}{20}}{c^5}+\frac{b\,x^5\,\mathrm{atan}\left(c\,x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atan(c*x)),x)
```

```
[Out] (a*x^5)/5 - ((b*log(c^2*x^2 + 1))/10 - (b*c^2*x^2)/10 + (b*c^4*x^4)/20)/c^5
 + (b*x^5*atan(c*x))/5
```

3.3 $\int x^3(a + b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=48

$$\frac{bx}{4c^3} - \frac{bx^3}{12c} - \frac{b\text{ArcTan}(cx)}{4c^4} + \frac{1}{4}x^4(a + b\text{ArcTan}(cx))$$

```
[Out] 1/4*b*x/c^3-1/12*b*x^3/c-1/4*b*arctan(c*x)/c^4+1/4*x^4*(a+b*arctan(c*x))
```

Rubi [A]
time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4946, 308, 209}

$$\frac{1}{4}x^4(a + b\text{ArcTan}(cx)) - \frac{b\text{ArcTan}(cx)}{4c^4} + \frac{bx}{4c^3} - \frac{bx^3}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*x)/(4*c^3) - (b*x^3)/(12*c) - (b*ArcTan[c*x])/(4*c^4) + (x^4*(a + b*ArcT
an[c*x]))/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^3(a+b\tan^{-1}(cx))\,dx &= \frac{1}{4}x^4(a+b\tan^{-1}(cx)) - \frac{1}{4}(bc)\int \frac{x^4}{1+c^2x^2}\,dx \\
&= \frac{1}{4}x^4(a+b\tan^{-1}(cx)) - \frac{1}{4}(bc)\int \left(-\frac{1}{c^4}+\frac{x^2}{c^2}+\frac{1}{c^4(1+c^2x^2)}\right)dx \\
&= \frac{bx}{4c^3} - \frac{bx^3}{12c} + \frac{1}{4}x^4(a+b\tan^{-1}(cx)) - \frac{b\int \frac{1}{1+c^2x^2}\,dx}{4c^3} \\
&= \frac{bx}{4c^3} - \frac{bx^3}{12c} - \frac{b\tan^{-1}(cx)}{4c^4} + \frac{1}{4}x^4(a+b\tan^{-1}(cx))
\end{aligned}
$$

Mathematica [A]
time = 0.00, size = 53, normalized size = 1.10

$$
\frac{bx}{4c^3} - \frac{bx^3}{12c} + \frac{ax^4}{4} - \frac{b\text{ArcTan}(cx)}{4c^4} + \frac{1}{4}bx^4\text{ArcTan}(cx)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x]),x]
```

```
[Out] (b*x)/(4*c^3) - (b*x^3)/(12*c) + (a*x^4)/4 - (b*ArcTan[c*x])/(4*c^4) + (b*x
^4*ArcTan[c*x])/4
```

Maple [A]
time = 0.10, size = 49, normalized size = 1.02

method	result	size
derivativedivides	$\frac{\frac{c^4x^4a}{4}+\frac{c^4x^4b\arctan(cx)}{4}-\frac{b\,c^3x^3}{12}+\frac{xbc}{4}-\frac{b\arctan(cx)}{4}}{c^4}$	49
default	$\frac{\frac{c^4x^4a}{4}+\frac{c^4x^4b\arctan(cx)}{4}-\frac{b\,c^3x^3}{12}+\frac{xbc}{4}-\frac{b\arctan(cx)}{4}}{c^4}$	49
risch	$-\frac{ib\,x^4\ln(icx+1)}{8}+\frac{ib\,x^4\ln(-icx+1)}{8}+\frac{a\,x^4}{4}-\frac{b\,x^3}{12c}+\frac{bx}{4c^3}-\frac{b\arctan(cx)}{4c^4}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/4*c^4*x^4*a+1/4*c^4*x^4*b*arctan(c*x)-1/12*b*c^3*x^3+1/4*x*b*c-1/4
*b*arctan(c*x))
```

Maxima [A]
time = 0.47, size = 48, normalized size = 1.00

$$
\frac{1}{4}\,ax^4 + \frac{1}{12}\left(3\,x^4\arctan\left(cx\right) - c\left(\frac{c^2x^3-3\,x}{c^4} + \frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)b
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x
)/c^5))*b
```

Fricas [A]
time = 6.43, size = 47, normalized size = 0.98

$$\frac{3\,ac^4x^4 - bc^3x^3 + 3\,bcx + 3\,(bc^4x^4 - b)\arctan\left(cx\right)}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*x^4 - b*c^3*x^3 + 3*b*c*x + 3*(b*c^4*x^4 - b)*arctan(c*x))/c^
4
```

Sympy [A]
time = 0.21, size = 53, normalized size = 1.10

$$\begin{cases} \frac{ax^4}{4} + \frac{bx^4\operatorname{atan}\left(cx\right)}{4} - \frac{bx^3}{12c} + \frac{bx}{4c^3} - \frac{b\operatorname{atan}\left(cx\right)}{4c^4} & \text{for}\: c \neq 0 \\ \frac{ax^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*x**4/4 + b*x**4*atan(c*x)/4 - b*x**3/(12*c) + b*x/(4*c**3) - b
*atan(c*x)/(4*c**4), Ne(c, 0)), (a*x**4/4, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.21, size = 44, normalized size = 0.92

$$\frac{a\,x^4}{4} - \frac{\frac{b\,\mathrm{atan}(c\,x)}{4} + \frac{b\,c^3\,x^3}{12} - \frac{b\,c\,x}{4}}{c^4} + \frac{b\,x^4\,\mathrm{atan}(c\,x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x)),x)
```

```
[Out] (a*x^4)/4 - ((b*atan(c*x))/4 + (b*c^3*x^3)/12 - (b*c*x)/4)/c^4 + (b*x^4*ata
n(c*x))/4
```

3.4 $\int x^2(a+b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=45

$$-\frac{bx^2}{6c}+\frac{1}{3}x^3(a+b\text{ArcTan}(cx))+\frac{b\log\left(1+c^2x^2\right)}{6c^3}$$

```
[Out] -1/6*b*x^2/c+1/3*x^3*(a+b*arctan(c*x))+1/6*b*ln(c^2*x^2+1)/c^3
```

Rubi [A]
time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {4946, 272, 45}

$$\frac{1}{3}x^3(a+b\text{ArcTan}(cx))+\frac{b\log\left(c^2x^2+1\right)}{6c^3}-\frac{bx^2}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/6*(b*x^2)/c + (x^3*(a + b*ArcTan[c*x]))/3 + (b*Log[1 + c^2*x^2])/(6*c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(a+b\tan^{-1}(cx)\right)\,dx &= \frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)-\frac{1}{3}(bc)\int\frac{x^3}{1+c^2x^2}\,dx\\
&= \frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)-\frac{1}{6}(bc)\text{Subst}\left(\int\frac{x}{1+c^2x}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)-\frac{1}{6}(bc)\text{Subst}\left(\int\left(\frac{1}{c^2}-\frac{1}{c^2\left(1+c^2x\right)}\right)\,dx,x,x^2\right)\\
&= -\frac{bx^2}{6c}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)+\frac{b\log\left(1+c^2x^2\right)}{6c^3}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 50, normalized size = 1.11

$$
-\frac{bx^2}{6c}+\frac{ax^3}{3}+\frac{1}{3}bx^3\text{ArcTan}(cx)+\frac{b\log\left(1+c^2x^2\right)}{6c^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/6*(b*x^2)/c + (a*x^3)/3 + (b*x^3*ArcTan[c*x])/3 + (b*Log[1 + c^2*x^2])/(
6*c^3)
```

Maple [A]
time = 0.05, size = 50, normalized size = 1.11

method	result	size
derivativedivides	$\frac{\frac{c^3x^3a}{3}+\frac{b\,c^3x^3\arctan(cx)}{3}-\frac{b\,c^2x^2}{6}+\frac{b\ln\left(c^2x^2+1\right)}{6}}{c^3}$	50
default	$\frac{\frac{c^3x^3a}{3}+\frac{b\,c^3x^3\arctan(cx)}{3}-\frac{b\,c^2x^2}{6}+\frac{b\ln\left(c^2x^2+1\right)}{6}}{c^3}$	50
risch	$-\frac{ix^3b\ln(icx+1)}{6}+\frac{ib\,x^3\ln(-icx+1)}{6}+\frac{a\,x^3}{3}-\frac{b\,x^2}{6c}+\frac{b\ln\left(-c^2x^2-1\right)}{6c^3}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/3*c^3*x^3*a+1/3*b*c^3*x^3*arctan(c*x)-1/6*b*c^2*x^2+1/6*b*ln(c^2*x
^2+1))
```

Maxima [A]
time = 0.26, size = 46, normalized size = 1.02

$$
\frac{1}{3}\,ax^3+\frac{1}{6}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)b
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b
```

Fricas [A]
time = 4.57, size = 49, normalized size = 1.09

$$\frac{2\,bc^3x^3\arctan\left(cx\right)+2\,ac^3x^3-bc^2x^2+b\log\left(c^2x^2+1\right)}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*c^3*x^3*arctan(c*x) + 2*a*c^3*x^3 - b*c^2*x^2 + b*log(c^2*x^2 + 1)
)/c^3
```

Sympy [A]
time = 0.19, size = 49, normalized size = 1.09

$$\begin{cases}\frac{ax^3}{3}+\frac{bx^3\operatorname{atan}\left(cx\right)}{3}-\frac{bx^2}{6c}+\frac{b\log\left(x^2+\frac{1}{c^2}\right)}{6c^3} & \text{for}\ c\neq 0\\ \frac{ax^3}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x)),x)
```

```
[Out] Piecewise((a*x**3/3 + b*x**3*atan(c*x)/3 - b*x**2/(6*c) + b*log(x**2 + c**(
-2))/(6*c**3), Ne(c, 0)), (a*x**3/3, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.37, size = 42, normalized size = 0.93

$$\frac{a\,x^3}{3}+\frac{b\,x^3\,\mathrm{atan}(c\,x)}{3}+\frac{b\,\ln\left(c^2\,x^2+1\right)}{6\,c^3}-\frac{b\,x^2}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x)),x)
```

```
[Out] (a*x^3)/3 + (b*x^3*atan(c*x))/3 + (b*log(c^2*x^2 + 1))/(6*c^3) - (b*x^2)/(6
*c)
```

3.5 $\int x(a+b\textbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=37

$$-\frac{bx}{2c}+\frac{b\text{ArcTan}(cx)}{2c^2}+\frac{1}{2}x^2(a+b\text{ArcTan}(cx))$$

```
[Out] -1/2*b*x/c+1/2*b*arctan(c*x)/c^2+1/2*x^2*(a+b*arctan(c*x))
```

Rubi [A]
time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {4946, 327, 209}

$$\frac{1}{2}x^2(a+b\text{ArcTan}(cx))+\frac{b\text{ArcTan}(cx)}{2c^2}-\frac{bx}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x]),x]
```

```
[Out] -1/2*(b*x)/c + (b*ArcTan[c*x])/(2*c^2) + (x^2*(a + b*ArcTan[c*x]))/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\tan^{-1}(cx)\right)\,dx &= \frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)-\frac{1}{2}(bc)\int\frac{x^2}{1+c^2x^2}\,dx\\ &= -\frac{bx}{2c}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)+\frac{b\int\frac{1}{1+c^2x^2}\,dx}{2c}\\ &= -\frac{bx}{2c}+\frac{b\tan^{-1}(cx)}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 42, normalized size = 1.14

$$-\frac{bx}{2c}+\frac{ax^2}{2}+\frac{b\text{ArcTan}(cx)}{2c^2}+\frac{1}{2}bx^2\text{ArcTan}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c*x]),x]
[Out] -1/2*(b*x)/c + (a*x^2)/2 + (b*ArcTan[c*x])/(2*c^2) + (b*x^2*ArcTan[c*x])/2
```

Maple [A]
time = 0.11, size = 40, normalized size = 1.08

method	result	size
derivativedivides	$\frac{\frac{c^2x^2a}{2}+\frac{\arctan(cx)b\,c^2x^2}{2}-\frac{xbc}{2}+\frac{b\arctan(cx)}{2}}{c^2}$	40
default	$\frac{\frac{c^2x^2a}{2}+\frac{\arctan(cx)b\,c^2x^2}{2}-\frac{xbc}{2}+\frac{b\arctan(cx)}{2}}{c^2}$	40
risch	$-\frac{ix^2b\ln(icx+1)}{4}+\frac{ib\,x^2\ln(-icx+1)}{4}+\frac{a\,x^2}{2}-\frac{bx}{2c}+\frac{b\arctan(cx)}{2c^2}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/c^2*(1/2*c^2*x^2*a+1/2*arctan(c*x)*b*c^2*x^2-1/2*x*b*c+1/2*b*arctan(c*x))
```

Maxima [A]
time = 0.46, size = 37, normalized size = 1.00

$$\frac{1}{2}\,ax^2+\frac{1}{2}\left(x^2\arctan\left(cx\right)-c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b
```

Fricas [A]
time = 3.43, size = 34, normalized size = 0.92

$$\frac{ac^2x^2 - bcx + (bc^2x^2 + b)\arctan(cx)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x)),x, algorithm="fricas")
[Out] 1/2*(a*c^2*x^2 - b*c*x + (b*c^2*x^2 + b)*arctan(c*x))/c^2
```

Sympy [A]
time = 0.14, size = 42, normalized size = 1.14

$$\begin{cases} \frac{ax^2}{2} + \frac{bx^2\operatorname{atan}(cx)}{2} - \frac{bx}{2c} + \frac{b\operatorname{atan}(cx)}{2c^2} & \text{for}\ c \neq 0 \\ \frac{ax^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x)),x)
[Out] Piecewise((a*x**2/2 + b*x**2*atan(c*x)/2 - b*x/(2*c) + b*atan(c*x)/(2*c**2)
, Ne(c, 0)), (a*x**2/2, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B]
time = 0.12, size = 34, normalized size = 0.92

$$\frac{a\,x^2}{2} + \frac{b\operatorname{atan}(c\,x)}{2\,c^2} + \frac{b\,x^2\operatorname{atan}(c\,x)}{2} - \frac{b\,x}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x)),x)
[Out] (a*x^2)/2 + (b*atan(c*x))/(2*c^2) + (b*x^2*atan(c*x))/2 - (b*x)/(2*c)
```

3.6 $\int (a + b\mathbf{ArcTan}(cx))\,dx$

Optimal. Leaf size=29

$$ax + bx\text{ArcTan}(cx) - \frac{b\log(1 + c^2x^2)}{2c}$$

```
[Out] a*x+b*x*arctan(c*x)-1/2*b*ln(c^2*x^2+1)/c
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4930, 266}

$$ax + bx\text{ArcTan}(cx) - \frac{b\log(c^2x^2 + 1)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTan[c*x],x]
```

```
[Out] a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \left(a + b\tan^{-1}(cx)\right)\,dx &= ax + b\int \tan^{-1}(cx)\,dx\\ &= ax + bx\tan^{-1}(cx) - (bc)\int \frac{x}{1 + c^2x^2}\,dx\\ &= ax + bx\tan^{-1}(cx) - \frac{b\log(1 + c^2x^2)}{2c}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 29, normalized size = 1.00

$$ax + bx\text{ArcTan}(cx) - \frac{b\log(1 + c^2x^2)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTan[c*x],x]
```

```
[Out] a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c)
```

Maple [A]
time = 0.08, size = 28, normalized size = 0.97

method	result	size
default	$ax + bx\arctan(cx) - \frac{b\ln(c^2x^2+1)}{2c}$	28
derivativedivides	$\frac{acx+bcx\arctan(cx)-\frac{b\ln(c^2x^2+1)}{2}}{c}$	31
risch	$ax - \frac{ibx\ln(icx+1)}{2} + \frac{ibx\ln(-icx+1)}{2} - \frac{b\ln(-c^2x^2-1)}{2c}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctan(c*x),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*x*arctan(c*x)-1/2*b*ln(c^2*x^2+1)/c
```

Maxima [A]
time = 0.26, size = 31, normalized size = 1.07

$$ax + \frac{(2\,cx\arctan(cx) - \log(c^2x^2 + 1))b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b/c
```

Fricas [A]
time = 3.29, size = 33, normalized size = 1.14

$$\frac{2\,bcx\arctan(cx) + 2\,acx - b\log(c^2x^2 + 1)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*c*x*arctan(c*x) + 2*a*c*x - b*log(c^2*x^2 + 1))/c
```

Sympy [A]
time = 0.09, size = 26, normalized size = 0.90

$$ax + b\left(\begin{cases} x\operatorname{atan}(cx) - \frac{\log(c^2x^2+1)}{2c} & \text{for } c \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atan(c*x),x)
[Out] a*x + b*Piecewise((x*atan(c*x) - log(c**2*x**2 + 1)/(2*c), Ne(c, 0)), (0, T
rue))
```

Giac [A]
time = 0.41, size = 31, normalized size = 1.07

$$ax + \frac{(2\,cx\arctan(cx) - \log(c^2x^2+1))b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x),x, algorithm="giac")
[Out] a*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b/c
```

Mupad [B]
time = 0.10, size = 27, normalized size = 0.93

$$a\,x - \frac{b\,\ln(c^2\,x^2+1)}{2\,c} + b\,x\,\mathrm{atan}(c\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atan(c*x),x)
[Out] a*x - (b*log(c^2*x^2 + 1))/(2*c) + b*x*atan(c*x)
```

3.7 $\int \frac{a+b\text{ArcTan}(cx)}{x}\,dx$

Optimal. Leaf size=35

$$a\log(x) + \frac{1}{2}ib\text{PolyLog}(2, -icx) - \frac{1}{2}ib\text{PolyLog}(2, icx)$$

```
[Out] a*ln(x)+1/2*I*b*polylog(2,-I*c*x)-1/2*I*b*polylog(2,I*c*x)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4940, 2438}

$$a\log(x) + \frac{1}{2}ib\text{Li}_2(-icx) - \frac{1}{2}ib\text{Li}_2(icx)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/x,x]
```

```
[Out] a*Log[x] + (I/2)*b*PolyLog[2, (-I)*c*x] - (I/2)*b*PolyLog[2, I*c*x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}(cx)}{x}\,dx &= a\log(x) + \frac{1}{2}(ib)\int \frac{\log(1-icx)}{x}\,dx - \frac{1}{2}(ib)\int \frac{\log(1+icx)}{x}\,dx\\ &= a\log(x) + \frac{1}{2}ib\text{Li}_2(-icx) - \frac{1}{2}ib\text{Li}_2(icx)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 35, normalized size = 1.00

$$a\log(x) + \frac{1}{2}ib\text{PolyLog}(2, -icx) - \frac{1}{2}ib\text{PolyLog}(2, icx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/x,x]
```

```
[Out] a*Log[x] + (I/2)*b*PolyLog[2, (-I)*c*x] - (I/2)*b*PolyLog[2, I*c*x]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(27) = 54$.
time = 0.10, size = 74, normalized size = 2.11

method	result
risch	$-\frac{ib\,\mathrm{dilog}(-icx+1)}{2} + a\ln(-icx) + \frac{ib\,\mathrm{dilog}(icx+1)}{2}$
derivativedivides	$a\ln(cx) + b\ln(cx)\arctan(cx) + \frac{ib\ln(cx)\ln(icx+1)}{2} - \frac{ib\ln(cx)\ln(-icx+1)}{2} + \frac{ib\,\mathrm{dilog}(icx+1)}{2} - \frac{ib\,\mathrm{di}}{}$
default	$a\ln(cx) + b\ln(cx)\arctan(cx) + \frac{ib\ln(cx)\ln(icx+1)}{2} - \frac{ib\ln(cx)\ln(-icx+1)}{2} + \frac{ib\,\mathrm{dilog}(icx+1)}{2} - \frac{ib\,\mathrm{di}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(c*x)+b*ln(c*x)*arctan(c*x)+1/2*I*b*ln(c*x)*ln(1+I*c*x)-1/2*I*b*ln(c*x)
*ln(1-I*c*x)+1/2*I*b*dilog(1+I*c*x)-1/2*I*b*dilog(1-I*c*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x,x, algorithm="maxima")
```

```
[Out] b*integrate(arctan(c*x)/x, x) + a*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x) + a)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{atan}(cx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x,x)
```

```
[Out] Integral((a + b*atan(c*x))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.29, size = 28, normalized size = 0.80

$$a\,\ln\left(x\right)-\frac{b\,\left(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/x,x)
```

```
[Out] a*log(x) - (b*(dilog(1 - c*x*1i) - dilog(c*x*1i + 1))*1i)/2
```

3.8 $\int \frac{a+b\text{ArcTan}(cx)}{x^2}\,dx$

Optimal. Leaf size=35

$$-\frac{a+b\text{ArcTan}(cx)}{x}+bc\log(x)-\frac{1}{2}bc\log\left(1+c^2x^2\right)$$

```
[Out] (-a-b*arctan(c*x))/x+b*c*ln(x)-1/2*b*c*ln(c^2*x^2+1)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4946, 272, 36, 29, 31}

$$-\frac{a+b\text{ArcTan}(cx)}{x}-\frac{1}{2}bc\log\left(c^2x^2+1\right)+bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/x^2,x]
```

```
[Out] -((a + b*ArcTan[c*x])/x) + b*c*Log[x] - (b*c*Log[1 + c^2*x^2])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx &= -\frac{a+b\tan^{-1}(cx)}{x}+(bc)\int \frac{1}{x(1+c^2x^2)}\,dx\\ &= -\frac{a+b\tan^{-1}(cx)}{x}+\frac{1}{2}(bc)\text{Subst}\left(\int \frac{1}{x(1+c^2x)}\,dx,x,x^2\right)\\ &= -\frac{a+b\tan^{-1}(cx)}{x}+\frac{1}{2}(bc)\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)-\frac{1}{2}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1+c^2x}\,dx,x,\right.\\ &= -\frac{a+b\tan^{-1}(cx)}{x}+bc\log(x)-\frac{1}{2}bc\log\left(1+c^2x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 38, normalized size = 1.09

$$-\frac{a}{x}-\frac{b\text{ArcTan}(cx)}{x}+bc\log(x)-\frac{1}{2}bc\log\left(1+c^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/x^2,x]
[Out] -(a/x) - (b*ArcTan[c*x])/x + b*c*Log[x] - (b*c*Log[1 + c^2*x^2])/2
```

Maple [A]
time = 0.08, size = 45, normalized size = 1.29

method	result	size
derivativedivides	$c\left(-\frac{a}{cx}-\frac{b\arctan(cx)}{cx}-\frac{b\ln(c^2x^2+1)}{2}+b\ln(cx)\right)$	45
default	$c\left(-\frac{a}{cx}-\frac{b\arctan(cx)}{cx}-\frac{b\ln(c^2x^2+1)}{2}+b\ln(cx)\right)$	45
risch	$\frac{ib\ln(icx+1)}{2x}-\frac{-2bc\ln(x)x+bc\ln(-c^2x^2-1)x+ib\ln(-icx+1)+2a}{2x}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
[Out] c*(-a/c/x-b/c/x*arctan(c*x)-1/2*b*ln(c^2*x^2+1)+b*ln(c*x))
```

Maxima [A]
time = 0.27, size = 39, normalized size = 1.11

$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\,\arctan\left(cx\right)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2,x, algorithm="maxima")
[Out] -1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b - a/x
```

Fricas [A]
time = 0.92, size = 37, normalized size = 1.06

$$-\frac{bcx\log\left(c^2x^2+1\right)-2\,bcx\log\left(x\right)+2\,b\arctan\left(cx\right)+2\,a}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2,x, algorithm="fricas")
[Out] -1/2*(b*c*x*log(c^2*x^2 + 1) - 2*b*c*x*log(x) + 2*b*arctan(c*x) + 2*a)/x
```

Sympy [A]
time = 0.27, size = 37, normalized size = 1.06

$$\begin{cases} -\frac{a}{x}+bc\log\left(x\right)-\frac{bc\log\left(x^2+\frac{1}{c^2}\right)}{2}-\frac{b\,\mathrm{atan}\left(cx\right)}{x} & \text{for}\ c\neq 0\\ -\frac{a}{x} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**2,x)
[Out] Piecewise((-a/x + b*c*log(x) - b*c*log(x**2 + c**(-2))/2 - b*atan(c*x)/x, N
e(c, 0)), (-a/x, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^2,x, algorithm="giac")
[Out] sage0*x
```

Mupad [B]
time = 0.32, size = 36, normalized size = 1.03

$$b\,c\,\ln\left(x\right)-\frac{a}{x}-\frac{b\,\mathrm{atan}(c\,x)}{x}-\frac{b\,c\,\ln\left(c^2\,x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/x^2,x)
[Out] b*c*log(x) - a/x - (b*atan(c*x))/x - (b*c*log(c^2*x^2 + 1))/2
```

3.9 $\int \frac{a+b\textbf{ArcTan}(cx)}{x^3}\,dx$

Optimal. Leaf size=37

$$-\frac{bc}{2x}-\frac{1}{2}bc^2\text{ArcTan}(cx)-\frac{a+b\text{ArcTan}(cx)}{2x^2}$$

```
[Out] -1/2*b*c/x-1/2*b*c^2*arctan(c*x)+1/2*(-a-b*arctan(c*x))/x^2
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {4946, 331, 209}

$$-\frac{a+b\text{ArcTan}(cx)}{2x^2}-\frac{1}{2}bc^2\text{ArcTan}(cx)-\frac{bc}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/x^3,x]
```

```
[Out] -1/2*(b*c)/x - (b*c^2*ArcTan[c*x])/2 - (a + b*ArcTan[c*x])/(2*x^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx = -\frac{a+b\tan^{-1}(cx)}{2x^2} + \frac{1}{2}(bc)\int \frac{1}{x^2\left(1+c^2x^2\right)}\,dx$$
$$= -\frac{bc}{2x} - \frac{a+b\tan^{-1}(cx)}{2x^2} - \frac{1}{2}\left(bc^3\right)\int \frac{1}{1+c^2x^2}\,dx$$
$$= -\frac{bc}{2x} - \frac{1}{2}bc^2\tan^{-1}(cx) - \frac{a+b\tan^{-1}(cx)}{2x^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.00, size = 46, normalized size = 1.24

$$-\frac{a}{2x^2} - \frac{b\text{ArcTan}(cx)}{2x^2} - \frac{bc\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-c^2x^2\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/x^3,x]
[Out] -1/2*a/x^2 - (b*ArcTan[c*x])/(2*x^2) - (b*c*Hypergeometric2F1[-1/2, 1, 1/2,
 -(c^2*x^2)])/(2*x)
```

Maple [A]
time = 0.12, size = 44, normalized size = 1.19

method	result	size
derivativedivides	$c^2\left(-\frac{a}{2c^2x^2}-\frac{b\arctan(cx)}{2c^2x^2}-\frac{b\arctan(cx)}{2}-\frac{b}{2cx}\right)$	44
default	$c^2\left(-\frac{a}{2c^2x^2}-\frac{b\arctan(cx)}{2c^2x^2}-\frac{b\arctan(cx)}{2}-\frac{b}{2cx}\right)$	44
risch	$\frac{ib\ln(icx+1)}{4x^2}-\frac{ic^2b\ln(-cx-i)x^2-ic^2b\ln(-cx+i)x^2+ib\ln(-icx+1)+2xbc+2a}{4x^2}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
[Out] c^2*(-1/2*a/c^2/x^2-1/2*b/c^2/x^2*arctan(c*x)-1/2*b*arctan(c*x)-1/2*b/c/x)
```

Maxima [A]
time = 0.47, size = 31, normalized size = 0.84

$$-\frac{1}{2}\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)b-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3,x, algorithm="maxima")
[Out] -1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b - 1/2*a/x^2
```

Fricas [A]
time = 0.71, size = 26, normalized size = 0.70

$$-\frac{bcx + (bc^2x^2 + b)\arctan(cx) + a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3,x, algorithm="fricas")
[Out] -1/2*(b*c*x + (b*c^2*x^2 + b)*arctan(c*x) + a)/x^2
```

Sympy [A]
time = 0.22, size = 37, normalized size = 1.00

$$-\frac{a}{2x^2} - \frac{bc^2 \operatorname{atan}(cx)}{2} - \frac{bc}{2x} - \frac{b \operatorname{atan}(cx)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**3,x)
[Out] -a/(2*x**2) - b*c**2*atan(c*x)/2 - b*c/(2*x) - b*atan(c*x)/(2*x**2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [B]
time = 0.33, size = 42, normalized size = 1.14

$$-\frac{\frac{a}{2} + \frac{b \operatorname{atan}(cx)}{2} + \frac{bcx}{2}}{x^2} - \frac{bc \operatorname{atan}\left(\frac{c^2 x}{\sqrt{c^2}}\right) \sqrt{c^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/x^3,x)
[Out] - (a/2 + (b*atan(c*x))/2 + (b*c*x)/2)/x^2 - (b*c*atan((c^2*x)/(c^2)^(1/2))*
(c^2)^(1/2))/2
```

3.10 $\int \frac{a+b\text{ArcTan}(cx)}{x^4}\,dx$

Optimal. Leaf size=53

$$-\frac{bc}{6x^2}-\frac{a+b\text{ArcTan}(cx)}{3x^3}-\frac{1}{3}bc^3\log(x)+\frac{1}{6}bc^3\log\left(1+c^2x^2\right)$$

```
[Out] -1/6*b*c/x^2+1/3*(-a-b*arctan(c*x))/x^3-1/3*b*c^3*ln(x)+1/6*b*c^3*ln(c^2*x^
2+1)
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4946, 272, 46}

$$-\frac{a+b\text{ArcTan}(cx)}{3x^3}-\frac{1}{3}bc^3\log(x)+\frac{1}{6}bc^3\log\left(c^2x^2+1\right)-\frac{bc}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/x^4,x]
```

```
[Out] -1/6*(b*c)/x^2 - (a + b*ArcTan[c*x])/(3*x^3) - (b*c^3*Log[x])/3 + (b*c^3*Lo
g[1 + c^2*x^2])/6
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}(cx)}{x^4}\,dx &= -\frac{a+b\tan^{-1}(cx)}{3x^3}+\frac{1}{3}(bc)\int \frac{1}{x^3\left(1+c^2x^2\right)}\,dx\\ &= -\frac{a+b\tan^{-1}(cx)}{3x^3}+\frac{1}{6}(bc)\text{Subst}\left(\int \frac{1}{x^2\left(1+c^2x\right)}\,dx,x,x^2\right)\\ &= -\frac{a+b\tan^{-1}(cx)}{3x^3}+\frac{1}{6}(bc)\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{c^2}{x}+\frac{c^4}{1+c^2x}\right)dx,x,x^2\right)\\ &= -\frac{bc}{6x^2}-\frac{a+b\tan^{-1}(cx)}{3x^3}-\frac{1}{3}bc^3\log(x)+\frac{1}{6}bc^3\log\left(1+c^2x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 1.02

$$-\frac{a}{3x^3}-\frac{b\text{ArcTan}(cx)}{3x^3}+\frac{1}{6}bc\left(-\frac{1}{x^2}-2c^2\log(x)+c^2\log\left(1+c^2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (b*ArcTan[c*x])/(3*x^3) + (b*c*(-x^(-2) - 2*c^2*Log[x] + c^2*L
og[1 + c^2*x^2]))/6
```

Maple [A]
time = 0.08, size = 57, normalized size = 1.08

method	result	size
derivativedivides	$c^3\left(-\frac{a}{3c^3x^3}-\frac{b\arctan(cx)}{3c^3x^3}+\frac{b\ln(c^2x^2+1)}{6}-\frac{b}{6c^2x^2}-\frac{b\ln(cx)}{3}\right)$	57
default	$c^3\left(-\frac{a}{3c^3x^3}-\frac{b\arctan(cx)}{3c^3x^3}+\frac{b\ln(c^2x^2+1)}{6}-\frac{b}{6c^2x^2}-\frac{b\ln(cx)}{3}\right)$	57
risch	$\frac{ib\ln(icx+1)}{6x^3}-\frac{2bc^3\ln(x)x^3-bc^3\ln(c^2x^2+1)x^3+ib\ln(-icx+1)+xbc+2a}{6x^3}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/3*a/c^3/x^3-1/3*b/c^3/x^3*arctan(c*x)+1/6*b*ln(c^2*x^2+1)-1/6*b/c^2
/x^2-1/3*b*ln(c*x))
```

Maxima [A]
time = 0.27, size = 51, normalized size = 0.96

$$\frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b
 - 1/3*a/x^3
```

Fricas [A]
time = 1.26, size = 50, normalized size = 0.94

$$\frac{bc^3x^3 \log\left(c^2x^2+1\right) - 2\,bc^3x^3\log\left(x\right) - bcx - 2\,b\arctan\left(cx\right) - 2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(b*c^3*x^3*log(c^2*x^2 + 1) - 2*b*c^3*x^3*log(x) - b*c*x - 2*b*arctan(c
*x) - 2*a)/x^3
```

Sympy [A]
time = 0.38, size = 61, normalized size = 1.15

$$\begin{cases} -\frac{a}{3x^3} - \frac{bc^3\log\left(x\right)}{3} + \frac{bc^3\log\left(x^2+\frac{1}{c^2}\right)}{6} - \frac{bc}{6x^2} - \frac{b\,\mathrm{atan}\left(cx\right)}{3x^3} & \text{for}\ c \neq 0 \\ -\frac{a}{3x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3) - b*c**3*log(x)/3 + b*c**3*log(x**2 + c**(-2))/6 - b
*c/(6*x**2) - b*atan(c*x)/(3*x**3), Ne(c, 0)), (-a/(3*x**3), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.12, size = 46, normalized size = 0.87

$$\frac{b\,c^3\,\ln\left(c^2\,x^2+1\right)}{6} - \frac{\frac{a}{3} + \frac{b\,\mathrm{atan}(c\,x)}{3} + \frac{b\,c\,x}{6}}{x^3} - \frac{b\,c^3\,\ln\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/x^4,x)
```

```
[Out] (b*c^3*log(c^2*x^2 + 1))/6 - (a/3 + (b*atan(c*x))/3 + (b*c*x)/6)/x^3 - (b*c
^3*log(x))/3
```

3.11 $\int \frac{a+b\text{ArcTan}(cx)}{x^5}\,dx$

Optimal. Leaf size=48

$$-\frac{bc}{12x^3}+\frac{bc^3}{4x}+\frac{1}{4}bc^4\text{ArcTan}(cx)-\frac{a+b\text{ArcTan}(cx)}{4x^4}$$

```
[Out] -1/12*b*c/x^3+1/4*b*c^3/x+1/4*b*c^4*arctan(c*x)+1/4*(-a-b*arctan(c*x))/x^4
```

Rubi [A]

time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4946, 331, 209}

$$-\frac{a+b\text{ArcTan}(cx)}{4x^4}+\frac{1}{4}bc^4\text{ArcTan}(cx)+\frac{bc^3}{4x}-\frac{bc}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/x^5,x]
```

```
[Out] -1/12*(b*c)/x^3 + (b*c^3)/(4*x) + (b*c^4*ArcTan[c*x])/4 - (a + b*ArcTan[c*x
])/(4*x^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^5}\,dx = -\frac{a+b\tan^{-1}(cx)}{4x^4}+\frac{1}{4}(bc)\int \frac{1}{x^4\left(1+c^2x^2\right)}\,dx$$
$$= -\frac{bc}{12x^3}-\frac{a+b\tan^{-1}(cx)}{4x^4}-\frac{1}{4}\left(bc^3\right)\int \frac{1}{x^2\left(1+c^2x^2\right)}\,dx$$
$$= -\frac{bc}{12x^3}+\frac{bc^3}{4x}-\frac{a+b\tan^{-1}(cx)}{4x^4}+\frac{1}{4}\left(bc^5\right)\int \frac{1}{1+c^2x^2}\,dx$$
$$= -\frac{bc}{12x^3}+\frac{bc^3}{4x}+\frac{1}{4}bc^4\tan^{-1}(cx)-\frac{a+b\tan^{-1}(cx)}{4x^4}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.00, size = 46, normalized size = 0.96

$$-\frac{a}{4x^4}-\frac{b\text{ArcTan}(cx)}{4x^4}-\frac{bc\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-c^2x^2\right)}{12x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/x^5,x]
```

```
[Out] -1/4*a/x^4 - (b*ArcTan[c*x])/(4*x^4) - (b*c*Hypergeometric2F1[-3/2, 1, -1/2
, -(c^2*x^2)])/(12*x^3)
```

Maple [A]
time = 0.13, size = 53, normalized size = 1.10

method	result	size
derivativedivides	$c^4\left(-\frac{a}{4c^4x^4}-\frac{b\arctan(cx)}{4c^4x^4}+\frac{b\arctan(cx)}{4}-\frac{b}{12c^3x^3}+\frac{b}{4cx}\right)$	53
default	$c^4\left(-\frac{a}{4c^4x^4}-\frac{b\arctan(cx)}{4c^4x^4}+\frac{b\arctan(cx)}{4}-\frac{b}{12c^3x^3}+\frac{b}{4cx}\right)$	53
risch	$\frac{ib\ln(icx+1)}{8x^4}-\frac{-3ic^4b\ln(-cx-i)x^4+3ic^4b\ln(-cx+i)x^4-6b\,c^3x^3+3ib\ln(-icx+1)+2xbc+6a}{24x^4}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(-1/4*a/c^4/x^4-1/4*b/c^4/x^4*arctan(c*x)+1/4*b*arctan(c*x)-1/12*b/c^3/
x^3+1/4*b/c/x)
```

Maxima [A]
time = 0.47, size = 46, normalized size = 0.96

$$\frac{1}{12}\left(\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\,\arctan\left(cx\right)}{x^4}\right)b-\frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b -
1/4*a/x^4
```

Fricas [A]
time = 0.63, size = 41, normalized size = 0.85

$$\frac{3\,bc^3x^3 - bcx + 3\,(bc^4x^4 - b)\arctan\left(cx\right) - 3\,a}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*c^3*x^3 - b*c*x + 3*(b*c^4*x^4 - b)*arctan(c*x) - 3*a)/x^4
```

Sympy [A]
time = 0.30, size = 46, normalized size = 0.96

$$-\frac{a}{4x^4} + \frac{bc^4\,\mathrm{atan}\left(cx\right)}{4} + \frac{bc^3}{4x} - \frac{bc}{12x^3} - \frac{b\,\mathrm{atan}\left(cx\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**5,x)
```

```
[Out] -a/(4*x**4) + b*c**4*atan(c*x)/4 + b*c**3/(4*x) - b*c/(12*x**3) - b*atan(c*
x)/(4*x**4)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.36, size = 42, normalized size = 0.88

$$\frac{b\,c^4\,\mathrm{atan}(c\,x)}{4} - \frac{-b\,c^3\,x^3 + \frac{b\,c\,x}{3} + a}{4\,x^4} - \frac{b\,\mathrm{atan}(c\,x)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/x^5,x)
```

```
[Out] (b*c^4*atan(c*x))/4 - (a - b*c^3*x^3 + (b*c*x)/3)/(4*x^4) - (b*atan(c*x))/(
4*x^4)
```

3.12 $\int \frac{a+b\text{ArcTan}(cx)}{x^6}\,dx$

Optimal. Leaf size=64

$$-\frac{bc}{20x^4}+\frac{bc^3}{10x^2}-\frac{a+b\text{ArcTan}(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)-\frac{1}{10}bc^5\log\left(1+c^2x^2\right)$$

```
[Out] -1/20*b*c/x^4+1/10*b*c^3/x^2+1/5*(-a-b*arctan(c*x))/x^5+1/5*b*c^5*ln(x)-1/1
0*b*c^5*ln(c^2*x^2+1)
```

Rubi [A]
time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4946, 272, 46}

$$-\frac{a+b\text{ArcTan}(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)+\frac{bc^3}{10x^2}-\frac{1}{10}bc^5\log\left(c^2x^2+1\right)-\frac{bc}{20x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])/x^6,x]
```

```
[Out] -1/20*(b*c)/x^4 + (b*c^3)/(10*x^2) - (a + b*ArcTan[c*x])/(5*x^5) + (b*c^5*L
og[x])/5 - (b*c^5*Log[1 + c^2*x^2])/10
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx)}{x^6}\,dx = -\frac{a+b\tan^{-1}(cx)}{5x^5}+\frac{1}{5}(bc)\int \frac{1}{x^5(1+c^2x^2)}\,dx$$
$$= -\frac{a+b\tan^{-1}(cx)}{5x^5}+\frac{1}{10}(bc)\text{Subst}\left(\int \frac{1}{x^3(1+c^2x)}\,dx,x,x^2\right)$$
$$= -\frac{a+b\tan^{-1}(cx)}{5x^5}+\frac{1}{10}(bc)\text{Subst}\left(\int \left(\frac{1}{x^3}-\frac{c^2}{x^2}+\frac{c^4}{x}-\frac{c^6}{1+c^2x}\right)dx,x,x^2\right)$$
$$= -\frac{bc}{20x^4}+\frac{bc^3}{10x^2}-\frac{a+b\tan^{-1}(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)-\frac{1}{10}bc^5\log\left(1+c^2x^2\right)$$

Mathematica [A]
time = 0.02, size = 64, normalized size = 1.00

$$-\frac{a}{5x^5}-\frac{b\text{ArcTan}(cx)}{5x^5}+\frac{1}{10}bc\left(-\frac{1}{2x^4}+\frac{c^2}{x^2}+2c^4\log(x)-c^4\log\left(1+c^2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])/x^6,x]
```

```
[Out] -1/5*a/x^5 - (b*ArcTan[c*x])/(5*x^5) + (b*c*(-1/2*1/x^4 + c^2/x^2 + 2*c^4*L
og[x] - c^4*Log[1 + c^2*x^2]))/10
```

Maple [A]
time = 0.09, size = 66, normalized size = 1.03

method	result	size
derivativedivides	$c^5\left(-\frac{a}{5c^5x^5}-\frac{b\arctan(cx)}{5c^5x^5}-\frac{b\ln(c^2x^2+1)}{10}-\frac{b}{20c^4x^4}+\frac{b\ln(cx)}{5}+\frac{b}{10c^2x^2}\right)$	66
default	$c^5\left(-\frac{a}{5c^5x^5}-\frac{b\arctan(cx)}{5c^5x^5}-\frac{b\ln(c^2x^2+1)}{10}-\frac{b}{20c^4x^4}+\frac{b\ln(cx)}{5}+\frac{b}{10c^2x^2}\right)$	66
risch	$\frac{ib\ln(icx+1)}{10x^5}-\frac{-4bc^5\ln(x)x^5+2bc^5\ln(-c^2x^2-1)x^5-2bc^3x^3+2ib\ln(-icx+1)+xbc+4a}{20x^5}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c^5*(-1/5*a/c^5/x^5-1/5*b/c^5/x^5*arctan(c*x)-1/10*b*ln(c^2*x^2+1)-1/20*b/c
^4/x^4+1/5*b*ln(c*x)+1/10*b/c^2/x^2)
```

Maxima [A]
time = 0.26, size = 62, normalized size = 0.97

$$-\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\,\arctan\left(cx\right)}{x^5}\right)b-\frac{a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
4*arctan(c*x)/x^5)*b - 1/5*a/x^5
```

Fricas [A]
time = 1.30, size = 59, normalized size = 0.92

$$-\frac{2\,bc^5x^5\log\left(c^2x^2+1\right)-4\,bc^5x^5\log\left(x\right)-2\,bc^3x^3+bcx+4\,b\arctan\left(cx\right)+4\,a}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(2*b*c^5*x^5*log(c^2*x^2 + 1) - 4*b*c^5*x^5*log(x) - 2*b*c^3*x^3 + b*
c*x + 4*b*arctan(c*x) + 4*a)/x^5
```

Sympy [A]
time = 0.54, size = 71, normalized size = 1.11

$$\begin{cases} -\frac{a}{5x^5}+\frac{bc^5\log\left(x\right)}{5}-\frac{bc^5\log\left(x^2+\frac{1}{c^2}\right)}{10}+\frac{bc^3}{10x^2}-\frac{bc}{20x^4}-\frac{b\,\mathrm{atan}\left(cx\right)}{5x^5} & \text{for}\; c\neq 0\\ -\frac{a}{5x^5} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))/x**6,x)
```

```
[Out] Piecewise((-a/(5*x**5) + b*c**5*log(x)/5 - b*c**5*log(x**2 + c**(-2))/10 +
b*c**3/(10*x**2) - b*c/(20*x**4) - b*atan(c*x)/(5*x**5), Ne(c, 0)), (-a/(5*
x**5), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.37, size = 56, normalized size = 0.88

$$\frac{b\,c^5\,\ln\left(x\right)}{5}-\frac{b\,\mathrm{atan}(c\,x)}{5\,x^5}-\frac{b\,c^5\,\ln\left(c^2\,x^2+1\right)}{10}-\frac{-\frac{b\,c^3\,x^3}{2}+\frac{b\,c\,x}{4}+a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))/x^6,x)
```

```
[Out] (b*c^5*log(x))/5 - (b*atan(c*x))/(5*x^5) - (b*c^5*log(c^2*x^2 + 1))/10 - (a
 - (b*c^3*x^3)/2 + (b*c*x)/4)/(5*x^5)
```

3.13 $\int x^5(a+b\mathbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=144

$$-\frac{abx}{3c^5}-\frac{4b^2x^2}{45c^4}+\frac{b^2x^4}{60c^2}-\frac{b^2x\text{ArcTan}(cx)}{3c^5}+\frac{bx^3(a+b\text{ArcTan}(cx))}{9c^3}-\frac{bx^5(a+b\text{ArcTan}(cx))}{15c}+\frac{(a+b\text{ArcTan}(cx))^2}{6c^6}$$

```
[Out] -1/3*a*b*x/c^5-4/45*b^2*x^2/c^4+1/60*b^2*x^4/c^2-1/3*b^2*x*arctan(c*x)/c^5+
1/9*b*x^3*(a+b*arctan(c*x))/c^3-1/15*b*x^5*(a+b*arctan(c*x))/c+1/6*(a+b*arc
tan(c*x))^2/c^6+1/6*x^6*(a+b*arctan(c*x))^2+23/90*b^2*ln(c^2*x^2+1)/c^6
```

Rubi [A]
time = 0.22, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4946, 5036, 272, 45, 4930, 266, 5004}

$$\frac{(a+b\text{ArcTan}(cx))^2}{6c^6}+\frac{bx^3(a+b\text{ArcTan}(cx))}{9c^3}+\frac{1}{6}x^6(a+b\text{ArcTan}(cx))^2-\frac{bx^5(a+b\text{ArcTan}(cx))}{15c}-\frac{abx}{3c^5}-\frac{b^2x\text{ArcTan}(cx)}{3c^5}-\frac{4b^2x^2}{45c^4}+\frac{b^2x^4}{60c^2}+\frac{23b^2\log(c^2x^2+1)}{90c^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] -1/3*(a*b*x)/c^5 - (4*b^2*x^2)/(45*c^4) + (b^2*x^4)/(60*c^2) - (b^2*x*ArcTa
n[c*x])/(3*c^5) + (b*x^3*(a + b*ArcTan[c*x]))/(9*c^3) - (b*x^5*(a + b*ArcTa
n[c*x]))/(15*c) + (a + b*ArcTan[c*x])^2/(6*c^6) + (x^6*(a + b*ArcTan[c*x])^
2)/6 + (23*b^2*Log[1 + c^2*x^2])/(90*c^6)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
```

```
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\int x^5\left(a+b\tan^{-1}(cx)\right)^2\,dx=\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{3}(bc)\int\frac{x^6(a+b\tan^{-1}(cx))}{1+c^2x^2}\,dx$$

$$=\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^2-\frac{b\int x^4(a+b\tan^{-1}(cx))\,dx}{3c}+\frac{b\int\frac{x^4(a+b\tan^{-1}(cx))}{1+c^2x^2}\,dx}{3c}$$

$$=-\frac{bx^5(a+b\tan^{-1}(cx))}{15c}+\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{15}b^2\int\frac{x^5}{1+c^2x^2}\,dx+\frac{b\int$$

$$=\frac{bx^3(a+b\tan^{-1}(cx))}{9c^3}-\frac{bx^5(a+b\tan^{-1}(cx))}{15c}+\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{30}b$$

$$=-\frac{abx}{3c^5}+\frac{bx^3(a+b\tan^{-1}(cx))}{9c^3}-\frac{bx^5(a+b\tan^{-1}(cx))}{15c}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{6c^6}+$$

$$=-\frac{abx}{3c^5}-\frac{b^2x^2}{30c^4}+\frac{b^2x^4}{60c^2}-\frac{b^2x\tan^{-1}(cx)}{3c^5}+\frac{bx^3(a+b\tan^{-1}(cx))}{9c^3}-\frac{bx^5(a+b\,\mathrm{ta}}{15c}$$

$$=-\frac{abx}{3c^5}-\frac{4b^2x^2}{45c^4}+\frac{b^2x^4}{60c^2}-\frac{b^2x\tan^{-1}(cx)}{3c^5}+\frac{bx^3(a+b\tan^{-1}(cx))}{9c^3}-\frac{bx^5(a+b\,\mathrm{t}}{15}$$

Mathematica [A]
time = 0.09, size = 138, normalized size = 0.96

$$\frac{cx(30a^2c^5x^5+b^2cx(-16+3c^2x^2)-4ab(15-5c^2x^2+3c^4x^4))+4b(bcx(-15+5c^2x^2-3c^4x^4)+15a(1+c^6x^6))\operatorname{ArcTan}(cx)+30b^2(1+c^6x^6)\operatorname{ArcTan}(cx)^2+46b^2\log(1+c^2x^2)}{180c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (c*x*(30*a^2*c^5*x^5 + b^2*c*x*(-16 + 3*c^2*x^2) - 4*a*b*(15 - 5*c^2*x^2 +
3*c^4*x^4)) + 4*b*(b*c*x*(-15 + 5*c^2*x^2 - 3*c^4*x^4) + 15*a*(1 + c^6*x^6)
)*ArcTan[c*x] + 30*b^2*(1 + c^6*x^6)*ArcTan[c*x]^2 + 46*b^2*Log[1 + c^2*x^2
])/(180*c^6)
```

Maple [A]
time = 0.33, size = 171, normalized size = 1.19

method	result
derivativedivides	$\frac{\frac{c^6x^6a^2}{6}+\frac{b^2c^6x^6\arctan(cx)^2}{6}-\frac{b^2\arctan(cx)c^5x^5}{15}+\frac{b^2\arctan(cx)c^3x^3}{9}-\frac{b^2\arctan(cx)cx}{3}+\frac{b^2\arctan(cx)^2}{6}+\frac{b^2c^4x^4}{60}-\frac{4b^2c^2x^2}{45}+\frac{23b^2}{}}{c^6}$
default	$\frac{\frac{c^6x^6a^2}{6}+\frac{b^2c^6x^6\arctan(cx)^2}{6}-\frac{b^2\arctan(cx)c^5x^5}{15}+\frac{b^2\arctan(cx)c^3x^3}{9}-\frac{b^2\arctan(cx)cx}{3}+\frac{b^2\arctan(cx)^2}{6}+\frac{b^2c^4x^4}{60}-\frac{4b^2c^2x^2}{45}+\frac{23b^2}{}}{c^6}$
risch	$-\frac{b^2(c^6x^6+1)\ln(icx+1)^2}{24c^6}-\frac{ib(30c^6x^6a+15ib\,c^6x^6\ln(-icx+1)-6b\,c^5x^5+10b\,c^3x^3-30xbc+15ib\ln(-icx+1))\ln(icx+1)}{180c^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^6*(1/6*c^6*x^6*a^2+1/6*b^2*c^6*x^6*arctan(c*x)^2-1/15*b^2*arctan(c*x)*c
^5*x^5+1/9*b^2*arctan(c*x)*c^3*x^3-1/3*b^2*arctan(c*x)*c*x+1/6*b^2*arctan(c
*x)^2+1/60*b^2*c^4*x^4-4/45*b^2*c^2*x^2+23/90*b^2*ln(c^2*x^2+1)+1/3*a*b*c^6
*x^6*arctan(c*x)-1/15*c^5*x^5*a*b+1/9*a*b*c^3*x^3-1/3*a*b*c*x+1/3*a*b*arcta
n(c*x))
```

Maxima [A]
time = 0.48, size = 163, normalized size = 1.13

$$\frac{1}{6}b^2x^6\arctan(cx)^2+\frac{1}{6}a^2x^6+\frac{1}{45}\left(15x^6\arctan(cx)-c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\right)ab-\frac{1}{180}\left(4c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\arctan(cx)-\frac{3c^4x^4-16c^2x^2-30\arctan(cx)^2+46\log(c^2x^2+1)}{c^6}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6*arctan(c*x)^2 + 1/6*a^2*x^6 + 1/45*(15*x^6*arctan(c*x) - c*((3*
c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*a*b - 1/180*(4*c*((3
*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7)*arctan(c*x) - (3*c^4
*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^2 + 46*log(c^2*x^2 + 1))/c^6)*b^2
```

Fricas [A]
time = 0.83, size = 152, normalized size = 1.06

$$\frac{30\,a^2c^6x^6 - 12\,abc^5x^5 + 3\,b^2c^4x^4 + 20\,abc^3x^3 - 16\,b^2c^2x^2 - 60\,abcx + 30\,(b^2c^6x^6 + b^2)\arctan(cx)^2 + 46\,b^2\log(c^2x^2 + 1) + 4\,(15\,abc^6x^6 - 3\,b^2c^5x^5 + 5\,b^2c^3x^3 - 15\,b^2cx + 15\,ab)\arctan(cx)}{180\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/180*(30*a^2*c^6*x^6 - 12*a*b*c^5*x^5 + 3*b^2*c^4*x^4 + 20*a*b*c^3*x^3 - 1
6*b^2*c^2*x^2 - 60*a*b*c*x + 30*(b^2*c^6*x^6 + b^2)*arctan(c*x)^2 + 46*b^2*
log(c^2*x^2 + 1) + 4*(15*a*b*c^6*x^6 - 3*b^2*c^5*x^5 + 5*b^2*c^3*x^3 - 15*b
^2*c*x + 15*a*b)*arctan(c*x))/c^6
```

Sympy [A]
time = 0.46, size = 199, normalized size = 1.38

$$\begin{cases} \frac{a^2x^6}{6} + \frac{abx^6\operatorname{atan}(cx)}{3} - \frac{abx^5}{15c} + \frac{abx^3}{9c^3} - \frac{abx}{3c^5} + \frac{ab\operatorname{atan}(cx)}{3c^6} + \frac{b^2x^6\operatorname{atan}^2(cx)}{6} - \frac{b^2x^5\operatorname{atan}(cx)}{15c} + \frac{b^2x^4}{60c^2} + \frac{b^2x^3\operatorname{atan}(cx)}{9c^3} - \frac{4b^2x^2}{45c^4} - \frac{b^2x\operatorname{atan}(cx)}{3c^5} + \frac{23b^2\log\left(x^2+\frac{1}{c^2}\right)}{90c^6} + \frac{b^2\operatorname{atan}^2(cx)}{6c^6} & \text{for}\ c \neq 0 \\ \frac{a^2x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**6/6 + a*b*x**6*atan(c*x)/3 - a*b*x**5/(15*c) + a*b*x**3/
(9*c**3) - a*b*x/(3*c**5) + a*b*atan(c*x)/(3*c**6) + b**2*x**6*atan(c*x)**2
/6 - b**2*x**5*atan(c*x)/(15*c) + b**2*x**4/(60*c**2) + b**2*x**3*atan(c*x)
/(9*c**3) - 4*b**2*x**2/(45*c**4) - b**2*x*atan(c*x)/(3*c**5) + 23*b**2*log
(x**2 + c**(-2))/(90*c**6) + b**2*atan(c*x)**2/(6*c**6), Ne(c, 0)), (a**2*x
**6/6, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.67, size = 171, normalized size = 1.19

$$\frac{30\,b^2\operatorname{atan}(c\,x)^2 + 46\,b^2\,\ln(c^2\,x^2 + 1) + 30\,a^2\,c^6\,x^6 - 16\,b^2\,c^2\,x^2 + 3\,b^2\,c^4\,x^4 + 60\,a\,b\operatorname{atan}(c\,x) + 20\,b^2\,c^3\,x^3\operatorname{atan}(c\,x) - 12\,b^2\,c^5\,x^5\operatorname{atan}(c\,x) - 60\,b^2\,c\,x\operatorname{atan}(c\,x) + 30\,b^2\,c^6\,x^6\operatorname{atan}(c\,x)^2 + 20\,a\,b\,c^3\,x^3 - 12\,a\,b\,c^5\,x^5 - 60\,a\,b\,c\,x + 60\,a\,b\,c^6\,x^6\operatorname{atan}(c\,x)}{180\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atan(c*x))^2,x)
```

```
[Out] (30*b^2*atan(c*x)^2 + 46*b^2*log(c^2*x^2 + 1) + 30*a^2*c^6*x^6 - 16*b^2*c^2
*x^2 + 3*b^2*c^4*x^4 + 60*a*b*atan(c*x) + 20*b^2*c^3*x^3*atan(c*x) - 12*b^2
*c^5*x^5*atan(c*x) - 60*b^2*c*x*atan(c*x) + 30*b^2*c^6*x^6*atan(c*x)^2 + 20
*a*b*c^3*x^3 - 12*a*b*c^5*x^5 - 60*a*b*c*x + 60*a*b*c^6*x^6*atan(c*x))/(180
*c^6)
```

3.14 $\int x^4(a+b\mathbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=170

$$-\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}+\frac{3b^2\text{ArcTan}(cx)}{10c^5}+\frac{bx^2(a+b\text{ArcTan}(cx))}{5c^3}-\frac{bx^4(a+b\text{ArcTan}(cx))}{10c}+\frac{i(a+b\text{ArcTan}(cx))^2}{5c^5}+\frac{1}{5}x$$

```
[Out] -3/10*b^2*x/c^4+1/30*b^2*x^3/c^2+3/10*b^2*arctan(c*x)/c^5+1/5*b*x^2*(a+b*ar
ctan(c*x))/c^3-1/10*b*x^4*(a+b*arctan(c*x))/c+1/5*I*(a+b*arctan(c*x))^2/c^5
+1/5*x^5*(a+b*arctan(c*x))^2+2/5*b*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^5+1/
5*I*b^2*polylog(2,1-2/(1+I*c*x))/c^5
```

Rubi [A]
time = 0.21, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {4946, 5036, 308, 209, 327, 5040, 4964, 2449, 2352}

$$\frac{i(a+b\text{ArcTan}(cx))^2}{5c^5}+\frac{2b\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{5c^5}+\frac{bx^2(a+b\text{ArcTan}(cx))}{5c^3}+\frac{1}{5}x^5(a+b\text{ArcTan}(cx))^2-\frac{bx^4(a+b\text{ArcTan}(cx))}{10c}+\frac{3b^2\text{ArcTan}(cx)}{10c^5}+\frac{ib^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{5c^5}-\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (-3*b^2*x)/(10*c^4) + (b^2*x^3)/(30*c^2) + (3*b^2*ArcTan[c*x])/(10*c^5) + (
b*x^2*(a + b*ArcTan[c*x]))/(5*c^3) - (b*x^4*(a + b*ArcTan[c*x]))/(10*c) + (
(I/5)*(a + b*ArcTan[c*x])^2)/c^5 + (x^5*(a + b*ArcTan[c*x])^2)/5 + (2*b*(a
+ b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(5*c^5) + ((I/5)*b^2*PolyLog[2, 1 - 2/
(1 + I*c*x)])/c^5
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x^4\left(a+b\tan^{-1}(cx)\right)^2\,dx=\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{5}(2bc)\int\frac{x^5(a+b\tan^{-1}(cx))}{1+c^2x^2}\,dx$$

$$=\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)^2-\frac{(2b)\int x^3(a+b\tan^{-1}(cx))\,dx}{5c}+\frac{(2b)\int\frac{x^3(a+b\tan^{-1}(c}{1+c^2x^2}}{5c}$$

$$=-\frac{bx^4(a+b\tan^{-1}(cx))}{10c}+\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{10}b^2\int\frac{x^4}{1+c^2x^2}\,dx+\frac{(2b}{}$$

$$=\frac{bx^2(a+b\tan^{-1}(cx))}{5c^3}-\frac{bx^4(a+b\tan^{-1}(cx))}{10c}+\frac{i(a+b\tan^{-1}(cx))^2}{5c^5}+\frac{1}{5}x^5(a$$

$$=-\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}+\frac{bx^2(a+b\tan^{-1}(cx))}{5c^3}-\frac{bx^4(a+b\tan^{-1}(cx))}{10c}+\frac{i(a+b\tan^{-}}{5c^5}$$

$$=-\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}+\frac{3b^2\tan^{-1}(cx)}{10c^5}+\frac{bx^2(a+b\tan^{-1}(cx))}{5c^3}-\frac{bx^4(a+b\tan^{-1}(cx}{10c}$$

$$=-\frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}+\frac{3b^2\tan^{-1}(cx)}{10c^5}+\frac{bx^2(a+b\tan^{-1}(cx))}{5c^3}-\frac{bx^4(a+b\tan^{-1}(cx}{10c}$$

Mathematica [A]
time = 0.36, size = 169, normalized size = 0.99

$$\frac{9ab-9b^2cx+6abc^2x^2+b^2c^3x^3-3abc^4x^4+6a^2c^5x^5+6b^2(-i+c^5x^5)\operatorname{ArcTan}(cx)^2-3b\operatorname{ArcTan}(cx)\left(-4ac^5x^5+b(-3-2c^2x^2+c^4x^4)-4b\log\left(1+e^{2i\operatorname{ArcTan}(cx)}\right)\right)-6ab\log\left(1+c^2x^2\right)-6ib^2\operatorname{PolyLog}\left(2,-e^{2i\operatorname{ArcTan}(cx)}\right)}{30c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (9*a*b - 9*b^2*c*x + 6*a*b*c^2*x^2 + b^2*c^3*x^3 - 3*a*b*c^4*x^4 + 6*a^2*c^
5*x^5 + 6*b^2*(-I + c^5*x^5)*ArcTan[c*x]^2 - 3*b*ArcTan[c*x]*(-4*a*c^5*x^5
+ b*(-3 - 2*c^2*x^2 + c^4*x^4) - 4*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - 6*a*
b*Log[1 + c^2*x^2] - (6*I)*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(30*c^5)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(148) = 296.
time = 0.47, size = 312, normalized size = 1.84

method	result
derivativedivides	$\frac{c^5x^5a^2}{5}+\frac{c^5x^5b^2\arctan(cx)^2}{5}-\frac{b^2\arctan(cx)c^4x^4}{10}+\frac{b^2\arctan(cx)c^2x^2}{5}-\frac{b^2\arctan(cx)\ln\left(c^2x^2+1\right)}{5}+\frac{b^2c^3x^3}{30}-\frac{3b^2cx}{10}+\frac{3b^2\arctan(}{10}$
default	$\frac{c^5x^5a^2}{5}+\frac{c^5x^5b^2\arctan(cx)^2}{5}-\frac{b^2\arctan(cx)c^4x^4}{10}+\frac{b^2\arctan(cx)c^2x^2}{5}-\frac{b^2\arctan(cx)\ln\left(c^2x^2+1\right)}{5}+\frac{b^2c^3x^3}{30}-\frac{3b^2cx}{10}+\frac{3b^2\arctan(}{10}$

risch	$-\frac{b^2\ln(-icx+1)^2x^5}{20}-\frac{b^2\ln(icx+1)^2x^5}{20}+\frac{413ib^2}{2250c^5}+\frac{ia^2}{5c^5}+\frac{a^2x^5}{5}+\frac{137ab}{150c^5}-\frac{ab\,x^4}{10c}+\frac{ib^2\,\mathrm{dilog}\left(\frac{1}{2}-\frac{icx}{2}\right)}{5c^5}+\frac{ib^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(1/5*c^5*x^5*a^2+1/5*c^5*x^5*b^2*arctan(c*x)^2-1/10*b^2*arctan(c*x)*c
^4*x^4+1/5*b^2*arctan(c*x)*c^2*x^2-1/5*b^2*arctan(c*x)*ln(c^2*x^2+1)+1/30*b
^2*c^3*x^3-3/10*b^2*c*x+3/10*b^2*arctan(c*x)-1/20*I*b^2*ln(c*x+I)^2+1/20*I*
b^2*ln(c*x-I)^2-1/10*I*b^2*dilog(1/2*I*(c*x-I))+1/10*I*b^2*ln(c*x-I)*ln(-1/
2*I*(c*x+I))+1/10*I*b^2*ln(c*x+I)*ln(c^2*x^2+1)+1/10*I*b^2*dilog(-1/2*I*(c*
x+I))-1/10*I*b^2*ln(c*x-I)*ln(c^2*x^2+1)-1/10*I*b^2*ln(c*x+I)*ln(1/2*I*(c*x
-I))+2/5*c^5*x^5*a*b*arctan(c*x)-1/10*c^4*x^4*a*b+1/5*a*b*c^2*x^2-1/5*a*b*l
n(c^2*x^2+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*x^5 + 1/10*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^
2*x^2 + 1)/c^6))*a*b + 1/80*(4*x^5*arctan(c*x)^2 - x^5*log(c^2*x^2 + 1)^2 +
 80*integrate(1/80*(4*c^2*x^6*log(c^2*x^2 + 1) - 8*c*x^5*arctan(c*x) + 60*(
c^2*x^6 + x^4)*arctan(c*x)^2 + 5*(c^2*x^6 + x^4)*log(c^2*x^2 + 1)^2)/(c^2*x
^2 + 1), x))*b^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^4*arctan(c*x)^2 + 2*a*b*x^4*arctan(c*x) + a^2*x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4(a+b\,\mathrm{atan}\,(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral(x**4*(a + b*atan(c*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atan(c*x))^2,x)
```

```
[Out] int(x^4*(a + b*atan(c*x))^2, x)
```

3.15 $\int x^3(a+b\mathbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=112

$$\frac{abx}{2c^3}+\frac{b^2x^2}{12c^2}+\frac{b^2x\text{ArcTan}(cx)}{2c^3}-\frac{bx^3(a+b\text{ArcTan}(cx))}{6c}-\frac{(a+b\text{ArcTan}(cx))^2}{4c^4}+\frac{1}{4}x^4(a+b\text{ArcTan}(cx))^2-\frac{b^2\log(1}{3}$$

```
[Out] 1/2*a*b*x/c^3+1/12*b^2*x^2/c^2+1/2*b^2*x*arctan(c*x)/c^3-1/6*b*x^3*(a+b*arc
tan(c*x))/c-1/4*(a+b*arctan(c*x))^2/c^4+1/4*x^4*(a+b*arctan(c*x))^2-1/3*b^2
*ln(c^2*x^2+1)/c^4
```

Rubi [A]
time = 0.15, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4946, 5036, 272, 45, 4930, 266, 5004}

$$-\frac{(a+b\text{ArcTan}(cx))^2}{4c^4}+\frac{1}{4}x^4(a+b\text{ArcTan}(cx))^2-\frac{bx^3(a+b\text{ArcTan}(cx))}{6c}+\frac{abx}{2c^3}+\frac{b^2x\text{ArcTan}(cx)}{2c^3}+\frac{b^2x^2}{12c^2}-\frac{b^2\log(c^2x^2+1)}{3c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (a*b*x)/(2*c^3) + (b^2*x^2)/(12*c^2) + (b^2*x*ArcTan[c*x])/(2*c^3) - (b*x^3
*(a + b*ArcTan[c*x]))/(6*c) - (a + b*ArcTan[c*x])^2/(4*c^4) + (x^4*(a + b*A
rcTan[c*x])^2)/4 - (b^2*Log[1 + c^2*x^2])/(3*c^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
```

```
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx &= \frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{2}(bc)\int\frac{x^4(a+b\tan^{-1}(cx))}{1+c^2x^2}\,dx\\
&= \frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{b\int x^2(a+b\tan^{-1}(cx))\,dx}{2c}+\frac{b\int\frac{x^2(a+b\tan^{-1}(cx))}{1+c^2x^2}\,dx}{2c}\\
&= -\frac{bx^3(a+b\tan^{-1}(cx))}{6c}+\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{6}b^2\int\frac{x^3}{1+c^2x^2}\,dx+\frac{b\int(\\
&= \frac{abx}{2c^3}-\frac{bx^3(a+b\tan^{-1}(cx))}{6c}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4c^4}+\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^2+\\
&= \frac{abx}{2c^3}+\frac{b^2x\tan^{-1}(cx)}{2c^3}-\frac{bx^3(a+b\tan^{-1}(cx))}{6c}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4c^4}+\frac{1}{4}x^4(a-\\
&= \frac{abx}{2c^3}+\frac{b^2x^2}{12c^2}+\frac{b^2x\tan^{-1}(cx)}{2c^3}-\frac{bx^3(a+b\tan^{-1}(cx))}{6c}-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{4c^4}+
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 111, normalized size = 0.99

$$\frac{cx(6ab+b^2cx-2abc^2x^2+3a^2c^3x^3)-2b(bcx(-3+c^2x^2)+a(3-3c^4x^4))\operatorname{ArcTan}(cx)+3b^2(-1+c^4x^4)\operatorname{ArcTan}(cx)^2-4b^2\log\left(1+c^2x^2\right)}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (c*x*(6*a*b + b^2*c*x - 2*a*b*c^2*x^2 + 3*a^2*c^3*x^3) - 2*b*(b*c*x*(-3 + c
^2*x^2) + a*(3 - 3*c^4*x^4))*ArcTan[c*x] + 3*b^2*(-1 + c^4*x^4)*ArcTan[c*x]
^2 - 4*b^2*Log[1 + c^2*x^2])/(12*c^4)
```

Maple [A]
time = 0.16, size = 135, normalized size = 1.21

method	result
derivativedivides	$\frac{\frac{a^2c^4x^4}{4}+\frac{b^2c^4x^4\arctan(cx)^2}{4}-\frac{b^2\arctan(cx)c^3x^3}{6}+\frac{b^2\arctan(cx)cx}{2}-\frac{b^2\arctan(cx)^2}{4}+\frac{b^2c^2x^2}{12}-\frac{b^2\ln\left(c^2x^2+1\right)}{3}+\frac{c^4x^4ab\arctan(cx)}{2}}{c^4}$
default	$\frac{\frac{a^2c^4x^4}{4}+\frac{b^2c^4x^4\arctan(cx)^2}{4}-\frac{b^2\arctan(cx)c^3x^3}{6}+\frac{b^2\arctan(cx)cx}{2}-\frac{b^2\arctan(cx)^2}{4}+\frac{b^2c^2x^2}{12}-\frac{b^2\ln\left(c^2x^2+1\right)}{3}+\frac{c^4x^4ab\arctan(cx)}{2}}{c^4}$
risch	$-\frac{b^2(c^4x^4-1)\ln(icx+1)^2}{16c^4}-\frac{ib(6c^4x^4a+3ib\,c^4x^4\ln(-icx+1)-2b\,c^3x^3+6xbc-3ib\ln(-icx+1))\ln(icx+1)}{24c^4}-\frac{b^2x^4\ln(-}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/4*a^2*c^4*x^4+1/4*b^2*c^4*x^4*arctan(c*x)^2-1/6*b^2*arctan(c*x)*c^
3*x^3+1/2*b^2*arctan(c*x)*c*x-1/4*b^2*arctan(c*x)^2+1/12*b^2*c^2*x^2-1/3*b^
2*ln(c^2*x^2+1)+1/2*c^4*x^4*a*b*arctan(c*x)-1/6*a*b*c^3*x^3+1/2*a*b*c*x-1/2
*a*b*arctan(c*x))
```

Maxima [A]
time = 0.47, size = 136, normalized size = 1.21

$$\frac{1}{4}b^2x^4\arctan(cx)^2+\frac{1}{4}a^2x^4+\frac{1}{6}\left(3x^4\arctan(cx)-c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)ab-\frac{1}{12}\left(2c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\arctan(cx)-\frac{c^2x^2+3\arctan(cx)^2-4\log(c^2x^2+1)}{c^4}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctan(c*x)^2 + 1/4*a^2*x^4 + 1/6*(3*x^4*arctan(c*x) - c*((c^2*
x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4 +
 3*arctan(c*x)/c^5)*arctan(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^
2 + 1))/c^4)*b^2
```

Fricas [A]
time = 0.96, size = 121, normalized size = 1.08

$$\frac{3a^2c^4x^4-2abc^3x^3+b^2c^2x^2+6abcx+3(b^2c^4x^4-b^2)\arctan(cx)^2-4b^2\log(c^2x^2+1)+2(3abc^4x^4-b^2c^3x^3+3b^2cx-3ab)\arctan(cx)}{12c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a^2*c^4*x^4 - 2*a*b*c^3*x^3 + b^2*c^2*x^2 + 6*a*b*c*x + 3*(b^2*c^4*
x^4 - b^2)*arctan(c*x)^2 - 4*b^2*log(c^2*x^2 + 1) + 2*(3*a*b*c^4*x^4 - b^2*
c^3*x^3 + 3*b^2*c*x - 3*a*b)*arctan(c*x))/c^4
```

Sympy [A]
time = 0.32, size = 155, normalized size = 1.38

$$\begin{cases} \frac{a^2x^4}{4} + \frac{abx^4\operatorname{atan}(cx)}{2} - \frac{abx^3}{6c} + \frac{abx}{2c^3} - \frac{ab\operatorname{atan}(cx)}{2c^4} + \frac{b^2x^4\operatorname{atan}^2(cx)}{4} - \frac{b^2x^3\operatorname{atan}(cx)}{6c} + \frac{b^2x^2}{12c^2} + \frac{b^2x\operatorname{atan}(cx)}{2c^3} - \frac{b^2\log\left(x^2+\frac{1}{c^2}\right)}{3c^4} - \frac{b^2\operatorname{atan}^2(cx)}{4c^4} & \text{for}\ c \neq 0 \\ \frac{a^2x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**4/4 + a*b*x**4*atan(c*x)/2 - a*b*x**3/(6*c) + a*b*x/(2*c
**3) - a*b*atan(c*x)/(2*c**4) + b**2*x**4*atan(c*x)**2/4 - b**2*x**3*atan(c
*x)/(6*c) + b**2*x**2/(12*c**2) + b**2*x*atan(c*x)/(2*c**3) - b**2*log(x**2
 + c**(-2))/(3*c**4) - b**2*atan(c*x)**2/(4*c**4), Ne(c, 0)), (a**2*x**4/4,
 True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.32, size = 134, normalized size = 1.20

$$\frac{3\,a^2\,c^4\,x^4-4\,b^2\,\ln\left(c^2\,x^2+1\right)-3\,b^2\,\mathrm{atan}\left(c\,x\right)^2+b^2\,c^2\,x^2-6\,a\,b\,\mathrm{atan}\left(c\,x\right)-2\,b^2\,c^3\,x^3\,\mathrm{atan}\left(c\,x\right)+6\,b^2\,c\,x\,\mathrm{atan}\left(c\,x\right)+3\,b^2\,c^4\,x^4\,\mathrm{atan}\left(c\,x\right)^2-2\,a\,b\,c^3\,x^3+6\,a\,b\,c\,x+6\,a\,b\,c^4\,x^4\,\mathrm{atan}\left(c\,x\right)}{12\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x))^2,x)
```

```
[Out] (3*a^2*c^4*x^4 - 4*b^2*log(c^2*x^2 + 1) - 3*b^2*atan(c*x)^2 + b^2*c^2*x^2 -
 6*a*b*atan(c*x) - 2*b^2*c^3*x^3*atan(c*x) + 6*b^2*c*x*atan(c*x) + 3*b^2*c^
4*x^4*atan(c*x)^2 - 2*a*b*c^3*x^3 + 6*a*b*c*x + 6*a*b*c^4*x^4*atan(c*x))/(1
2*c^4)
```

3.16 $\int x^2(a+b\mathbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=138

$$\frac{b^2x}{3c^2}-\frac{b^2\text{ArcTan}(cx)}{3c^3}-\frac{bx^2(a+b\text{ArcTan}(cx))}{3c}-\frac{i(a+b\text{ArcTan}(cx))^2}{3c^3}+\frac{1}{3}x^3(a+b\text{ArcTan}(cx))^2-\frac{2b(a+b\text{ArcTa}}{}$$

```
[Out] 1/3*b^2*x/c^2-1/3*b^2*arctan(c*x)/c^3-1/3*b*x^2*(a+b*arctan(c*x))/c-1/3*I*(
a+b*arctan(c*x))^2/c^3+1/3*x^3*(a+b*arctan(c*x))^2-2/3*b*(a+b*arctan(c*x))*
ln(2/(1+I*c*x))/c^3-1/3*I*b^2*polylog(2,1-2/(1+I*c*x))/c^3
```

Rubi [A]
time = 0.14, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4946, 5036, 327, 209, 5040, 4964, 2449, 2352}

$$-\frac{i(a+b\text{ArcTan}(cx))^2}{3c^3}-\frac{2b\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{3c^3}+\frac{1}{3}x^3(a+b\text{ArcTan}(cx))^2-\frac{bx^2(a+b\text{ArcTan}(cx))}{3c}-\frac{b^2\text{ArcTan}(cx)}{3c^3}-\frac{ib^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{3c^3}+\frac{b^2x}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (b^2*x)/(3*c^2) - (b^2*ArcTan[c*x])/(3*c^3) - (b*x^2*(a + b*ArcTan[c*x]))/(
3*c) - ((I/3)*(a + b*ArcTan[c*x])^2)/c^3 + (x^3*(a + b*ArcTan[c*x])^2)/3 -
(2*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) - ((I/3)*b^2*PolyLog[2
, 1 - 2/(1 + I*c*x)])/c^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2\left(a+b\tan^{-1}(cx)\right)^2\,dx=\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{1}{3}(2bc)\int\frac{x^3\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx$$

$$=\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{(2b)\int x\left(a+b\tan^{-1}(cx)\right)\,dx}{3c}+\frac{(2b)\int\frac{x\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,d}{3c}$$

$$=-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^2+\frac{1}{3}b^2$$

$$=\frac{b^2x}{3c^2}-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^2-$$

$$=\frac{b^2x}{3c^2}-\frac{b^2\tan^{-1}(cx)}{3c^3}-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}+\frac{1}{3}x^3(a+b$$

$$=\frac{b^2x}{3c^2}-\frac{b^2\tan^{-1}(cx)}{3c^3}-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)}{3c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^2}{3c^3}+\frac{1}{3}x^3(a+b$$

Mathematica [A]
time = 0.20, size = 131, normalized size = 0.95

$$\frac{b^2cx-abc^2x^2+a^2c^3x^3+b^2(i+c^3x^3)\,\text{ArcTan}(cx)^2-b\text{ArcTan}(cx)\left(b+bc^2x^2-2ac^3x^3+2b\log\left(1+e^{2i\text{ArcTan}(cx)}\right)\right)+ab\log\left(1+c^2x^2\right)+ib^2\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (b^2*c*x - a*b*c^2*x^2 + a^2*c^3*x^3 + b^2*(I + c^3*x^3)*ArcTan[c*x]^2 - b*
ArcTan[c*x]*(b + b*c^2*x^2 - 2*a*c^3*x^3 + 2*b*Log[1 + E^((2*I)*ArcTan[c*x]
)]) + a*b*Log[1 + c^2*x^2] + I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(3*c
^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(120) = 240.
time = 0.34, size = 276, normalized size = 2.00

method	result
derivativedivides	$\frac{c^3x^3a^2}{3}+\frac{b^2c^3x^3\arctan(cx)^2}{3}-\frac{b^2\arctan(cx)c^2x^2}{3}+\frac{b^2\arctan(cx)\ln\left(c^2x^2+1\right)}{3}+\frac{b^2cx}{3}-\frac{b^2\arctan(cx)}{3}+\frac{ib^2\ln(cx+i)\ln\left(\frac{i(cx-i)}{2}\right)}{6}+i$
default	$\frac{c^3x^3a^2}{3}+\frac{b^2c^3x^3\arctan(cx)^2}{3}-\frac{b^2\arctan(cx)c^2x^2}{3}+\frac{b^2\arctan(cx)\ln\left(c^2x^2+1\right)}{3}+\frac{b^2cx}{3}-\frac{b^2\arctan(cx)}{3}+\frac{ib^2\ln(cx+i)\ln\left(\frac{i(cx-i)}{2}\right)}{6}+i$
risch	$\frac{ib^2\ln(-icx+1)^2}{12c^3}-\frac{ia^2}{3c^3}-\frac{iba\ln(icx+1)x^3}{3}+\frac{a^2x^3}{3}-\frac{17ib^2}{54c^3}+\frac{b^2\ln(icx+1)\ln(-icx+1)x^3}{6}+\frac{ib^2\ln(icx+1)x^2}{6c}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/3*c^3*x^3*a^2+1/3*b^2*c^3*x^3*arctan(c*x)^2-1/3*b^2*arctan(c*x)*c^
2*x^2+1/3*b^2*arctan(c*x)*ln(c^2*x^2+1)+1/3*b^2*c*x-1/3*b^2*arctan(c*x)+1/6
*I*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))+1/12*I*b^2*ln(c*x+I)^2-1/12*I*b^2*ln(c*x
-I)^2-1/6*I*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/6*I*b^2*dilog(-1/2*I*(c*x+I)
)-1/6*I*b^2*ln(c*x+I)*ln(c^2*x^2+1)+1/6*I*b^2*dilog(1/2*I*(c*x-I))+1/6*I*b^
2*ln(c*x-I)*ln(c^2*x^2+1)+2/3*a*b*c^3*x^3*arctan(c*x)-1/3*a*b*c^2*x^2+1/3*a
*b*ln(c^2*x^2+1))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*
a*b + 1/48*(4*x^3*arctan(c*x)^2 - x^3*log(c^2*x^2 + 1)^2 + 48*integrate(1/4
8*(4*c^2*x^4*log(c^2*x^2 + 1) - 8*c*x^3*arctan(c*x) + 36*(c^2*x^4 + x^2)*ar
ctan(c*x)^2 + 3*(c^2*x^4 + x^2)*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x))*b^2
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctan(c*x)^2 + 2*a*b*x^2*arctan(c*x) + a^2*x^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a+b\operatorname{atan}(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 (a + b \operatorname{atan}(c x))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x))^2,x)
```

```
[Out] int(x^2*(a + b*atan(c*x))^2, x)
```

3.17 $\int x(a+b\mathbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=76

$$-\frac{abx}{c}-\frac{b^2x\text{ArcTan}(cx)}{c}+\frac{(a+b\text{ArcTan}(cx))^2}{2c^2}+\frac{1}{2}x^2(a+b\text{ArcTan}(cx))^2+\frac{b^2\log\left(1+c^2x^2\right)}{2c^2}$$

```
[Out] -a*b*x/c-b^2*x*arctan(c*x)/c+1/2*(a+b*arctan(c*x))^2/c^2+1/2*x^2*(a+b*arcta
n(c*x))^2+1/2*b^2*ln(c^2*x^2+1)/c^2
```

Rubi [A]
time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.417$, Rules used = {4946, 5036, 4930, 266, 5004}

$$\frac{(a+b\text{ArcTan}(cx))^2}{2c^2}+\frac{1}{2}x^2(a+b\text{ArcTan}(cx))^2-\frac{abx}{c}-\frac{b^2x\text{ArcTan}(cx)}{c}+\frac{b^2\log\left(c^2x^2+1\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] -((a*b*x)/c) - (b^2*x*ArcTan[c*x])/c + (a + b*ArcTan[c*x])^2/(2*c^2) + (x^2
*(a + b*ArcTan[c*x])^2)/2 + (b^2*Log[1 + c^2*x^2])/(2*c^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
```

```
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\tan^{-1}(cx)\right)^2\,dx &= \frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^2-(bc)\int\frac{x^2\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\,dx\\ &= \frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{b\int\left(a+b\tan^{-1}(cx)\right)\,dx}{c}+\frac{b\int\frac{a+b\tan^{-1}(cx)}{1+c^2x^2}\,dx}{c}\\ &= -\frac{abx}{c}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{b^2\int\tan^{-1}(cx)\,dx}{c}\\ &= -\frac{abx}{c}-\frac{b^2x\tan^{-1}(cx)}{c}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^2+b^2\int\frac{}{1}\\ &= -\frac{abx}{c}-\frac{b^2x\tan^{-1}(cx)}{c}+\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^2+\frac{b^2\log(}{}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 75, normalized size = 0.99

$$\frac{acx(-2b+acx)+2b(a-bcx+ac^2x^2)\operatorname{ArcTan}(cx)+b^2(1+c^2x^2)\operatorname{ArcTan}(cx)^2+b^2\log\left(1+c^2x^2\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (a*c*x*(-2*b + a*c*x) + 2*b*(a - b*c*x + a*c^2*x^2)*ArcTan[c*x] + b^2*(1 +
c^2*x^2)*ArcTan[c*x]^2 + b^2*Log[1 + c^2*x^2])/(2*c^2)
```

Maple [A]
time = 0.19, size = 97, normalized size = 1.28

method	result
derivativedivides	$\frac{\frac{c^2x^2a^2}{2}+\frac{b^2c^2x^2\arctan(cx)^2}{2}+\frac{b^2\arctan(cx)^2}{2}-b^2\arctan(cx)cx+\frac{b^2\ln\left(c^2x^2+1\right)}{2}+ab\,c^2x^2\arctan(cx)+ab\arctan(cx)-abcx}{c^2}$

default	$\frac{\frac{c^2x^2a^2}{2}+\frac{b^2c^2x^2\arctan(cx)^2}{2}+\frac{b^2\arctan(cx)^2}{2}-b^2\arctan(cx)cx+\frac{b^2\ln(c^2x^2+1)}{2}+ab\,c^2x^2\arctan(cx)+ab\arctan(cx)-abcx}{c^2}$
risch	$-\frac{b^2(c^2x^2+1)\ln(icx+1)^2}{8c^2}-\frac{ib(2c^2x^2a+ib\,c^2x^2\ln(-icx+1)-2xbc+ib\ln(-icx+1))\ln(icx+1)}{4c^2}-\frac{b^2x^2\ln(-icx+1)^2}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/2*c^2*x^2*a^2+1/2*b^2*c^2*x^2*arctan(c*x)^2+1/2*b^2*arctan(c*x)^2-
b^2*arctan(c*x)*c*x+1/2*b^2*ln(c^2*x^2+1)+a*b*c^2*x^2*arctan(c*x)+a*b*arcta
n(c*x)-a*b*c*x)
```

Maxima [A]
time = 0.47, size = 104, normalized size = 1.37

$$\frac{1}{2}b^2x^2\arctan(cx)^2+\frac{1}{2}a^2x^2+\left(x^2\arctan(cx)-c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)ab-\frac{1}{2}\left(2c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\arctan(cx)+\frac{\arctan(cx)^2-\log(c^2x^2+1)}{c^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arctan(c*x)^2 + 1/2*a^2*x^2 + (x^2*arctan(c*x) - c*(x/c^2 - arc
tan(c*x)/c^3))*a*b - 1/2*(2*c*(x/c^2 - arctan(c*x)/c^3)*arctan(c*x) + (arct
an(c*x)^2 - log(c^2*x^2 + 1))/c^2)*b^2
```

Fricas [A]
time = 0.77, size = 83, normalized size = 1.09

$$\frac{a^2c^2x^2-2\,abcx+(b^2c^2x^2+b^2)\arctan(cx)^2+b^2\log(c^2x^2+1)+2\,(abc^2x^2-b^2cx+ab)\arctan(cx)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*c^2*x^2 - 2*a*b*c*x + (b^2*c^2*x^2 + b^2)*arctan(c*x)^2 + b^2*log(
c^2*x^2 + 1) + 2*(a*b*c^2*x^2 - b^2*c*x + a*b)*arctan(c*x))/c^2
```

Sympy [A]
time = 0.19, size = 107, normalized size = 1.41

$$\begin{cases} \frac{a^2x^2}{2}+abx^2\operatorname{atan}(cx)-\frac{abx}{c}+\frac{ab\operatorname{atan}(cx)}{c^2}+\frac{b^2x^2\operatorname{atan}^2(cx)}{2}-\frac{b^2x\operatorname{atan}(cx)}{c}+\frac{b^2\log\left(x^2+\frac{1}{c^2}\right)}{2c^2}+\frac{b^2\operatorname{atan}^2(cx)}{2c^2} & \text{for } c\neq 0 \\ \frac{a^2x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**2/2 + a*b*x**2*atan(c*x) - a*b*x/c + a*b*atan(c*x)/c**2
+ b**2*x**2*atan(c*x)**2/2 - b**2*x*atan(c*x)/c + b**2*log(x**2 + c**(-2))/
(2*c**2) + b**2*atan(c*x)**2/(2*c**2), Ne(c, 0)), (a**2*x**2/2, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 0.41, size = 88, normalized size = 1.16

$$\frac{\frac{b^2\,\mathrm{atan}(c\,x)^2}{2}+\frac{b^2\,\ln(c^2\,x^2+1)}{2}-c\,(x\,\mathrm{atan}(c\,x)\,b^2+a\,x\,b)+a\,b\,\mathrm{atan}(c\,x)}{c^2}+\frac{a^2\,x^2}{2}+\frac{b^2\,x^2\,\mathrm{atan}(c\,x)^2}{2}+a\,b\,x^2\,\mathrm{atan}(c\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x))^2,x)
```

```
[Out] ((b^2*atan(c*x)^2)/2 + (b^2*log(c^2*x^2 + 1))/2 - c*(b^2*x*atan(c*x) + a*b*
x) + a*b*atan(c*x))/c^2 + (a^2*x^2)/2 + (b^2*x^2*atan(c*x)^2)/2 + a*b*x^2*a
tan(c*x)
```

3.18 $\int (a+b\textbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=83

$$\frac{i(a+b\text{ArcTan}(cx))^2}{c}+x(a+b\text{ArcTan}(cx))^2+\frac{2b(a+b\text{ArcTan}(cx))\log\left(\frac{2}{1+icx}\right)}{c}+\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}$$

```
[Out] I*(a+b*arctan(c*x))^2/c+x*(a+b*arctan(c*x))^2+2*b*(a+b*arctan(c*x))*ln(2/(1
+I*c*x))/c+I*b^2*polylog(2,1-2/(1+I*c*x))/c
```

Rubi [A]
time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4930, 5040, 4964, 2449, 2352}

$$x(a+b\text{ArcTan}(cx))^2+\frac{i(a+b\text{ArcTan}(cx))^2}{c}+\frac{2b\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c}+\frac{ib^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (I*(a + b*ArcTan[c*x])^2)/c + x*(a + b*ArcTan[c*x])^2 + (2*b*(a + b*ArcTan[
c*x])*Log[2/(1 + I*c*x)])/c + (I*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
```

```
x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+b\tan^{-1}(cx)\right)^2\,dx = x\left(a+b\tan^{-1}(cx)\right)^2-(2bc)\int \frac{x(a+b\tan^{-1}(cx))}{1+c^2x^2}\,dx$$

$$= \frac{i(a+b\tan^{-1}(cx))^2}{c}+x\left(a+b\tan^{-1}(cx)\right)^2+(2b)\int \frac{a+b\tan^{-1}(cx)}{i-cx}\,dx$$

$$= \frac{i(a+b\tan^{-1}(cx))^2}{c}+x\left(a+b\tan^{-1}(cx)\right)^2+\frac{2b(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c}-(2$$

$$= \frac{i(a+b\tan^{-1}(cx))^2}{c}+x\left(a+b\tan^{-1}(cx)\right)^2+\frac{2b(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c}+\frac{(2}{}$$

$$= \frac{i(a+b\tan^{-1}(cx))^2}{c}+x\left(a+b\tan^{-1}(cx)\right)^2+\frac{2b(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right)}{c}+\frac{ib}{}$$

Mathematica [A]
time = 0.05, size = 90, normalized size = 1.08

$$\frac{b^2(-i+cx)\text{ArcTan}(cx)^2+2b\text{ArcTan}(cx)\left(acx+b\log\left(1+e^{2i\text{ArcTan}(cx)}\right)\right)+a(acx-b\log(1+c^2x^2))-ib^2\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2,x]
```

```
[Out] (b^2*(-I + c*x)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(a*c*x + b*Log[1 + E^((2*I)
*ArcTan[c*x])]) + a*(a*c*x - b*Log[1 + c^2*x^2]) - I*b^2*PolyLog[2, -E^((2*
I)*ArcTan[c*x])])/c
```

Maple [A]
time = 0.76, size = 123, normalized size = 1.48

method	result
derivativedivides	$\frac{cx\,a^2-i\arctan(cx)^2b^2+b^2cx\arctan(cx)^2+2\arctan(cx)\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right)b^2-i\,\mathrm{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right)b^2+2abcx\arctan(cx}{c}$

default	$\frac{cx\,a^2 - i\arctan(cx)^2 b^2 + b^2 cx\arctan(cx)^2 + 2\arctan(cx)\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right)b^2 - i\,\mathrm{polylog}\left(2, -\frac{(icx+1)^2}{c^2x^2+1}\right)b^2 + 2abcx\arctan(}{c}$
risch	$-\frac{ib^2\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-icx+1)}{c} + \frac{ib^2\ln(icx+1)^2}{4c} + \frac{ib^2\,\mathrm{dilog}\left(\frac{1}{2}-\frac{icx}{2}\right)}{c} - \frac{i\ln(-icx+1)^2 b^2}{4c} - iba\ln(icx+1)\,x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(c*x*a^2-I*arctan(c*x)^2*b^2+b^2*c*x*arctan(c*x)^2+2*arctan(c*x)*ln(1+(
1+I*c*x)^2/(c^2*x^2+1))*b^2-I*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))*b^2+2*a*b
*c*x*arctan(c*x)-a*b*ln(c^2*x^2+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*x*arctan(c*x)^2 + 192*c^2*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^2
 + 1), x) + 16*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x)
+ 64*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - x*log(c^2*
x^2 + 1)^2 + 4*arctan(c*x)^3/c - 128*c*integrate(1/16*x*arctan(c*x)/(c^2*x^
2 + 1), x) + 16*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x))*b^2 +
a^2*x + (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{atan}\,(cx))^2\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{atan}(c\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2,x)
```

```
[Out] int((a + b*atan(c*x))^2, x)
```

3.19 $\int \frac{(a+b\mathbf{ArcTan}(cx))^2}{x}\,dx$

Optimal. Leaf size=132

$$2(a+b\text{ArcTan}(cx))^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-ib(a+b\text{ArcTan}(cx))\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)+ib(a+b\text{ArcTan}$$

```
[Out] -2*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))-I*b*(a+b*arctan(c*x))*polylo
g(2,1-2/(1+I*c*x))+I*b*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-1/2*b^2*
polylog(3,1-2/(1+I*c*x))+1/2*b^2*polylog(3,-1+2/(1+I*c*x))
```

Rubi [A]
time = 0.18, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4942, 5108, 5004, 5114, 6745}

$$-ib\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))+ib\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))+2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2-\frac{1}{2}b^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)+\frac{1}{2}b^2\text{Li}_3\left(\frac{2}{icx+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/x,x]
```

```
[Out] 2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] - I*b*(a + b*ArcTan[c*x]
)*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2
/(1 + I*c*x)] - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*PolyLog[3, -1
+ 2/(1 + I*c*x)])/2
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{x}\,dx = 2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-(4bc)\int \frac{(a+b\tan^{-1}(cx))\tanh^{-1}\left(1\right.}{1+c^2x^2}$$

$$= 2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)+(2bc)\int \frac{(a+b\tan^{-1}(cx))\log\left(\frac{2}{1+icx}\right.}{1+c^2x^2}$$

$$= 2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-ib\left(a+b\tan^{-1}(cx)\right)\text{Li}_2\left(1-\frac{2}{1+ic}\right.$$

$$= 2\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-ib\left(a+b\tan^{-1}(cx)\right)\text{Li}_2\left(1-\frac{2}{1+ic}\right.$$

Mathematica [A]
time = 0.05, size = 144, normalized size = 1.09

$$2(a+b\text{ArcTan}(cx))^2\tanh^{-1}\left(\frac{i+cx}{-i+cx}\right)+\frac{1}{2}b\left(2i(a+b\text{ArcTan}(cx))\text{PolyLog}\left(2,\frac{i+cx}{i-cx}\right)-2i(a+b\text{ArcTan}(cx))\text{PolyLog}\left(2,\frac{i+cx}{-i+cx}\right)+b\left(\text{PolyLog}\left(3,\frac{i+cx}{i-cx}\right)-\text{PolyLog}\left(3,\frac{i+cx}{-i+cx}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/x,x]
```

```
[Out] 2*(a + b*ArcTan[c*x])^2*ArcTanh[(I + c*x)/(-I + c*x)] + (b*((2*I)*(a + b*Ar
cTan[c*x])*PolyLog[2, (I + c*x)/(I - c*x)] - (2*I)*(a + b*ArcTan[c*x])*Poly
Log[2, (I + c*x)/(-I + c*x)] + b*(PolyLog[3, (I + c*x)/(I - c*x)] - PolyLog
[3, (I + c*x)/(-I + c*x)])))/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 8.30, size = 1128, normalized size = 8.55

method	result	size

derivativedivides	Expression too large to display	1128
default	Expression too large to display	1128

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)
))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c
*x)^2+a^2*ln(c*x)+1/2*I*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c
*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^
2+1)))*arctan(c*x)^2-1/2*I*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(
I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+
I*a*b*dilog(1+I*c*x)+2*a*b*ln(c*x)*arctan(c*x)-I*a*b*dilog(1-I*c*x)+I*b^2*a
rctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-2*I*b^2*arctan(c*x)*polylog(
2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*b^2*Pi*arctan(c*x)^2-2*I*b^2*arctan(c
*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*b^2*Pi*csgn(I*((1+I*c*x)^2
/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2+1/2*I*b^2*Pi*csgn((
(1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2+1/2
*I*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3
*arctan(c*x)^2-1/2*I*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2
/(c^2*x^2+1)))^2*arctan(c*x)^2+I*a*b*ln(c*x)*ln(1+I*c*x)-I*a*b*ln(c*x)*ln(1
-I*c*x)-1/2*I*b^2*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^
2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+b^2*ln(c*x)*a
rctan(c*x)^2-b^2*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+b^2*arctan(c*x
)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2
*x^2+1)^(1/2))-1/2*b^2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+2*b^2*polylog(3,
(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*b^2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + 1/16*integrate((12*b^2*arctan(c*x)^2 + b^2*log(c^2*x^2 + 1)^2
+ 32*a*b*arctan(c*x))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atan}(cx))^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x,x)
```

```
[Out] Integral((a + b*atan(c*x))**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{atan}(c\,x))^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/x,x)
```

```
[Out] int((a + b*atan(c*x))^2/x, x)
```

3.20 $\int \frac{(a+b\mathbf{ArcTan}(cx))^2}{x^2}\,dx$

Optimal. Leaf size=82

$$-ic(a+b\text{ArcTan}(cx))^2-\frac{(a+b\text{ArcTan}(cx))^2}{x}+2bc(a+b\text{ArcTan}(cx))\log\left(2-\frac{2}{1-icx}\right)-ib^2c\text{PolyLog}\left(2,-1\right.$$

```
[Out] -I*c*(a+b*arctan(c*x))^2-(a+b*arctan(c*x))^2/x+2*b*c*(a+b*arctan(c*x))*ln(2
-2/(1-I*c*x))-I*b^2*c*polylog(2,-1+2/(1-I*c*x))
```

Rubi [A]
time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4946, 5044, 4988, 2497}

$$-ic(a+b\text{ArcTan}(cx))^2-\frac{(a+b\text{ArcTan}(cx))^2}{x}+2bc\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))-ib^2c\text{Li}_2\left(\frac{2}{1-icx}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/x^2,x]
```

```
[Out] (-I)*c*(a + b*ArcTan[c*x])^2 - (a + b*ArcTan[c*x])^2/x + 2*b*c*(a + b*ArcTa
n[c*x])*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{x^2}\,dx = -\frac{(a+b\tan^{-1}(cx))^2}{x} + (2bc)\int \frac{a+b\tan^{-1}(cx)}{x(1+c^2x^2)}\,dx$$

$$= -ic(a+b\tan^{-1}(cx))^2 - \frac{(a+b\tan^{-1}(cx))^2}{x} + (2ibc)\int \frac{a+b\tan^{-1}(cx)}{x(i+cx)}\,dx$$

$$= -ic(a+b\tan^{-1}(cx))^2 - \frac{(a+b\tan^{-1}(cx))^2}{x} + 2bc(a+b\tan^{-1}(cx))\log\left(2-\frac{2}{1-\right.$$

$$= -ic(a+b\tan^{-1}(cx))^2 - \frac{(a+b\tan^{-1}(cx))^2}{x} + 2bc(a+b\tan^{-1}(cx))\log\left(2-\frac{2}{1-\right.$$

Mathematica [A]
time = 0.10, size = 102, normalized size = 1.24

$$\frac{b^2(-1-icx)\mathrm{ArcTan}(cx)^2+2b\mathrm{ArcTan}(cx)\left(-a+bcx\log\left(1-e^{2i\mathrm{ArcTan}(cx)}\right)\right)-a(a-2bcx\log(cx)+bcx\log(1+c^2x^2))-ib^2cx\mathrm{PolyLog}\left(2,e^{2i\mathrm{ArcTan}(cx)}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/x^2,x]
```

```
[Out] (b^2*(-1 - I*c*x)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(-a + b*c*x*Log[1 - E^((2
*I)*ArcTan[c*x])]) - a*(a - 2*b*c*x*Log[c*x] + b*c*x*Log[1 + c^2*x^2]) - I*
b^2*c*x*PolyLog[2, E^((2*I)*ArcTan[c*x])])/x
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(78) = 156.
time = 0.80, size = 318, normalized size = 3.88

method	result
derivativedivides	$c\left(-\frac{a^2}{cx} - \frac{b^2\arctan(cx)^2}{cx} - b^2\arctan(cx)\ln(c^2x^2+1) + 2b^2\ln(cx)\arctan(cx) - ib^2\,\mathrm{dilog}(-\right.$
default	$c\left(-\frac{a^2}{cx} - \frac{b^2\arctan(cx)^2}{cx} - b^2\arctan(cx)\ln(c^2x^2+1) + 2b^2\ln(cx)\arctan(cx) - ib^2\,\mathrm{dilog}(-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-a^2/c/x-b^2/c/x*arctan(c*x)^2-b^2*arctan(c*x)*ln(c^2*x^2+1)+2*b^2*ln(c*
x)*arctan(c*x)+I*b^2*dilog(1+I*c*x)+1/2*I*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-
1/2*I*b^2*dilog(1/2*I*(c*x-I))-1/2*I*b^2*ln(c*x-I)*ln(c^2*x^2+1)+1/2*I*b^2*
dilog(-1/2*I*(c*x+I))-1/2*I*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))+I*b^2*ln(c*x)*l
n(1+I*c*x)-I*b^2*dilog(1-I*c*x)+1/4*I*b^2*ln(c*x-I)^2+1/2*I*b^2*ln(c*x+I)*l
n(c^2*x^2+1)-1/4*I*b^2*ln(c*x+I)^2-I*b^2*ln(c*x)*ln(1-I*c*x)-2*a*b/c/x*arct
an(c*x)-a*b*ln(c^2*x^2+1)+2*a*b*ln(c*x))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] -(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b + 1/16*(4*(c*arcta
n(c*x)^3 + 4*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x)
- 16*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) + 32*c*int
egrate(1/16*x*arctan(c*x)/(c^2*x^4 + x^2), x) + 48*integrate(1/16*arctan(c*
x)^2/(c^2*x^4 + x^2), x) + 4*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^4 + x
^2), x))*x - 4*arctan(c*x)^2 + log(c^2*x^2 + 1)^2)*b^2/x - a^2/x
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/x^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atan}(cx))^2}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**2/x**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/x^2,x)
```

```
[Out] int((a + b*atan(c*x))^2/x^2, x)
```

3.21 $\int \frac{(a+b\mathbf{ArcTan}(cx))^2}{x^3} dx$

Optimal. Leaf size=79

$$-\frac{bc(a+b\text{ArcTan}(cx))}{x}-\frac{1}{2}c^2(a+b\text{ArcTan}(cx))^2-\frac{(a+b\text{ArcTan}(cx))^2}{2x^2}+b^2c^2\log(x)-\frac{1}{2}b^2c^2\log\left(1+c^2x^2\right)$$

```
[Out] -b*c*(a+b*arctan(c*x))/x-1/2*c^2*(a+b*arctan(c*x))^2-1/2*(a+b*arctan(c*x))^
2/x^2+b^2*c^2*ln(x)-1/2*b^2*c^2*ln(c^2*x^2+1)
```

Rubi [A]
time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4946, 5038, 272, 36, 29, 31, 5004}

$$-\frac{1}{2}c^2(a+b\text{ArcTan}(cx))^2-\frac{(a+b\text{ArcTan}(cx))^2}{2x^2}-\frac{bc(a+b\text{ArcTan}(cx))}{x}-\frac{1}{2}b^2c^2\log\left(c^2x^2+1\right)+b^2c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/x^3,x]
```

```
[Out] -((b*c*(a + b*ArcTan[c*x]))/x) - (c^2*(a + b*ArcTan[c*x])^2)/2 - (a + b*Arc
Tan[c*x])^2/(2*x^2) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 + c^2*x^2])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^3}\,dx &= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+(bc)\int \frac{a+b\tan^{-1}(cx)}{x^2\left(1+c^2x^2\right)}\,dx\\
&= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+(bc)\int \frac{a+b\tan^{-1}(cx)}{x^2}\,dx-\left(bc^3\right)\int \frac{a+b\tan^{-1}(cx)}{1+c^2x^2}\,dx\\
&= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+\left(b^2c^2\right)\int\\
&= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+\frac{1}{2}\left(b^2c^2\right)\mathrm{S}\\
&= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+\frac{1}{2}\left(b^2c^2\right)\mathrm{S}\\
&= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{2x^2}+b^2c^2\log(x
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 90, normalized size = 1.14

$$-\frac{a^2+2abcx+2b\left(a+bcx+ac^2x^2\right)\mathrm{ArcTan}(cx)+b^2\left(1+c^2x^2\right)\mathrm{ArcTan}(cx)^2-2b^2c^2x^2\log(x)+b^2c^2x^2\log\left(1+c^2x^2\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/x^3,x]
```

```
[Out] -1/2*(a^2 + 2*a*b*c*x + 2*b*(a + b*c*x + a*c^2*x^2)*ArcTan[c*x] + b^2*(1 +
c^2*x^2)*ArcTan[c*x]^2 - 2*b^2*c^2*x^2*Log[x] + b^2*c^2*x^2*Log[1 + c^2*x^2
])/x^2
```

Maple [A]
time = 0.24, size = 115, normalized size = 1.46

method	result
derivativedivides	$c^2\left(-\frac{a^2}{2c^2x^2}-\frac{b^2\arctan(cx)^2}{2c^2x^2}-\frac{b^2\arctan(cx)^2}{2}-\frac{b^2\arctan(cx)}{cx}-\frac{b^2\ln(c^2x^2+1)}{2}+b^2\ln(cx)-\frac{ab\arctan}{c^2x^2}\right.$
default	$c^2\left(-\frac{a^2}{2c^2x^2}-\frac{b^2\arctan(cx)^2}{2c^2x^2}-\frac{b^2\arctan(cx)^2}{2}-\frac{b^2\arctan(cx)}{cx}-\frac{b^2\ln(c^2x^2+1)}{2}+b^2\ln(cx)-\frac{ab\arctan}{c^2x^2}\right.$
risch	$\frac{b^2(c^2x^2+1)\ln(icx+1)^2}{8x^2}+\frac{ib(ibc^2x^2\ln(-icx+1)+2xbc+2a+ib\ln(-icx+1))\ln(icx+1)}{4x^2}-\frac{-4i\ln((-3ibc-ac)x-3b+}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*a^2/c^2/x^2-1/2*b^2/c^2/x^2*arctan(c*x)^2-1/2*b^2*arctan(c*x)^2-b
^2*arctan(c*x)/c/x-1/2*b^2*ln(c^2*x^2+1)+b^2*ln(c*x)-a*b/c^2/x^2*arctan(c*x
)-a*b*arctan(c*x)-a*b/c/x)
```

Maxima [A]
time = 0.47, size = 98, normalized size = 1.24

$$-\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)ab+\frac{1}{2}\left(\left(\arctan(cx)^2-\log(c^2x^2+1)+2\log(x)\right)c^2-2\left(c\arctan(cx)+\frac{1}{x}\right)c\arctan(cx)\right)b^2-\frac{b^2\arctan(cx)^2}{2x^2}-\frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] -((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b + 1/2*((arctan(c*x)^2 - lo
g(c^2*x^2 + 1) + 2*log(x))*c^2 - 2*(c*arctan(c*x) + 1/x)*c*arctan(c*x))*b^2
 - 1/2*b^2*arctan(c*x)^2/x^2 - 1/2*a^2/x^2
```

Fricas [A]
time = 0.78, size = 94, normalized size = 1.19

$$-\frac{b^2c^2x^2\log(c^2x^2+1)-2\,b^2c^2x^2\log(x)+2\,abcx+(b^2c^2x^2+b^2)\arctan(cx)^2+a^2+2\,(abc^2x^2+b^2cx+ab)\arctan(cx)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2*x^2*log(c^2*x^2 + 1) - 2*b^2*c^2*x^2*log(x) + 2*a*b*c*x + (b^
2*c^2*x^2 + b^2)*arctan(c*x)^2 + a^2 + 2*(a*b*c^2*x^2 + b^2*c*x + a*b)*arct
an(c*x))/x^2
```

Sympy [A]
time = 0.34, size = 119, normalized size = 1.51

$$\begin{cases} -\frac{a^2}{2x^2} - abc^2 \operatorname{atan}(cx) - \frac{abc}{x} - \frac{ab \operatorname{atan}(cx)}{x^2} + b^2c^2 \log(x) - \frac{b^2c^2 \log\left(x^2 + \frac{1}{c^2}\right)}{2} - \frac{b^2c^2 \operatorname{atan}^2(cx)}{2} - \frac{b^2c \operatorname{atan}(cx)}{x} - \frac{b^2 \operatorname{atan}^2(cx)}{2x^2} & \text{for } c \neq 0 \\ -\frac{a^2}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**3,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - a*b*c**2*atan(c*x) - a*b*c/x - a*b*atan(c*x)/x*
*2 + b**2*c**2*log(x) - b**2*c**2*log(x**2 + c**(-2))/2 - b**2*c**2*atan(c*
x)**2/2 - b**2*c*atan(c*x)/x - b**2*atan(c*x)**2/(2*x**2), Ne(c, 0)), (-a**
2/(2*x**2), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 2.31, size = 140, normalized size = 1.77

$$b^2\,c^2\,\ln(x) - \frac{a^2}{2\,x^2} - \frac{b^2\,c^2\,\mathrm{atan}(c\,x)^2}{2} - \frac{b^2\,c^2\,\ln(c\,x + 1\mathrm{i})}{2} - \frac{b^2\,c^2\,\ln(1 + c\,x\,1\mathrm{i})}{2} - \frac{b^2\,\mathrm{atan}(c\,x)^2}{2\,x^2} - \frac{a\,b\,c}{x} - \frac{a\,b\,\mathrm{atan}(c\,x)}{x^2} - \frac{b^2\,c\,\mathrm{atan}(c\,x)}{x} - \frac{a\,b\,c^2\,\ln(c\,x + 1\mathrm{i})\,1\mathrm{i}}{2} + \frac{a\,b\,c^2\,\ln(1 + c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/x^3,x)
```

```
[Out] b^2*c^2*log(x) - a^2/(2*x^2) - (b^2*c^2*atan(c*x)^2)/2 - (b^2*c^2*log(c*x +
 1i))/2 - (b^2*c^2*log(c*x*1i + 1))/2 - (b^2*atan(c*x)^2)/(2*x^2) - (a*b*c)
/x - (a*b*atan(c*x))/x^2 - (a*b*c^2*log(c*x + 1i)*1i)/2 + (a*b*c^2*log(c*x*
1i + 1)*1i)/2 - (b^2*c*atan(c*x))/x
```

3.22 $\int \frac{(a+b\mathbf{ArcTan}(cx))^2}{x^4}\,dx$

Optimal. Leaf size=140

$$-\frac{b^2c^2}{3x}-\frac{1}{3}b^2c^3\text{ArcTan}(cx)-\frac{bc(a+b\text{ArcTan}(cx))}{3x^2}+\frac{1}{3}ic^3(a+b\text{ArcTan}(cx))^2-\frac{(a+b\text{ArcTan}(cx))^2}{3x^3}-\frac{2}{3}bc^3(a+b$$

```
[Out] -1/3*b^2*c^2/x-1/3*b^2*c^3*arctan(c*x)-1/3*b*c*(a+b*arctan(c*x))/x^2+1/3*I*
c^3*(a+b*arctan(c*x))^2-1/3*(a+b*arctan(c*x))^2/x^3-2/3*b*c^3*(a+b*arctan(c
*x))*ln(2-2/(1-I*c*x))+1/3*I*b^2*c^3*polylog(2,-1+2/(1-I*c*x))
```

Rubi [A]
time = 0.17, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4946, 5038, 331, 209, 5044, 4988, 2497}

$$\frac{1}{3}ic^3(a+b\text{ArcTan}(cx))^2-\frac{2}{3}bc^3\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))-\frac{(a+b\text{ArcTan}(cx))^2}{3x^3}-\frac{bc(a+b\text{ArcTan}(cx))}{3x^2}-\frac{1}{3}b^2c^3\text{ArcTan}(cx)+\frac{1}{3}ib^2c^3\text{Li}_2\left(\frac{2}{1-icx}-1\right)-\frac{b^2c^2}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/x^4,x]
```

```
[Out] -1/3*(b^2*c^2)/x - (b^2*c^3*ArcTan[c*x])/3 - (b*c*(a + b*ArcTan[c*x]))/(3*x
^2) + (I/3)*c^3*(a + b*ArcTan[c*x])^2 - (a + b*ArcTan[c*x])^2/(3*x^3) - (2*
b*c^3*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/3 + (I/3)*b^2*c^3*PolyLog
[2, -1 + 2/(1 - I*c*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^4}\,dx &= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}(2bc)\int \frac{a+b\tan^{-1}(cx)}{x^3\left(1+c^2x^2\right)}\,dx\\
&= -\frac{\left(a+b\tan^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}(2bc)\int \frac{a+b\tan^{-1}(cx)}{x^3}\,dx-\frac{1}{3}\left(2bc^3\right)\int \frac{a+b\tan^{-1}(cx}{x\left(1+c^2x^2\right)}\\
&= -\frac{bc\left(a+b\tan^{-1}(cx)\right)}{3x^2}+\frac{1}{3}ic^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}\left(b^2c^2\right)\int\\
&= -\frac{b^2c^2}{3x}-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{3x^2}+\frac{1}{3}ic^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{\left(a+b\tan^{-1}(cx)\right)^2}{3x^3}-\frac{2}{3}\\
&= -\frac{b^2c^2}{3x}-\frac{1}{3}b^2c^3\tan^{-1}(cx)-\frac{bc\left(a+b\tan^{-1}(cx)\right)}{3x^2}+\frac{1}{3}ic^3\left(a+b\tan^{-1}(cx)\right)^2-\frac{(a+}{}
\end{aligned}
$$

Mathematica [A]
time = 0.27, size = 153, normalized size = 1.09

$$-\frac{a^2+abcx+b^2c^2x^2+b^2(1-ic^3x^3)\operatorname{ArcTan}(cx)^2+b\operatorname{ArcTan}(cx)\left(2a+bcx+bc^3x^3+2bc^3x^3\log\left(1-e^{2i\operatorname{ArcTan}(cx)}\right)\right)+2abc^3x^3\log(cx)-abc^3x^3\log\left(1+c^2x^2\right)-ib^2c^3x^3\operatorname{PolyLog}\left(2,e^{2i\operatorname{ArcTan}(cx)}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/x^4,x]
```

```
[Out] -1/3*(a^2 + a*b*c*x + b^2*c^2*x^2 + b^2*(1 - I*c^3*x^3)*ArcTan[c*x]^2 + b*A
rcTan[c*x]*(2*a + b*c*x + b*c^3*x^3 + 2*b*c^3*x^3*Log[1 - E^((2*I)*ArcTan[c
*x])]) + 2*a*b*c^3*x^3*Log[c*x] - a*b*c^3*x^3*Log[1 + c^2*x^2] - I*b^2*c^3*
x^3*PolyLog[2, E^((2*I)*ArcTan[c*x])])/x^3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(122) = 244.
time = 0.84, size = 365, normalized size = 2.61

method	result
derivativedivides	$c^3\left(-\frac{a^2}{3c^3x^3}-\frac{b^2\arctan(cx)^2}{3c^3x^3}+\frac{b^2\arctan(cx)\ln(c^2x^2+1)}{3}-\frac{b^2\arctan(cx)}{3c^2x^2}-\frac{2b^2\ln(cx)\arctan(cx)}{3}+\frac{ib^2\ln(c\right.$
default	$c^3\left(-\frac{a^2}{3c^3x^3}-\frac{b^2\arctan(cx)^2}{3c^3x^3}+\frac{b^2\arctan(cx)\ln(c^2x^2+1)}{3}-\frac{b^2\arctan(cx)}{3c^2x^2}-\frac{2b^2\ln(cx)\arctan(cx)}{3}+\frac{ib^2\ln(c\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/3*a^2/c^3/x^3-1/3*b^2/c^3/x^3*arctan(c*x)^2+1/3*b^2*arctan(c*x)*ln(
c^2*x^2+1)-1/3*b^2*arctan(c*x)/c^2/x^2-2/3*b^2*ln(c*x)*arctan(c*x)+1/3*I*b^
2*ln(c*x)*ln(1-I*c*x)+1/3*I*b^2*dilog(1-I*c*x)-1/3*I*b^2*ln(c*x)*ln(1+I*c*x
)+1/6*I*b^2*ln(c*x-I)*ln(c^2*x^2+1)-1/6*I*b^2*ln(c*x+I)*ln(c^2*x^2+1)+1/6*I
*b^2*ln(c*x+I)*ln(1/2*I*(c*x-I))-1/3*I*b^2*dilog(1+I*c*x)-1/12*I*b^2*ln(c*x
-I)^2-1/3*b^2*arctan(c*x)-1/3*b^2/c/x+1/6*I*b^2*dilog(1/2*I*(c*x-I))-1/6*I*
b^2*dilog(-1/2*I*(c*x+I))-1/6*I*b^2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/12*I*b^2
*ln(c*x+I)^2-2/3*a*b/c^3/x^3*arctan(c*x)+1/3*a*b*ln(c^2*x^2+1)-1/3*a*b/c^2/
x^2-2/3*a*b*ln(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*a
*b + 1/48*(48*x^3*integrate(-1/48*(4*c^2*x^2*log(c^2*x^2 + 1) - 8*c*x*arcta
n(c*x) - 36*(c^2*x^2 + 1)*arctan(c*x)^2 - 3*(c^2*x^2 + 1)*log(c^2*x^2 + 1)^
2)/(c^2*x^6 + x^4), x) - 4*arctan(c*x)^2 + log(c^2*x^2 + 1)^2)*b^2/x^3 - 1/
3*a^2/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{atan}(cx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**4,x)
```

```
[Out] Integral((a + b*atan(c*x))**2/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^4,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\operatorname{atan}(c\,x))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/x^4,x)
```

```
[Out] int((a + b*atan(c*x))^2/x^4, x)
```

3.23 $\int \frac{(a+b\mathbf{ArcTan}(cx))^2}{x^5}\,dx$

Optimal. Leaf size=116

$$-\frac{b^2c^2}{12x^2}-\frac{bc(a+b\text{ArcTan}(cx))}{6x^3}+\frac{bc^3(a+b\text{ArcTan}(cx))}{2x}+\frac{1}{4}c^4(a+b\text{ArcTan}(cx))^2-\frac{(a+b\text{ArcTan}(cx))^2}{4x^4}-\frac{2}{3}b^2c$$

```
[Out] -1/12*b^2*c^2/x^2-1/6*b*c*(a+b*arctan(c*x))/x^3+1/2*b*c^3*(a+b*arctan(c*x))
/x+1/4*c^4*(a+b*arctan(c*x))^2-1/4*(a+b*arctan(c*x))^2/x^4-2/3*b^2*c^4*ln(x
)+1/3*b^2*c^4*ln(c^2*x^2+1)
```

Rubi [A]
time = 0.16, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4946, 5038, 272, 46, 36, 29, 31, 5004}

$$\frac{1}{4}c^4(a+b\text{ArcTan}(cx))^2+\frac{bc^3(a+b\text{ArcTan}(cx))}{2x}-\frac{(a+b\text{ArcTan}(cx))^2}{4x^4}-\frac{bc(a+b\text{ArcTan}(cx))}{6x^3}-\frac{2}{3}b^2c^4\log(x)-\frac{b^2c^2}{12x^2}+\frac{1}{3}b^2c^4\log\left(c^2x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^2/x^5,x]
```

```
[Out] -1/12*(b^2*c^2)/x^2 - (b*c*(a + b*ArcTan[c*x]))/(6*x^3) + (b*c^3*(a + b*Arc
Tan[c*x]))/(2*x) + (c^4*(a + b*ArcTan[c*x])^2)/4 - (a + b*ArcTan[c*x])^2/(4
*x^4) - (2*b^2*c^4*Log[x])/3 + (b^2*c^4*Log[1 + c^2*x^2])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx))^2}{x^5}\,dx = -\frac{(a+b\tan^{-1}(cx))^2}{4x^4} + \frac{1}{2}(bc)\int \frac{a+b\tan^{-1}(cx)}{x^4\,(1+c^2x^2)}\,dx$$
$$= -\frac{(a+b\tan^{-1}(cx))^2}{4x^4} + \frac{1}{2}(bc)\int \frac{a+b\tan^{-1}(cx)}{x^4}\,dx - \frac{1}{2}(bc^3)\int \frac{a+b\tan^{-1}(cx)}{x^2\,(1+c^2x^2)}$$
$$= -\frac{bc(a+b\tan^{-1}(cx))}{6x^3} - \frac{(a+b\tan^{-1}(cx))^2}{4x^4} + \frac{1}{6}(b^2c^2)\int \frac{1}{x^3\,(1+c^2x^2)}\,dx - \frac{1}{2}(b$$
$$= -\frac{bc(a+b\tan^{-1}(cx))}{6x^3} + \frac{bc^3(a+b\tan^{-1}(cx))}{2x} + \frac{1}{4}c^4(a+b\tan^{-1}(cx))^2 - \frac{(a+b}{}$$
$$= -\frac{bc(a+b\tan^{-1}(cx))}{6x^3} + \frac{bc^3(a+b\tan^{-1}(cx))}{2x} + \frac{1}{4}c^4(a+b\tan^{-1}(cx))^2 - \frac{(a+b}{}$$
$$= -\frac{b^2c^2}{12x^2} - \frac{bc(a+b\tan^{-1}(cx))}{6x^3} + \frac{bc^3(a+b\tan^{-1}(cx))}{2x} + \frac{1}{4}c^4(a+b\tan^{-1}(cx))^2 -$$
$$= -\frac{b^2c^2}{12x^2} - \frac{bc(a+b\tan^{-1}(cx))}{6x^3} + \frac{bc^3(a+b\tan^{-1}(cx))}{2x} + \frac{1}{4}c^4(a+b\tan^{-1}(cx))^2 -$$

Mathematica [A]
time = 0.05, size = 128, normalized size = 1.10

$$\frac{-3a^2-2abcx-b^2c^2x^2+6abc^3x^3+2b(bcx(-1+3c^2x^2)+3a(-1+c^4x^4))\operatorname{ArcTan}(cx)+3b^2(-1+c^4x^4)\operatorname{ArcTan}(cx)^2-8b^2c^4x^4\log(x)+4b^2c^4x^4\log(1+c^2x^2)}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^2/x^5,x]
```

```
[Out] (-3*a^2 - 2*a*b*c*x - b^2*c^2*x^2 + 6*a*b*c^3*x^3 + 2*b*(b*c*x*(-1 + 3*c^2*
x^2) + 3*a*(-1 + c^4*x^4))*ArcTan[c*x] + 3*b^2*(-1 + c^4*x^4)*ArcTan[c*x]^2
 - 8*b^2*c^4*x^4*Log[x] + 4*b^2*c^4*x^4*Log[1 + c^2*x^2])/(12*x^4)
```

Maple [A]
time = 0.25, size = 152, normalized size = 1.31

method	result
derivativedivides	$c^4\Big(-\frac{a^2}{4c^4x^4} - \frac{b^2\arctan(cx)^2}{4c^4x^4} + \frac{b^2\arctan(cx)^2}{4} - \frac{b^2\arctan(cx)}{6c^3x^3} + \frac{b^2\arctan(cx)}{2cx} + \frac{b^2\ln(c^2x^2+1)}{3} - \frac{b^2}{12c^2x^2}$
default	$c^4\Big(-\frac{a^2}{4c^4x^4} - \frac{b^2\arctan(cx)^2}{4c^4x^4} + \frac{b^2\arctan(cx)^2}{4} - \frac{b^2\arctan(cx)}{6c^3x^3} + \frac{b^2\arctan(cx)}{2cx} + \frac{b^2\ln(c^2x^2+1)}{3} - \frac{b^2}{12c^2x^2}$
risch	$-\frac{b^2(c^4x^4-1)\ln(icx+1)^2}{16x^4} + \frac{ib(-3ib\,c^4x^4\ln(-icx+1)-6b\,c^3x^3+2xbc+6a+3ib\ln(-icx+1))\ln(icx+1)}{24x^4} - \frac{-12i\ln((4}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(-1/4*a^2/c^4/x^4-1/4*b^2/c^4/x^4*arctan(c*x)^2+1/4*b^2*arctan(c*x)^2-1
/6*b^2*arctan(c*x)/c^3/x^3+1/2*b^2*arctan(c*x)/c/x+1/3*b^2*ln(c^2*x^2+1)-1/
12*b^2/c^2/x^2-2/3*b^2*ln(c*x)-1/2*a*b/c^4/x^4*arctan(c*x)+1/2*a*b*arctan(c
*x)-1/6*a*b/c^3/x^3+1/2*a*b/c/x)
```

Maxima [A]
time = 0.48, size = 152, normalized size = 1.31

$$\frac{1}{6}\left(\left(3\,c^{3}\arctan\left(cx\right)+\frac{3\,c^{2}x^{2}-1}{x^{3}}\right)c-\frac{3\,\arctan\left(cx\right)}{x^{4}}\right)ab+\frac{1}{12}\left(2\left(3\,c^{3}\arctan\left(cx\right)+\frac{3\,c^{2}x^{2}-1}{x^{3}}\right)c\arctan\left(cx\right)-\frac{\left(3\,c^{2}x^{2}\arctan\left(cx\right)^{2}-4\,c^{2}x^{2}\log\left(c^{2}x^{2}+1\right)+8\,c^{2}x^{2}\log\left(x\right)+1\right)c^{2}}{x^{2}}\right)b^{2}-\frac{b^{2}\arctan\left(cx\right)^{2}}{4\,x^{4}}-\frac{a^{2}}{4\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*a*b +
 1/12*(2*(3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c*arctan(c*x) - (3*c^2*x
^2*arctan(c*x)^2 - 4*c^2*x^2*log(c^2*x^2 + 1) + 8*c^2*x^2*log(x) + 1)*c^2/x
^2)*b^2 - 1/4*b^2*arctan(c*x)^2/x^4 - 1/4*a^2/x^4
```

Fricas [A]
time = 1.15, size = 135, normalized size = 1.16

$$\frac{4\,b^{2}c^{4}x^{4}\log\left(c^{2}x^{2}+1\right)-8\,b^{2}c^{4}x^{4}\log\left(x\right)+6\,abc^{3}x^{3}-b^{2}c^{2}x^{2}-2\,abcx+3\,\left(b^{2}c^{4}x^{4}-b^{2}\right)\arctan\left(cx\right)^{2}-3\,a^{2}+2\,\left(3\,abc^{4}x^{4}+3\,b^{2}c^{3}x^{3}-b^{2}cx-3\,ab\right)\arctan\left(cx\right)}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^2*c^4*x^4*log(c^2*x^2 + 1) - 8*b^2*c^4*x^4*log(x) + 6*a*b*c^3*x^3
 - b^2*c^2*x^2 - 2*a*b*c*x + 3*(b^2*c^4*x^4 - b^2)*arctan(c*x)^2 - 3*a^2 +
2*(3*a*b*c^4*x^4 + 3*b^2*c^3*x^3 - b^2*c*x - 3*a*b)*arctan(c*x))/x^4
```

Sympy [A]
time = 0.51, size = 170, normalized size = 1.47

$$\begin{cases} - \frac{a^{2}}{4 x^{4}} + \frac{a b c^{4} \operatorname{atan}{\left(c x \right)}}{2} + \frac{a b c^{3}}{2 x} - \frac{a b c}{6 x^{3}} - \frac{a b \operatorname{atan}{\left(c x \right)}}{2 x^{4}} - \frac{2 b^{2} c^{4} \log{\left(x \right)}}{3} + \frac{b^{2} c^{4} \log{\left(x^{2} + \frac{1}{c^{2}} \right)}}{3} + \frac{b^{2} c^{4} \operatorname{atan}^{2}{\left(c x \right)}}{4} + \frac{b^{2} c^{3} \operatorname{atan}{\left(c x \right)}}{2 x} - \frac{b^{2} c^{2}}{12 x^{2}} - \frac{b^{2} c \operatorname{atan}{\left(c x \right)}}{6 x^{3}} - \frac{b^{2} \operatorname{atan}^{2}{\left(c x \right)}}{4 x^{4}} & \text{for}\: c \neq 0 \\- \frac{a^{2}}{4 x^{4}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**2/x**5,x)
```

```
[Out] Piecewise((-a**2/(4*x**4) + a*b*c**4*atan(c*x)/2 + a*b*c**3/(2*x) - a*b*c/(
6*x**3) - a*b*atan(c*x)/(2*x**4) - 2*b**2*c**4*log(x)/3 + b**2*c**4*log(x**
2 + c**(-2))/3 + b**2*c**4*atan(c*x)**2/4 + b**2*c**3*atan(c*x)/(2*x) - b**
2*c**2/(12*x**2) - b**2*c*atan(c*x)/(6*x**3) - b**2*atan(c*x)**2/(4*x**4),
Ne(c, 0)), (-a**2/(4*x**4), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^2/x^5,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [B]
time = 2.26, size = 171, normalized size = 1.47

$$\frac{b^2\,c^4\,\mathrm{atan}(c\,x)^2}{4}-\frac{2\,b^2\,c^4\,\ln\left(x\right)}{3}-\frac{\frac{b^2\,\mathrm{atan}(c\,x)^2}{4}+\frac{a^2}{4}+x\,\left(\frac{c\,\mathrm{atan}(c\,x)\,b^2}{6}+\frac{a\,c\,b}{6}\right)-x^3\,\left(\frac{b^2\,c^3\,\mathrm{atan}(c\,x)}{2}+\frac{a\,b\,c^3}{2}\right)+\frac{b^2\,c^2\,x^2}{12}+\frac{a\,b\,\mathrm{atan}(c\,x)}{2}}{x^4}+\frac{b^2\,c^4\,\ln\left(c\,x+1{}\mathrm{i}\right)}{3}+\frac{b^2\,c^4\,\ln\left(1+c\,x\,1{}\mathrm{i}\right)}{3}+\frac{a\,b\,c^4\,\ln\left(c\,x+1{}\mathrm{i}\right)\,1{}\mathrm{i}}{4}-\frac{a\,b\,c^4\,\ln\left(1+c\,x\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2/x^5,x)
```

```
[Out] (b^2*c^4*atan(c*x)^2)/4 - (2*b^2*c^4*log(x))/3 - ((b^2*atan(c*x)^2)/4 + a^2
/4 + x*((b^2*c*atan(c*x))/6 + (a*b*c)/6) - x^3*((b^2*c^3*atan(c*x))/2 + (a*
b*c^3)/2) + (b^2*c^2*x^2)/12 + (a*b*atan(c*x))/2)/x^4 + (b^2*c^4*log(c*x +
1i))/3 + (b^2*c^4*log(c*x*1i + 1))/3 + (a*b*c^4*log(c*x + 1i)*1i)/4 - (a*b*
c^4*log(c*x*1i + 1)*1i)/4
```

3.24 $\int x^5(a + b\text{ArcTan}(cx))^3\,dx$

Optimal. Leaf size=255

$$\frac{19b^3x}{60c^5}-\frac{b^3x^3}{60c^3}-\frac{19b^3\text{ArcTan}(cx)}{60c^6}-\frac{4b^2x^2(a+b\text{ArcTan}(cx))}{15c^4}+\frac{b^2x^4(a+b\text{ArcTan}(cx))}{20c^2}-\frac{23ib(a+b\text{ArcTan}(cx))}{30c^6}$$

```
[Out] 19/60*b^3*x/c^5-1/60*b^3*x^3/c^3-19/60*b^3*arctan(c*x)/c^6-4/15*b^2*x^2*(a+
b*arctan(c*x))/c^4+1/20*b^2*x^4*(a+b*arctan(c*x))/c^2-23/30*I*b*(a+b*arctan
(c*x))^2/c^6-1/2*b*x*(a+b*arctan(c*x))^2/c^5+1/6*b*x^3*(a+b*arctan(c*x))^2/
c^3-1/10*b*x^5*(a+b*arctan(c*x))^2/c+1/6*(a+b*arctan(c*x))^3/c^6+1/6*x^6*(a
+b*arctan(c*x))^3-23/15*b^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^6-23/30*I*b
^3*polylog(2,1-2/(1+I*c*x))/c^6
```

Rubi [A]
time = 0.65, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {4946, 5036, 308, 209, 327, 5040, 4964, 2449, 2352, 4930, 5004}

$$-\frac{23b^2\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{15c^6}-\frac{4b^2x^2(a+b\text{ArcTan}(cx))}{15c^4}+\frac{b^2x^4(a+b\text{ArcTan}(cx))}{20c^2}+\frac{(a+b\text{ArcTan}(cx))^3}{6c^6}-\frac{23ib(a+b\text{ArcTan}(cx))^2}{30c^6}-\frac{bx(a+b\text{ArcTan}(cx))^2}{2c^5}+\frac{bx^3(a+b\text{ArcTan}(cx))^2}{6c^3}+\frac{1}{6}x^6(a+b\text{ArcTan}(cx))^3-\frac{bx^5(a+b\text{ArcTan}(cx))^2}{10c}-\frac{19b^3\text{ArcTan}(cx)}{60c^6}-\frac{23ib^3\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{30c^6}+\frac{19b^3x}{60c^5}-\frac{b^3x^3}{60c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (19*b^3*x)/(60*c^5) - (b^3*x^3)/(60*c^3) - (19*b^3*ArcTan[c*x])/(60*c^6) -
(4*b^2*x^2*(a + b*ArcTan[c*x]))/(15*c^4) + (b^2*x^4*(a + b*ArcTan[c*x]))/(2
0*c^2) - (((23*I)/30)*b*(a + b*ArcTan[c*x])^2)/c^6 - (b*x*(a + b*ArcTan[c*x
])^2)/(2*c^5) + (b*x^3*(a + b*ArcTan[c*x])^2)/(6*c^3) - (b*x^5*(a + b*ArcTa
n[c*x])^2)/(10*c) + (a + b*ArcTan[c*x])^3/(6*c^6) + (x^6*(a + b*ArcTan[c*x]
)^3)/6 - (23*b^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(15*c^6) - (((23*I
)/30)*b^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^6
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^5\left(a+b\tan^{-1}(cx)\right)^3\,dx &= \frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^3-\frac{1}{2}(bc)\int\frac{x^6\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx\\
&= \frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^3-\frac{b\int x^4\left(a+b\tan^{-1}(cx)\right)^2\,dx}{2c}+\frac{b\int\frac{x^4\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx}{2c}\\
&= -\frac{bx^5\left(a+b\tan^{-1}(cx)\right)^2}{10c}+\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^3+\frac{1}{5}b^2\int\frac{x^5(a+b\tan^{-1}(cx))}{1+c^2x^2}\\
&= \frac{bx^3\left(a+b\tan^{-1}(cx)\right)^2}{6c^3}-\frac{bx^5\left(a+b\tan^{-1}(cx)\right)^2}{10c}+\frac{1}{6}x^6\left(a+b\tan^{-1}(cx)\right)^3-\frac{b\int}{}\\
&= \frac{b^2x^4\left(a+b\tan^{-1}(cx)\right)}{20c^2}-\frac{bx\left(a+b\tan^{-1}(cx)\right)^2}{2c^5}+\frac{bx^3\left(a+b\tan^{-1}(cx)\right)^2}{6c^3}-\frac{bx^5(}{}\\
&= -\frac{4b^2x^2\left(a+b\tan^{-1}(cx)\right)}{15c^4}+\frac{b^2x^4\left(a+b\tan^{-1}(cx)\right)}{20c^2}-\frac{23ib\left(a+b\tan^{-1}(cx)\right)^2}{30c^6}-\\
&= \frac{19b^3x}{60c^5}-\frac{b^3x^3}{60c^3}-\frac{4b^2x^2\left(a+b\tan^{-1}(cx)\right)}{15c^4}+\frac{b^2x^4\left(a+b\tan^{-1}(cx)\right)}{20c^2}-\frac{23ib(a+b}{30}\\
&= \frac{19b^3x}{60c^5}-\frac{b^3x^3}{60c^3}-\frac{19b^3\tan^{-1}(cx)}{60c^6}-\frac{4b^2x^2\left(a+b\tan^{-1}(cx)\right)}{15c^4}+\frac{b^2x^4(a+b\tan^{-1}(}{20c^2}\\
&= \frac{19b^3x}{60c^5}-\frac{b^3x^3}{60c^3}-\frac{19b^3\tan^{-1}(cx)}{60c^6}-\frac{4b^2x^2\left(a+b\tan^{-1}(cx)\right)}{15c^4}+\frac{b^2x^4(a+b\tan^{-1}(}{20c^2}
\end{aligned}$$

Mathematica [A]
time = 0.55, size = 291, normalized size = 1.14

$$\frac{-19ab^2-30a^2bcx+19b^3cx-16ab^2c^2x^2+10a^2bc^3x^3-b^3c^3x^3+3ab^2c^4x^4-6a^2bc^5x^5+10a^3c^6x^6+2b^2(b(23i-15cx+5c^3x^3-3c^5x^5)+15a(1+c^6x^6))\operatorname{ArcTan}(cx)^2+10b^3(1+c^6x^6)\operatorname{ArcTan}(cx)^3+b\operatorname{ArcTan}(cx)\,(b^2(-19-16c^2x^2+3c^4x^4)-4abcx(15-5c^2x^2+3c^4x^4)+30a^2(1+c^6x^6)-92b^2\log\left(1+e^{2i\operatorname{ArcTan}(cx)}\right))+46ab^2\log\left(1+c^2x^2\right)+46ib^3\operatorname{PolyLog}\left(2,-e^{2i\operatorname{ArcTan}(cx)}\right)}{60c^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (-19*a*b^2 - 30*a^2*b*c*x + 19*b^3*c*x - 16*a*b^2*c^2*x^2 + 10*a^2*b*c^3*x^
3 - b^3*c^3*x^3 + 3*a*b^2*c^4*x^4 - 6*a^2*b*c^5*x^5 + 10*a^3*c^6*x^6 + 2*b^
2*(b*(23*I - 15*c*x + 5*c^3*x^3 - 3*c^5*x^5) + 15*a*(1 + c^6*x^6))*ArcTan[c
*x]^2 + 10*b^3*(1 + c^6*x^6)*ArcTan[c*x]^3 + b*ArcTan[c*x]*(b^2*(-19 - 16*c
^2*x^2 + 3*c^4*x^4) - 4*a*b*c*x*(15 - 5*c^2*x^2 + 3*c^4*x^4) + 30*a^2*(1 +
c^6*x^6) - 92*b^2*Log[1 + E^((2*I)*ArcTan[c*x])]) + 46*a*b^2*Log[1 + c^2*x^
2] + (46*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(60*c^6)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(225) = 450$.
time = 0.47, size = 494, normalized size = 1.94

method	result
derivativedivides	$\frac{\frac{c^6x^6a^3}{6}-\frac{4b^3\arctan(cx)c^2x^2}{15}+\frac{b^3c^6x^6\arctan(cx)^3}{6}-\frac{a^2bcx}{2}+\frac{a\,b^2c^4x^4}{20}-\frac{4a\,b^2c^2x^2}{15}-\frac{b^3\arctan(cx)^2c^5x^5}{10}+\frac{b^3\arctan(cx)^2c^3x^3}{6}-b}{}$
default	$\frac{\frac{c^6x^6a^3}{6}-\frac{4b^3\arctan(cx)c^2x^2}{15}+\frac{b^3c^6x^6\arctan(cx)^3}{6}-\frac{a^2bcx}{2}+\frac{a\,b^2c^4x^4}{20}-\frac{4a\,b^2c^2x^2}{15}-\frac{b^3\arctan(cx)^2c^5x^5}{10}+\frac{b^3\arctan(cx)^2c^3x^3}{6}-b}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^6*(1/6*c^6*x^6*a^3-23/60*I*b^3*ln(c*x+I)*ln(c^2*x^2+1)+23/60*I*b^3*ln(c
*x-I)*ln(c^2*x^2+1)-4/15*b^3*arctan(c*x)*c^2*x^2+1/6*b^3*c^6*x^6*arctan(c*x
)^3+23/60*I*b^3*ln(c*x+I)*ln(1/2*I*(c*x-I))-23/60*I*b^3*ln(c*x-I)*ln(-1/2*I
*(c*x+I))-1/2*a^2*b*c*x+1/20*a*b^2*c^4*x^4-4/15*a*b^2*c^2*x^2-1/10*b^3*arct
an(c*x)^2*c^5*x^5+1/6*b^3*arctan(c*x)^2*c^3*x^3-1/2*b^3*arctan(c*x)^2*c*x+1
/20*b^3*arctan(c*x)*c^4*x^4-1/10*c^5*x^5*a^2*b+1/6*a^2*b*c^3*x^3+19/60*b^3*
c*x-1/60*b^3*c^3*x^3+23/30*b^3*arctan(c*x)*ln(c^2*x^2+1)-23/60*I*b^3*dilog(
-1/2*I*(c*x+I))+23/120*I*b^3*ln(c*x+I)^2+23/60*I*b^3*dilog(1/2*I*(c*x-I))+1
/2*a*b^2*arctan(c*x)^2+23/30*a*b^2*ln(c^2*x^2+1)-23/120*I*b^3*ln(c*x-I)^2+1
/2*a^2*b*arctan(c*x)+1/6*b^3*arctan(c*x)^3-19/60*b^3*arctan(c*x)+1/2*a^2*b*
c^6*x^6*arctan(c*x)+1/2*a*b^2*c^6*x^6*arctan(c*x)^2-1/5*a*b^2*c^5*x^5*arcta
n(c*x)+1/3*a*b^2*c^3*x^3*arctan(c*x)-a*b^2*c*x*arctan(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*b^2*x^6*arctan(c*x)^2 + 1/6*a^3*x^6 + 1/30*(15*x^6*arctan(c*x) - c*((
3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*a^2*b - 1/60*(4*c*
((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7)*arctan(c*x) - (3*
c^4*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^2 + 46*log(c^2*x^2 + 1))/c^6)*a*b^2 +
 1/480*(20*(5760*c^7*integrate(1/480*x^7*arctan(c*x)^3/(c^7*x^2 + c^5), x)
- 1440*c^6*integrate(1/480*x^6*arctan(c*x)^2/(c^7*x^2 + c^5), x) - 360*c^6*
integrate(1/480*x^6*log(c^2*x^2 + 1)^2/(c^7*x^2 + c^5), x) - 288*c^6*integr
ate(1/480*x^6*log(c^2*x^2 + 1)/(c^7*x^2 + c^5), x) + 5760*c^5*integrate(1/4
80*x^5*arctan(c*x)^3/(c^7*x^2 + c^5), x) + 576*c^5*integrate(1/480*x^5*arct
an(c*x)/(c^7*x^2 + c^5), x) + 480*c^4*integrate(1/480*x^4*log(c^2*x^2 + 1)/
(c^7*x^2 + c^5), x) - 960*c^3*integrate(1/480*x^3*arctan(c*x)/(c^7*x^2 + c^
5), x) - 1440*c^2*integrate(1/480*x^2*log(c^2*x^2 + 1)/(c^7*x^2 + c^5), x)
+ 2880*c*integrate(1/480*x*arctan(c*x)/(c^7*x^2 + c^5), x) - arctan(c*x)^3/
c^6 - 360*integrate(1/480*log(c^2*x^2 + 1)^2/(c^7*x^2 + c^5), x))*c^6 + 40*
(c^6*x^6 + 1)*arctan(c*x)^3 - 4*(3*c^5*x^5 - 5*c^3*x^3 + 15*c*x)*arctan(c*x
)^2 + (3*c^5*x^5 - 5*c^3*x^3 + 15*c*x)*log(c^2*x^2 + 1)^2)*b^3/c^6
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^5*arctan(c*x)^3 + 3*a*b^2*x^5*arctan(c*x)^2 + 3*a^2*b*x^5*ar
ctan(c*x) + a^3*x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5(a + b\,\mathrm{atan}\,(cx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral(x**5*(a + b*atan(c*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,(a+b\,\mathrm{atan}(c\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atan(c*x))^3,x)
```

```
[Out] int(x^5*(a + b*atan(c*x))^3, x)
```

3.25 $\int x^4(a+b\mathbf{ArcTan}(cx))^3\,dx$

Optimal. Leaf size=271

$$-\frac{9ab^2x}{10c^4}-\frac{b^3x^2}{20c^3}-\frac{9b^3x\mathrm{ArcTan}(cx)}{10c^4}+\frac{b^2x^3(a+b\mathrm{ArcTan}(cx))}{10c^2}+\frac{9b(a+b\mathrm{ArcTan}(cx))^2}{20c^5}+\frac{3bx^2(a+b\mathrm{ArcTan}(cx))}{10c^3}$$

```
[Out] -9/10*a*b^2*x/c^4-1/20*b^3*x^2/c^3-9/10*b^3*x*arctan(c*x)/c^4+1/10*b^2*x^3*
(a+b*arctan(c*x))/c^2+9/20*b*(a+b*arctan(c*x))^2/c^5+3/10*b*x^2*(a+b*arctan
(c*x))^2/c^3-3/20*b*x^4*(a+b*arctan(c*x))^2/c+1/5*I*(a+b*arctan(c*x))^3/c^5
+1/5*x^5*(a+b*arctan(c*x))^3+3/5*b*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^5+
1/2*b^3*ln(c^2*x^2+1)/c^5+3/5*I*b^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*
x))/c^5+3/10*b^3*polylog(3,1-2/(1+I*c*x))/c^5
```

Rubi [A]
time = 0.51, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {4946, 5036, 272, 45, 4930, 266, 5004, 5040, 4964, 5114, 6745}

$$\frac{3ib^2\mathrm{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\mathrm{ArcTan}(cx))}{5c^5}+\frac{b^2x^3(a+b\mathrm{ArcTan}(cx))}{10c^2}+\frac{i(a+b\mathrm{ArcTan}(cx))^3}{5c^5}+\frac{9b(a+b\mathrm{ArcTan}(cx))^2}{20c^5}+\frac{3b\log\left(\frac{2}{1+icx}\right)(a+b\mathrm{ArcTan}(cx))^2}{5c^5}+\frac{3bx^2(a+b\mathrm{ArcTan}(cx))^2}{10c^3}+\frac{1}{5}x^5(a+b\mathrm{ArcTan}(cx))^3-\frac{3bx^4(a+b\mathrm{ArcTan}(cx))^2}{20c}-\frac{9ab^2x}{10c^4}-\frac{9b^3x\mathrm{ArcTan}(cx)}{10c^4}+\frac{3b^3\mathrm{Li}_3\left(1-\frac{2}{icx+1}\right)}{10c^5}-\frac{b^3x^2}{20c^3}+\frac{b^3\log\left(c^2x^2+1\right)}{2c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (-9*a*b^2*x)/(10*c^4) - (b^3*x^2)/(20*c^3) - (9*b^3*x*ArcTan[c*x])/(10*c^4)
 + (b^2*x^3*(a + b*ArcTan[c*x]))/(10*c^2) + (9*b*(a + b*ArcTan[c*x])^2)/(20
*c^5) + (3*b*x^2*(a + b*ArcTan[c*x])^2)/(10*c^3) - (3*b*x^4*(a + b*ArcTan[c
*x])^2)/(20*c) + ((I/5)*(a + b*ArcTan[c*x])^3)/c^5 + (x^5*(a + b*ArcTan[c*x
])^3)/5 + (3*b*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(5*c^5) + (b^3*Log
[1 + c^2*x^2])/(2*c^5) + (((3*I)/5)*b^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 -
2/(1 + I*c*x)])/c^5 + (3*b^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/(10*c^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int x^4\left(a+b\tan^{-1}(cx)\right)^3\,dx=\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)^3-\frac{1}{5}(3bc)\int\frac{x^5\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx$$

$$=\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)^3-\frac{(3b)\int x^3\left(a+b\tan^{-1}(cx)\right)^2\,dx}{5c}+\frac{(3b)\int\frac{x^3(a+b\tan^{-1}(cx}{1+c^2x^2}}{5c}$$

$$=-\frac{3bx^4\left(a+b\tan^{-1}(cx)\right)^2}{20c}+\frac{1}{5}x^5\left(a+b\tan^{-1}(cx)\right)^3+\frac{1}{10}\left(3b^2\right)\int\frac{x^4(a+b\tan^-}{1+c^2x^2}$$

$$=\frac{3bx^2\left(a+b\tan^{-1}(cx)\right)^2}{10c^3}-\frac{3bx^4\left(a+b\tan^{-1}(cx)\right)^2}{20c}+\frac{i\left(a+b\tan^{-1}(cx)\right)^3}{5c^5}+\frac{1}{5}x^5$$

$$=\frac{b^2x^3\left(a+b\tan^{-1}(cx)\right)}{10c^2}+\frac{3bx^2\left(a+b\tan^{-1}(cx)\right)^2}{10c^3}-\frac{3bx^4\left(a+b\tan^{-1}(cx)\right)^2}{20c}+\frac{i(}{}$$

$$=-\frac{9ab^2x}{10c^4}+\frac{b^2x^3\left(a+b\tan^{-1}(cx)\right)}{10c^2}+\frac{9b\left(a+b\tan^{-1}(cx)\right)^2}{20c^5}+\frac{3bx^2(a+b\tan^{-1}(c}{10c^3}$$

$$=-\frac{9ab^2x}{10c^4}-\frac{9b^3x\tan^{-1}(cx)}{10c^4}+\frac{b^2x^3\left(a+b\tan^{-1}(cx)\right)}{10c^2}+\frac{9b\left(a+b\tan^{-1}(cx)\right)^2}{20c^5}+\frac{3}{}$$

$$=-\frac{9ab^2x}{10c^4}-\frac{b^3x^2}{20c^3}-\frac{9b^3x\tan^{-1}(cx)}{10c^4}+\frac{b^2x^3\left(a+b\tan^{-1}(cx)\right)}{10c^2}+\frac{9b(a+b\tan^{-1}(c}{20c^5}$$

Mathematica [**A**]
time = 0.64, size = 396, normalized size = 1.46

$$\frac{-b^3-18ab^2cx+6a^2bc^2x^2-b^3c^2x^2+2ab^2c^3x^3-3a^2bc^4x^4+4a^3c^5x^5+18ab^2\text{ArcTan}(cx)-18b^3cx\text{ArcTan}(cx)+12ab^2c^2x^2\text{ArcTan}(cx)+2b^3c^3x^3\text{ArcTan}(cx)-6ab^2c^4x^4\text{ArcTan}(cx)+12a^2bc^5x^5\text{ArcTan}(cx)-12iab^2\text{ArcTan}(cx)^2+9b^3\text{ArcTan}(cx)^2+6b^3c^2x^2\text{ArcTan}(cx)^2-3b^3c^4x^4\text{ArcTan}(cx)^2+12ab^2c^5x^5\text{ArcTan}(cx)^2-4ib^3\text{ArcTan}(cx)^3+4b^3c^5x^5\text{ArcTan}(cx)^3+24ab^2\text{ArcTan}(cx)\log\left(1+e^{2i\text{ArcTan}(cx)}\right)+12b^3\text{ArcTan}(cx)^2\log\left(1+e^{2i\text{ArcTan}(cx)}\right)-6a^2b\log\left(1+c^2x^2\right)+10b^3\log\left(1+c^2x^2\right)-12ib^2(a+b\text{ArcTan}(cx))\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)+6b^3\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(cx)}\right)}{20c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (-b^3 - 18*a*b^2*c*x + 6*a^2*b*c^2*x^2 - b^3*c^2*x^2 + 2*a*b^2*c^3*x^3 - 3*
a^2*b*c^4*x^4 + 4*a^3*c^5*x^5 + 18*a*b^2*ArcTan[c*x] - 18*b^3*c*x*ArcTan[c*
x] + 12*a*b^2*c^2*x^2*ArcTan[c*x] + 2*b^3*c^3*x^3*ArcTan[c*x] - 6*a*b^2*c^4
*x^4*ArcTan[c*x] + 12*a^2*b*c^5*x^5*ArcTan[c*x] - (12*I)*a*b^2*ArcTan[c*x]^
2 + 9*b^3*ArcTan[c*x]^2 + 6*b^3*c^2*x^2*ArcTan[c*x]^2 - 3*b^3*c^4*x^4*ArcTa
n[c*x]^2 + 12*a*b^2*c^5*x^5*ArcTan[c*x]^2 - (4*I)*b^3*ArcTan[c*x]^3 + 4*b^3
*c^5*x^5*ArcTan[c*x]^3 + 24*a*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])
] + 12*b^3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - 6*a^2*b*Log[1 + c
^2*x^2] + 10*b^3*Log[1 + c^2*x^2] - (12*I)*b^2*(a + b*ArcTan[c*x])*PolyLog[
2, -E^((2*I)*ArcTan[c*x])] + 6*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x])])/(20*
c^5)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 18.19, size = 2951, normalized size = 10.89

method	result	size
derivativedivides	Expression too large to display	2951
default	Expression too large to display	2951

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(9/80*b^3*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)*csgn(I*(1+I
*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*Pi*c*x+9/160*b^3*arc
tan(c*x)^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*Pi*c*x-1/20*b^3-3/160*b^
3*arctan(c*x)^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I*(1+(1+I*c*x)^2
/(c^2*x^2+1))^2)*Pi*c^3*x^3-9/160*b^3*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c^2
*x^2+1)+I)^2*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I
)*Pi*c*x-3/80*b^3*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)*csgn(I*(1
+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*Pi*c^3*x^3+9/160*b
^3*arctan(c*x)^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I*(1+(1+I*c*x)^
2/(c^2*x^2+1))^2)*Pi*c*x-9/80*b^3*arctan(c*x)^2*csgn(I*(1+(1+I*c*x)^2/(c^2*
x^2+1)))*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*Pi*c*x+3/160*b^3*arctan(c*
x)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2
*I*(1+I*c*x)^2/(c^2*x^2+1)+I)*Pi*c^3*x^3+9/10*a*b^2*arctan(c*x)-3/10*a^2*b*
ln(c^2*x^2+1)+3/160*I*b^3*arctan(c*x)^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^
2)^3*Pi+21/160*I*b^3*arctan(c*x)^2*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+
I*c*x)^2/(c^2*x^2+1)+I)^3*Pi-3/20*I*b^3*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c
^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*Pi-1/20*b^3*c^2*x^2-3/10*a*b^2*c
^4*x^4*arctan(c*x)+3/5*a*b^2*c^2*x^2*arctan(c*x)-3/20*I*b^3*arctan(c*x)^2*c
sgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*Pi+3/5*c^5*x^5*a*b^2*arctan(c*x)^2+3/5*c^5
*x^5*a^2*b*arctan(c*x)-3/10*I*a*b^2*ln(c*x-I)*ln(c^2*x^2+1)+3/10*I*a*b^2*ln
(c*x-I)*ln(-1/2*I*(c*x+I))+3/10*I*a*b^2*ln(c*x+I)*ln(c^2*x^2+1)-3/10*I*a*b^
```

```
2*ln(c*x+I)*ln(1/2*I*(c*x-I))+3/5*b^3*arctan(c*x)^2*ln((1+I*c*x)/(c^2*x^2+1
)^(1/2))+I*b^3*arctan(c*x)-3/10*b^3*arctan(c*x)^2*ln(c^2*x^2+1)+3/5*b^3*ln(
2)*arctan(c*x)^2-1/5*I*b^3*arctan(c*x)^3+1/5*c^5*x^5*a^3+1/10*a*b^2*c^3*x^3
-9/10*a*b^2*c*x-3/5*a*b^2*arctan(c*x)*ln(c^2*x^2+1)+3/10*I*a*b^2*dilog(-1/2
*I*(c*x+I))+3/20*I*a*b^2*ln(c*x-I)^2-3/10*I*a*b^2*dilog(1/2*I*(c*x-I))-3/20
*I*a*b^2*ln(c*x+I)^2-3/20*a^2*b*c^4*x^4+3/10*a^2*b*c^2*x^2+1/5*c^5*x^5*b^3*
arctan(c*x)^3+3/80*b^3*arctan(c*x)^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))*cs
gn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*Pi*c^3*x^3+3/160*b^3*arctan(c*x)^2*cs
gn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^3*Pi*c^3*x^3-
3/160*b^3*arctan(c*x)^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*Pi*c^3*x^3-
9/160*b^3*arctan(c*x)^2*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c
^2*x^2+1)+I)^3*Pi*c*x-3/5*I*b^3*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2
+1))+9/160*I*b^3*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*csgn(I*(
1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)*Pi*c^2*x^2-9/80*I*b
^3*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)*csgn(I*(1+I*c*x)^4/(c^2*
x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*Pi*c^2*x^2-9/160*I*b^3*arctan(c*x
)^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)
)^2)*Pi*c^2*x^2+9/80*I*b^3*arctan(c*x)^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))
)*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*Pi*c^2*x^2-3/20*I*b^3*arctan(c*x)
^2*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*cs
gn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*Pi+9/160*I*b^3*
arctan(c*x)^2*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+
I)^3*Pi*c^2*x^2-9/160*I*b^3*arctan(c*x)^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)
)^2)^3*Pi*c^2*x^2-3/20*b^3*c^4*x^4*arctan(c*x)^2+3/10*b^3*c^2*x^2*arctan(c*
x)^2-9/10*b^3*arctan(c*x)*c*x+1/10*b^3*arctan(c*x)*c^3*x^3-3/20*I*b^3*arcta
n(c*x)^2*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+
1))*Pi+3/10*I*b^3*arctan(c*x)^2*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*
(1+I*c*x)^2/(c^2*x^2+1))^2*Pi+3/20*I*b^3*arctan(c*x)^2*csgn(I/(1+(1+I*c*x)^
2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)
)^2)^2*Pi+21/160*I*b^3*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*cs
gn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)*Pi-21/80*I*b^
3*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)*csgn(I*(1+I*c*x)^4/(c^2*x
^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*Pi+3/20*I*b^3*arctan(c*x)^2*csgn(I
*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^
2*x^2+1))^2)^2*Pi+3/160*I*b^3*arctan(c*x)^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+
1)))^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*Pi-3/80*I*b^3*arctan(c*x)^2*cs
gn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*P
i+9/20*b^3*arctan(c*x)^2+3/10*b^3*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-b^3*l
n(1+(1+I*c*x)^2/(c^2*x^2+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/40*b^3*x^5*arctan(c*x)^3 - 3/160*b^3*x^5*arctan(c*x)*log(c^2*x^2 + 1)^2 +
 1/5*a^3*x^5 + 3/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c
^2*x^2 + 1)/c^6))*a^2*b + integrate(1/160*(12*b^3*c^2*x^6*arctan(c*x)*log(c
^2*x^2 + 1) + 140*(b^3*c^2*x^6 + b^3*x^4)*arctan(c*x)^3 + 12*(40*a*b^2*c^2*
x^6 - b^3*c*x^5 + 40*a*b^2*x^4)*arctan(c*x)^2 + 3*(b^3*c*x^5 + 5*(b^3*c^2*x
^6 + b^3*x^4)*arctan(c*x))*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^4*arctan(c*x)^3 + 3*a*b^2*x^4*arctan(c*x)^2 + 3*a^2*b*x^4*ar
ctan(c*x) + a^3*x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{4}(a+b\operatorname{atan}(cx))^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral(x**4*(a + b*atan(c*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atan(c*x))^3,x)
```

```
[Out] int(x^4*(a + b*atan(c*x))^3, x)
```

3.26 $\int x^3(a+b\text{ArcTan}(cx))^3\,dx$

Optimal. Leaf size=194

$$-\frac{b^3x}{4c^3}+\frac{b^3\text{ArcTan}(cx)}{4c^4}+\frac{b^2x^2(a+b\text{ArcTan}(cx))}{4c^2}+\frac{ib(a+b\text{ArcTan}(cx))^2}{c^4}+\frac{3bx(a+b\text{ArcTan}(cx))^2}{4c^3}-\frac{bx^3(a+b$$

```
[Out] -1/4*b^3*x/c^3+1/4*b^3*arctan(c*x)/c^4+1/4*b^2*x^2*(a+b*arctan(c*x))/c^2+I*
b*(a+b*arctan(c*x))^2/c^4+3/4*b*x*(a+b*arctan(c*x))^2/c^3-1/4*b*x^3*(a+b*ar
ctan(c*x))^2/c-1/4*(a+b*arctan(c*x))^3/c^4+1/4*x^4*(a+b*arctan(c*x))^3+2*b^
2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4+I*b^3*polylog(2,1-2/(1+I*c*x))/c^4
```

Rubi [A]
time = 0.38, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.714$, Rules used = {4946, 5036, 327, 209, 5040, 4964, 2449, 2352, 4930, 5004}

$$\frac{2b^2\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c^4}+\frac{b^2x^2(a+b\text{ArcTan}(cx))}{4c^2}-\frac{(a+b\text{ArcTan}(cx))^3}{4c^4}+\frac{ib(a+b\text{ArcTan}(cx))^2}{c^4}+\frac{3bx(a+b\text{ArcTan}(cx))^2}{4c^3}+\frac{1}{4}x^4(a+b\text{ArcTan}(cx))^3-\frac{bx^3(a+b\text{ArcTan}(cx))^2}{4c}+\frac{b^3\text{ArcTan}(cx)}{4c^4}+\frac{ib^3\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{c^4}-\frac{b^3x}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] -1/4*(b^3*x)/c^3 + (b^3*ArcTan[c*x])/(4*c^4) + (b^2*x^2*(a + b*ArcTan[c*x])
)/(4*c^2) + (I*b*(a + b*ArcTan[c*x])^2)/c^4 + (3*b*x*(a + b*ArcTan[c*x])^2)
/(4*c^3) - (b*x^3*(a + b*ArcTan[c*x])^2)/(4*c) - (a + b*ArcTan[c*x])^3/(4*c
^4) + (x^4*(a + b*ArcTan[c*x])^3)/4 + (2*b^2*(a + b*ArcTan[c*x])*Log[2/(1 +
 I*c*x)])/c^4 + (I*b^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+b\tan^{-1}(cx)\right)^3\,dx&=\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^3-\frac{1}{4}(3bc)\int\frac{x^4\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx\\&=\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^3-\frac{(3b)\int x^2\left(a+b\tan^{-1}(cx)\right)^2\,dx}{4c}+\frac{(3b)\int\frac{x^2(a+b\tan^{-1}(cx}{1+c^2x^2}}{4c}\\&=-\frac{bx^3\left(a+b\tan^{-1}(cx)\right)^2}{4c}+\frac{1}{4}x^4\left(a+b\tan^{-1}(cx)\right)^3+\frac{1}{2}b^2\int\frac{x^3\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\\&=\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{4c^3}-\frac{bx^3\left(a+b\tan^{-1}(cx)\right)^2}{4c}-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{4c^4}+\frac{1}{4}x^4(a\\&=\frac{b^2x^2\left(a+b\tan^{-1}(cx)\right)}{4c^2}+\frac{ib\left(a+b\tan^{-1}(cx)\right)^2}{c^4}+\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{4c^3}-\frac{bx^3(a}{}\\&=-\frac{b^3x}{4c^3}+\frac{b^2x^2\left(a+b\tan^{-1}(cx)\right)}{4c^2}+\frac{ib\left(a+b\tan^{-1}(cx)\right)^2}{c^4}+\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{4c^3}\\&=-\frac{b^3x}{4c^3}+\frac{b^3\tan^{-1}(cx)}{4c^4}+\frac{b^2x^2\left(a+b\tan^{-1}(cx)\right)}{4c^2}+\frac{ib\left(a+b\tan^{-1}(cx)\right)^2}{c^4}+\frac{3bx(a}{}\\&=-\frac{b^3x}{4c^3}+\frac{b^3\tan^{-1}(cx)}{4c^4}+\frac{b^2x^2\left(a+b\tan^{-1}(cx)\right)}{4c^2}+\frac{ib\left(a+b\tan^{-1}(cx)\right)^2}{c^4}+\frac{3bx(a}{}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 225, normalized size = 1.16

$$\frac{ab^2+3a^2bcx-b^3cx+ab^2c^2x^2-a^2bc^3x^3+a^3c^4x^4-b^2(b(4i-3cx+c^3x^3)+a(3-3c^4x^4))\,\mathrm{ArcTan}(cx)^2+b^3(-1+c^4x^4)\,\mathrm{ArcTan}(cx)^3+b\mathrm{ArcTan}(cx)\left(-2abcx(-3+c^2x^2)+b^2(1+c^2x^2)+3a^2(-1+c^4x^4)+8b^2\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right)\right)-4ab^2\log\left(1+c^2x^2\right)-4ib^3\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (a*b^2 + 3*a^2*b*c*x - b^3*c*x + a*b^2*c^2*x^2 - a^2*b*c^3*x^3 + a^3*c^4*x^
4 - b^2*(b*(4*I - 3*c*x + c^3*x^3) + a*(3 - 3*c^4*x^4))*ArcTan[c*x]^2 + b^3
*(-1 + c^4*x^4)*ArcTan[c*x]^3 + b*ArcTan[c*x]*(-2*a*b*c*x*(-3 + c^2*x^2) +
b^2*(1 + c^2*x^2) + 3*a^2*(-1 + c^4*x^4) + 8*b^2*Log[1 + E^((2*I)*ArcTan[c*
x])]) - 4*a*b^2*Log[1 + c^2*x^2] - (4*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*
x])])/(4*c^4)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(176) = 352.
time = 0.54, size = 411, normalized size = 2.12 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(-1/2*I*b^3*ln(c*x+I)*ln(1/2*I*(c*x-I))+1/2*I*b^3*ln(c*x-I)*ln(-1/2*I
*(c*x+I))+1/2*I*b^3*ln(c*x+I)*ln(c^2*x^2+1)-1/2*I*b^3*ln(c*x-I)*ln(c^2*x^2+
1)+1/4*b^3*arctan(c*x)*c^2*x^2+3/4*a^2*b*c*x+1/4*a*b^2*c^2*x^2-1/4*b^3*arct
an(c*x)^2*c^3*x^3+3/4*b^3*arctan(c*x)^2*c*x-1/4*a^2*b*c^3*x^3-1/4*b^3*c*x-b
^3*arctan(c*x)*ln(c^2*x^2+1)-3/4*a*b^2*arctan(c*x)^2-a*b^2*ln(c^2*x^2+1)-3/
4*a^2*b*arctan(c*x)+1/4*I*b^3*ln(c*x-I)^2+1/2*I*b^3*dilog(-1/2*I*(c*x+I))-1
/4*I*b^3*ln(c*x+I)^2-1/2*I*b^3*dilog(1/2*I*(c*x-I))+1/4*c^4*x^4*a^3+3/4*c^4
*x^4*a^2*b*arctan(c*x)+3/4*a*b^2*c^4*x^4*arctan(c*x)^2-1/4*b^3*arctan(c*x)^
3+1/4*b^3*arctan(c*x)+1/4*c^4*x^4*b^3*arctan(c*x)^3-1/2*a*b^2*c^3*x^3*arcta
n(c*x)+3/2*a*b^2*c*x*arctan(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 3/4*a*b^2*x^4*arctan(c*x)^2 + 1/4*a^3*x^4 + 1/4*(3*x^4*arctan(c*x) - c*((c^
2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a^2*b - 1/4*(2*c*((c^2*x^3 - 3*x)/c^
4 + 3*arctan(c*x)/c^5)*arctan(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2
*x^2 + 1))/c^4)*a*b^2 + 1/64*(4*(512*c^5*integrate(1/64*x^5*arctan(c*x)^3/(
c^5*x^2 + c^3), x) - 192*c^4*integrate(1/64*x^4*arctan(c*x)^2/(c^5*x^2 + c^
3), x) - 48*c^4*integrate(1/64*x^4*log(c^2*x^2 + 1)^2/(c^5*x^2 + c^3), x) -
 64*c^4*integrate(1/64*x^4*log(c^2*x^2 + 1)/(c^5*x^2 + c^3), x) + 512*c^3*i
ntegrate(1/64*x^3*arctan(c*x)^3/(c^5*x^2 + c^3), x) + 128*c^3*integrate(1/6
4*x^3*arctan(c*x)/(c^5*x^2 + c^3), x) + 192*c^2*integrate(1/64*x^2*log(c^2*
x^2 + 1)/(c^5*x^2 + c^3), x) - 384*c*integrate(1/64*x*arctan(c*x)/(c^5*x^2
+ c^3), x) + arctan(c*x)^3/c^4 + 48*integrate(1/64*log(c^2*x^2 + 1)^2/(c^5*
x^2 + c^3), x))*c^4 + 8*(c^4*x^4 - 1)*arctan(c*x)^3 - 4*(c^3*x^3 - 3*c*x)*a
rctan(c*x)^2 + (c^3*x^3 - 3*c*x)*log(c^2*x^2 + 1)^2)*b^3/c^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctan(c*x)^3 + 3*a*b^2*x^3*arctan(c*x)^2 + 3*a^2*b*x^3*ar
ctan(c*x) + a^3*x^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a + b\,\mathrm{atan}\,(cx))^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral(x**3*(a + b*atan(c*x))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,{\left(a+b\,\mathrm{atan}(c\,x)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x))^3,x)
```

```
[Out] int(x^3*(a + b*atan(c*x))^3, x)
```

3.27 $\int x^2(a+b\text{ArcTan}(cx))^3\,dx$

Optimal. Leaf size=206

$$\frac{ab^2x}{c^2}+\frac{b^3x\text{ArcTan}(cx)}{c^2}-\frac{b(a+b\text{ArcTan}(cx))^2}{2c^3}-\frac{bx^2(a+b\text{ArcTan}(cx))^2}{2c}-\frac{i(a+b\text{ArcTan}(cx))^3}{3c^3}+\frac{1}{3}x^3(a+b\text{Ar}$$

```
[Out] a*b^2*x/c^2+b^3*x*arctan(c*x)/c^2-1/2*b*(a+b*arctan(c*x))^2/c^3-1/2*b*x^2*(
a+b*arctan(c*x))^2/c-1/3*I*(a+b*arctan(c*x))^3/c^3+1/3*x^3*(a+b*arctan(c*x)
)^3-b*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^3-1/2*b^3*ln(c^2*x^2+1)/c^3-I*b
^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^3-1/2*b^3*polylog(3,1-2/(1+
I*c*x))/c^3
```

Rubi [A]
time = 0.31, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {4946, 5036, 4930, 266, 5004, 5040, 4964, 5114, 6745}

$$-\frac{ib^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{c^3}-\frac{b(a+b\text{ArcTan}(cx))^2}{2c^3}-\frac{i(a+b\text{ArcTan}(cx))^3}{3c^3}-\frac{b\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{c^3}+\frac{1}{3}x^3(a+b\text{ArcTan}(cx))^3-\frac{bx^2(a+b\text{ArcTan}(cx))^2}{2c}+\frac{ab^2x}{c^2}+\frac{b^3x\text{ArcTan}(cx)}{c^2}-\frac{b^3\text{Li}_3\left(1-\frac{2}{icx+1}\right)}{2c^3}-\frac{b^3\log\left(c^2x^2+1\right)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (a*b^2*x)/c^2 + (b^3*x*ArcTan[c*x])/c^2 - (b*(a + b*ArcTan[c*x])^2)/(2*c^3)
 - (b*x^2*(a + b*ArcTan[c*x])^2)/(2*c) - ((I/3)*(a + b*ArcTan[c*x])^3)/c^3
+ (x^3*(a + b*ArcTan[c*x])^3)/3 - (b*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x
)])/c^3 - (b^3*Log[1 + c^2*x^2])/(2*c^3) - (I*b^2*(a + b*ArcTan[c*x])*PolyL
og[2, 1 - 2/(1 + I*c*x)])/c^3 - (b^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
```

```
IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int x^2\left(a+b\tan^{-1}(cx)\right)^3\,dx=\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^3-(bc)\int\frac{x^3\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx$$

$$=\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^3-\frac{b\int x\left(a+b\tan^{-1}(cx)\right)^2\,dx}{c}+\frac{b\int\frac{x\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx}{c}$$

$$=-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^3}{3c^3}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^3+b^2$$

$$=-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^3}{3c^3}+\frac{1}{3}x^3\left(a+b\tan^{-1}(cx)\right)^3-\frac{b(}{}$$

$$=\frac{ab^2x}{c^2}-\frac{b\left(a+b\tan^{-1}(cx)\right)^2}{2c^3}-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{i\left(a+b\tan^{-1}(cx)\right)^3}{3c^3}+$$

$$=\frac{ab^2x}{c^2}+\frac{b^3x\tan^{-1}(cx)}{c^2}-\frac{b\left(a+b\tan^{-1}(cx)\right)^2}{2c^3}-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{i(a-}{}$$

$$=\frac{ab^2x}{c^2}+\frac{b^3x\tan^{-1}(cx)}{c^2}-\frac{b\left(a+b\tan^{-1}(cx)\right)^2}{2c^3}-\frac{bx^2\left(a+b\tan^{-1}(cx)\right)^2}{2c}-\frac{i(a-}{}$$

Mathematica [A]
time = 0.42, size = 269, normalized size = 1.31

$$\frac{-3a^2bc^2x^2+2a^3c^3x^3+6a^2bc^3x^3\mathrm{ArcTan}(cx)+3a^2b\log\left(1+c^2x^2\right)+6ab^2\left(cx+\left(i+c^3x^3\right)\mathrm{ArcTan}(cx)^2-\mathrm{ArcTan}(cx)\left(1+c^2x^2+2\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right)\right)+i\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right)\right)+b^3\left(6cx\mathrm{ArcTan}(cx)-3\mathrm{ArcTan}(cx)^2-3c^2x^2\mathrm{ArcTan}(cx)^2+2i\mathrm{ArcTan}(cx)^3+2c^3x^3\mathrm{ArcTan}(cx)^3-6\mathrm{ArcTan}(cx)^2\log\left(1+e^{2i\mathrm{ArcTan}(cx)}\right)-3\log\left(1+c^2x^2\right)+6i\mathrm{ArcTan}(cx)\mathrm{PolyLog}\left(2,-e^{2i\mathrm{ArcTan}(cx)}\right)-3\mathrm{PolyLog}\left(3,-e^{2i\mathrm{ArcTan}(cx)}\right)\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (-3*a^2*b*c^2*x^2 + 2*a^3*c^3*x^3 + 6*a^2*b*c^3*x^3*ArcTan[c*x] + 3*a^2*b*L
og[1 + c^2*x^2] + 6*a*b^2*(c*x + (I + c^3*x^3)*ArcTan[c*x]^2 - ArcTan[c*x]*
(1 + c^2*x^2 + 2*Log[1 + E^((2*I)*ArcTan[c*x])]) + I*PolyLog[2, -E^((2*I)*A
rcTan[c*x])]) + b^3*(6*c*x*ArcTan[c*x] - 3*ArcTan[c*x]^2 - 3*c^2*x^2*ArcTan
[c*x]^2 + (2*I)*ArcTan[c*x]^3 + 2*c^3*x^3*ArcTan[c*x]^3 - 6*ArcTan[c*x]^2*L
og[1 + E^((2*I)*ArcTan[c*x])] - 3*Log[1 + c^2*x^2] + (6*I)*ArcTan[c*x]*Poly
Log[2, -E^((2*I)*ArcTan[c*x])] - 3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/(6*
c^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 5.49, size = 1921, normalized size = 9.33

method	result	size
derivativedivides	Expression too large to display	1921
default	Expression too large to display	1921

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-1/4*b^3*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)*csgn(I*(1+I
*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*Pi*c*x-1/8*b^3*arcta
n(c*x)^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*Pi*c*x+a*b^2*c^3*x^3*arcta
n(c*x)^2+1/2*I*a*b^2*ln(c*x-I)*ln(c^2*x^2+1)-1/2*I*a*b^2*ln(c*x-I)*ln(-1/2*
I*(c*x+I))-1/2*I*a*b^2*ln(c*x+I)*ln(c^2*x^2+1)+1/2*I*a*b^2*ln(c*x+I)*ln(1/2
*I*(c*x-I))+a^2*b*c^3*x^3*arctan(c*x)-1/8*I*b^3*csgn(I*(1+I*c*x)^4/(c^2*x^2
+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^3*arctan(c*x)^2*Pi+1/4*I*b^3*arctan(c*
x)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*Pi+1/4
*I*b^3*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*Pi+1/3*c^3*x^3*a^3+1
/8*b^3*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*csgn(I*(1+I*c*x)^4
/(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)*Pi*c*x-1/8*b^3*arctan(c*x)^2*
csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)
*Pi*c*x+1/4*b^3*arctan(c*x)^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1
+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*Pi*c*x-a*b^2*arctan(c*x)+1/2*a^2*b*ln(c^2*x^
2+1)+1/4*I*b^3*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*arctan(c*x)^2*csgn(I*(
1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c
^2*x^2+1))*Pi-I*b^3*arctan(c*x)+1/3*I*b^3*arctan(c*x)^3-1/8*I*b^3*arctan(c*
x)^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*Pi-a*b^2*c^2*x^2*arctan(c*x)+I
*b^3*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/3*b^3*c^3*x^3*arctan
(c*x)^3-b^3*arctan(c*x)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*b^3*arctan(c*
x)^2*ln(c^2*x^2+1)-b^3*ln(2)*arctan(c*x)^2+a*b^2*c*x+a*b^2*arctan(c*x)*ln(c
^2*x^2+1)-1/2*a^2*b*c^2*x^2-1/4*I*b^3*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)
*arctan(c*x)^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2
)^2*Pi-1/8*I*b^3*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)+I)^2*csgn(I*(1+I*c*x)^4/(c^
2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)*arctan(c*x)^2*Pi+1/4*I*b^3*csgn(I
*(1+I*c*x)^2/(c^2*x^2+1)+I)*csgn(I*(1+I*c*x)^4/(c^2*x^2+1)^2+2*I*(1+I*c*x)^
2/(c^2*x^2+1)+I)^2*arctan(c*x)^2*Pi-1/4*I*b^3*arctan(c*x)^2*csgn(I*(1+I*c*x
)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*csgn(I*(1+I*c*x)^2/(c^2*x^
2+1))*Pi+1/4*I*b^3*arctan(c*x)^2*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn
(I*(1+I*c*x)^2/(c^2*x^2+1))*Pi-1/2*I*b^3*arctan(c*x)^2*csgn(I*(1+I*c*x)/(c^
2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^2*Pi-1/8*I*b^3*arctan(c*x)^
2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^
2)*Pi+1/4*I*b^3*arctan(c*x)^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1
+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*Pi+1/8*b^3*arctan(c*x)^2*csgn(I*(1+I*c*x)^4/
(c^2*x^2+1)^2+2*I*(1+I*c*x)^2/(c^2*x^2+1)+I)^3*Pi*c*x-1/2*I*a*b^2*dilog(-1/
2*I*(c*x+I))-1/4*I*a*b^2*ln(c*x-I)^2+1/2*I*a*b^2*dilog(1/2*I*(c*x-I))+1/4*I
*a*b^2*ln(c*x+I)^2-1/2*b^3*c^2*x^2*arctan(c*x)^2+b^3*arctan(c*x)*c*x-1/2*b^
3*arctan(c*x)^2-1/2*b^3*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+b^3*ln(1+(1+I*c
*x)^2/(c^2*x^2+1)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*x^3*arctan(c*x)^3 - 1/32*b^3*x^3*arctan(c*x)*log(c^2*x^2 + 1)^2 +
1/3*a^3*x^3 + 1/2*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*
a^2*b + integrate(1/32*(4*b^3*c^2*x^4*arctan(c*x)*log(c^2*x^2 + 1) + 28*(b^
3*c^2*x^4 + b^3*x^2)*arctan(c*x)^3 + 4*(24*a*b^2*c^2*x^4 - b^3*c*x^3 + 24*a
*b^2*x^2)*arctan(c*x)^2 + (b^3*c*x^3 + 3*(b^3*c^2*x^4 + b^3*x^2)*arctan(c*x
))*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctan(c*x)^3 + 3*a*b^2*x^2*arctan(c*x)^2 + 3*a^2*b*x^2*ar
ctan(c*x) + a^3*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a+b\,\mathrm{atan}\,(cx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral(x**2*(a + b*atan(c*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 (a + b \operatorname{atan}(c x))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x))^3,x)
```

```
[Out] int(x^2*(a + b*atan(c*x))^3, x)
```

3.28 $\int x(a+b\mathbf{ArcTan}(cx))^3\,dx$

Optimal. Leaf size=131

$$-\frac{3ib(a+b\text{ArcTan}(cx))^2}{2c^2}-\frac{3bx(a+b\text{ArcTan}(cx))^2}{2c}+\frac{(a+b\text{ArcTan}(cx))^3}{2c^2}+\frac{1}{2}x^2(a+b\text{ArcTan}(cx))^3-\frac{3b^2(a+$$

```
[Out] -3/2*I*b*(a+b*arctan(c*x))^2/c^2-3/2*b*x*(a+b*arctan(c*x))^2/c+1/2*(a+b*arc
tan(c*x))^3/c^2+1/2*x^2*(a+b*arctan(c*x))^3-3*b^2*(a+b*arctan(c*x))*ln(2/(1
+I*c*x))/c^2-3/2*I*b^3*polylog(2,1-2/(1+I*c*x))/c^2
```

Rubi [A]
time = 0.17, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {4946, 5036, 4930, 5040, 4964, 2449, 2352, 5004}

$$-\frac{3b^2\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))}{c^2}-\frac{3ib(a+b\text{ArcTan}(cx))^2}{2c^2}+\frac{(a+b\text{ArcTan}(cx))^3}{2c^2}+\frac{1}{2}x^2(a+b\text{ArcTan}(cx))^3-\frac{3bx(a+b\text{ArcTan}(cx))^2}{2c}-\frac{3ib^3\text{Li}_2\left(1-\frac{2}{icx+1}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (((-3*I)/2)*b*(a + b*ArcTan[c*x])^2)/c^2 - (3*b*x*(a + b*ArcTan[c*x])^2)/(2
*c) + (a + b*ArcTan[c*x])^3/(2*c^2) + (x^2*(a + b*ArcTan[c*x])^3)/2 - (3*b^
2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^2 - (((3*I)/2)*b^3*PolyLog[2, 1
 - 2/(1 + I*c*x)])/c^2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x\left(a+b\tan^{-1}(cx)\right)^3\,dx &= \frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^3-\frac{1}{2}(3bc)\int \frac{x^2\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx\\
&= \frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^3-\frac{(3b)\int \left(a+b\tan^{-1}(cx)\right)^2\,dx}{2c}+\frac{(3b)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,d}{2c}\\
&= -\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2\left(a+b\tan^{-1}(cx)\right)^3+\left(3b^2\right.\\
&= -\frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}-\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2(\\
&= -\frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}-\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2(\\
&= -\frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}-\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2(\\
&= -\frac{3ib\left(a+b\tan^{-1}(cx)\right)^2}{2c^2}-\frac{3bx\left(a+b\tan^{-1}(cx)\right)^2}{2c}+\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2c^2}+\frac{1}{2}x^2(
\end{aligned}
$$

Mathematica [A]
time = 0.19, size = 152, normalized size = 1.16

$$\frac{3b^2(a+ac^2x^2+b(i-cx))\operatorname{ArcTan}(cx)^2+b^3(1+c^2x^2)\operatorname{ArcTan}(cx)^3+3b\operatorname{ArcTan}(cx)\left(a(a-2bcx+ac^2x^2)-2b^2\log\left(1+e^{2i\operatorname{ArcTan}(cx)}\right)\right)+a(acx(-3b+acx)+3b^2\log(1+c^2x^2))+3ib^3\operatorname{PolyLog}\left(2,-e^{2i\operatorname{ArcTan}(cx)}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (3*b^2*(a + a*c^2*x^2 + b*(I - c*x))*ArcTan[c*x]^2 + b^3*(1 + c^2*x^2)*ArcT
an[c*x]^3 + 3*b*ArcTan[c*x]*(a*(a - 2*b*c*x + a*c^2*x^2) - 2*b^2*Log[1 + E^
((2*I)*ArcTan[c*x])]) + a*(a*c*x*(-3*b + a*c*x) + 3*b^2*Log[1 + c^2*x^2]) +
 (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(2*c^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(117) = 234.
time = 0.37, size = 323, normalized size = 2.47

method	result
derivativedivides	$\frac{c^2x^2a^3}{2}+\frac{b^3c^2x^2\arctan(cx)^3}{2}+\frac{b^3\arctan(cx)^3}{2}-\frac{3b^3\arctan(cx)^2cx}{2}+\frac{3b^3\arctan(cx)\ln\left(c^2x^2+1\right)}{2}+\frac{3ib^3\ln(cx+i)\ln\left(\frac{i(cx-i)}{2}\right)}{4}-\frac{3ib}{}$
default	$\frac{c^2x^2a^3}{2}+\frac{b^3c^2x^2\arctan(cx)^3}{2}+\frac{b^3\arctan(cx)^3}{2}-\frac{3b^3\arctan(cx)^2cx}{2}+\frac{3b^3\arctan(cx)\ln\left(c^2x^2+1\right)}{2}+\frac{3ib^3\ln(cx+i)\ln\left(\frac{i(cx-i)}{2}\right)}{4}-\frac{3ib}{}$

risch	$-\frac{3ib^3(-icx+1)^2\ln(-icx+1)}{32c^2}+\frac{3ib^3(-icx+1)^2\ln(-icx+1)^2}{32c^2}+\frac{3b^2\ln(-icx+1)(-icx+1)^2a}{8c^2}+\frac{3ia\,b^2\arctan(cx)}{8c^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/2*c^2*x^2*a^3+1/2*b^3*c^2*x^2*arctan(c*x)^3+1/2*b^3*arctan(c*x)^3-
3/2*b^3*arctan(c*x)^2*c*x+3/2*b^3*arctan(c*x)*ln(c^2*x^2+1)+3/4*I*b^3*ln(c*
x+I)*ln(1/2*I*(c*x-I))+3/8*I*b^3*ln(c*x+I)^2-3/8*I*b^3*ln(c*x-I)^2-3/4*I*b^
3*dilog(-1/2*I*(c*x+I))+3/4*I*b^3*dilog(1/2*I*(c*x-I))-3/4*I*b^3*ln(c*x+I)*
ln(c^2*x^2+1)-3/4*I*b^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))+3/4*I*b^3*ln(c*x-I)*ln
(c^2*x^2+1)+3/2*a*b^2*c^2*x^2*arctan(c*x)^2+3/2*a*b^2*arctan(c*x)^2-3*a*b^2
*c*x*arctan(c*x)+3/2*a*b^2*ln(c^2*x^2+1)+3/2*a^2*b*c^2*x^2*arctan(c*x)-3/2*
a^2*b*c*x+3/2*a^2*b*arctan(c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a*b^2*x^2*arctan(c*x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arctan(c*x) - c*(x/c^2
 - arctan(c*x)/c^3))*a^2*b - 3/2*(2*c*(x/c^2 - arctan(c*x)/c^3)*arctan(c*x)
 + (arctan(c*x)^2 - log(c^2*x^2 + 1))/c^2)*a*b^2 - 1/32*(12*c*x*arctan(c*x)
^2 - 8*(c^2*x^2 + 1)*arctan(c*x)^3 - 3*c*x*log(c^2*x^2 + 1)^2 - 4*(128*c^3*
integrate(1/32*x^3*arctan(c*x)^3/(c^3*x^2 + c), x) - 96*c^2*integrate(1/32*
x^2*arctan(c*x)^2/(c^3*x^2 + c), x) - 24*c^2*integrate(1/32*x^2*log(c^2*x^2
 + 1)^2/(c^3*x^2 + c), x) - 96*c^2*integrate(1/32*x^2*log(c^2*x^2 + 1)/(c^3
*x^2 + c), x) + 128*c*integrate(1/32*x*arctan(c*x)^3/(c^3*x^2 + c), x) + 19
2*c*integrate(1/32*x*arctan(c*x)/(c^3*x^2 + c), x) - arctan(c*x)^3/c^2 - 24
*integrate(1/32*log(c^2*x^2 + 1)^2/(c^3*x^2 + c), x))*c^2)*b^3/c^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctan(c*x)^3 + 3*a*b^2*x*arctan(c*x)^2 + 3*a^2*b*x*arctan(c
*x) + a^3*x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+b\,\mathrm{atan}\,(cx))^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral(x*(a + b*atan(c*x))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x))^3,x)
```

```
[Out] int(x*(a + b*atan(c*x))^3, x)
```

3.29 $\int (a+b\mathbf{ArcTan}(cx))^3\,dx$

Optimal. Leaf size=119

$$\frac{i(a+b\text{ArcTan}(cx))^3}{c}+x(a+b\text{ArcTan}(cx))^3+\frac{3b(a+b\text{ArcTan}(cx))^2\log\left(\frac{2}{1+icx}\right)}{c}+\frac{3ib^2(a+b\text{ArcTan}(cx))\text{PolyI}}{c}$$

```
[Out] I*(a+b*arctan(c*x))^3/c+x*(a+b*arctan(c*x))^3+3*b*(a+b*arctan(c*x))^2*ln(2/
(1+I*c*x))/c+3*I*b^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c+3/2*b^3*p
olylog(3,1-2/(1+I*c*x))/c
```

Rubi [A]
time = 0.15, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4930, 5040, 4964, 5004, 5114, 6745}

$$\frac{3ib^2\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))}{c}+x(a+b\text{ArcTan}(cx))^3+\frac{i(a+b\text{ArcTan}(cx))^3}{c}+\frac{3b\log\left(\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^2}{c}+\frac{3b^3\text{Li}_3\left(1-\frac{2}{icx+1}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3,x]
```

```
[Out] (I*(a + b*ArcTan[c*x])^3)/c + x*(a + b*ArcTan[c*x])^3 + (3*b*(a + b*ArcTan[
c*x])^2*Log[2/(1 + I*c*x)])/c + ((3*I)*b^2*(a + b*ArcTan[c*x])*PolyLog[2, 1
 - 2/(1 + I*c*x)])/c + (3*b^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\tan^{-1}(cx)\right)^3\,dx &= x\left(a+b\tan^{-1}(cx)\right)^3-(3bc)\int \frac{x\left(a+b\tan^{-1}(cx)\right)^2}{1+c^2x^2}\,dx\\
&= \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{c}+x\left(a+b\tan^{-1}(cx)\right)^3+(3b)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{i-cx}\,dx\\
&= \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{c}+x\left(a+b\tan^{-1}(cx)\right)^3+\frac{3b\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{c}-\\
&= \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{c}+x\left(a+b\tan^{-1}(cx)\right)^3+\frac{3b\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{c}+\\
&= \frac{i\left(a+b\tan^{-1}(cx)\right)^3}{c}+x\left(a+b\tan^{-1}(cx)\right)^3+\frac{3b\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{2}{1+icx}\right)}{c}+
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 192, normalized size = 1.61

$$a^3x+3a^2bx\text{ArcTan}(cx)-\frac{3a^2b\log\left(1+c^2x^2\right)}{2c}+\frac{3ab^2\left(-i\text{ArcTan}(cx)^2+cx\text{ArcTan}(cx)^2+2\text{ArcTan}(cx)\log\left(1+e^{2i\text{ArcTan}(cx)}\right)-i\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)\right)}{c}+\frac{b^3\left(-i\text{ArcTan}(cx)^3+cx\text{ArcTan}(cx)^3+3\text{ArcTan}(cx)^2\log\left(1+e^{2i\text{ArcTan}(cx)}\right)-3i\text{ArcTan}(cx)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx)}\right)+\frac{3}{2}\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(cx)}\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3,x]
```

```
[Out] a^3*x + 3*a^2*b*x*ArcTan[c*x] - (3*a^2*b*Log[1 + c^2*x^2])/(2*c) + (3*a*b^2
*((-I)*ArcTan[c*x]^2 + c*x*ArcTan[c*x]^2 + 2*ArcTan[c*x]*Log[1 + E^((2*I)*A
```

```
rcTan[c*x])] - I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c + (b^3*((-I)*ArcTan
[c*x]^3 + c*x*ArcTan[c*x]^3 + 3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])
] - (3*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (3*PolyLog[3, -E
^((2*I)*ArcTan[c*x])])/2))/c
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(112) = 224$.
time = 1.52, size = 254, normalized size = 2.13

method	result
derivativedivides	$\frac{c\,a^3x - ib^3\arctan(cx)^3 + b^3cx\arctan(cx)^3 + 3b^3\arctan(cx)^2\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right) - 3ib^3\arctan(cx)\operatorname{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right) + \frac{3b}{}}{}$
default	$\frac{c\,a^3x - ib^3\arctan(cx)^3 + b^3cx\arctan(cx)^3 + 3b^3\arctan(cx)^2\ln\left(1+\frac{(icx+1)^2}{c^2x^2+1}\right) - 3ib^3\arctan(cx)\operatorname{polylog}\left(2,-\frac{(icx+1)^2}{c^2x^2+1}\right) + \frac{3b}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(c*a^3*x-I*b^3*arctan(c*x)^3+b^3*c*x*arctan(c*x)^3+3*b^3*arctan(c*x)^2*
ln(1+(1+I*c*x)^2/(c^2*x^2+1))-3*I*b^3*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c
^2*x^2+1))+3/2*b^3*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-3*I*arctan(c*x)^2*a*
b^2+3*a*b^2*c*x*arctan(c*x)^2+6*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))*a
*b^2-3*I*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))*a*b^2+3*a^2*b*c*x*arctan(c*x)-
3/2*a^2*b*ln(c^2*x^2+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^3*x*arctan(c*x)^3 - 3/32*b^3*x*arctan(c*x)*log(c^2*x^2 + 1)^2 + 7/32*
b^3*arctan(c*x)^4/c + 28*b^3*c^2*integrate(1/32*x^2*arctan(c*x)^3/(c^2*x^2
+ 1), x) + 3*b^3*c^2*integrate(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2
*x^2 + 1), x) + 96*a*b^2*c^2*integrate(1/32*x^2*arctan(c*x)^2/(c^2*x^2 + 1)
, x) + 12*b^3*c^2*integrate(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2
+ 1), x) + a*b^2*arctan(c*x)^3/c - 12*b^3*c*integrate(1/32*x*arctan(c*x)^2/
(c^2*x^2 + 1), x) + 3*b^3*c*integrate(1/32*x*log(c^2*x^2 + 1)^2/(c^2*x^2 +
1), x) + a^3*x + 3*b^3*integrate(1/32*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x
^2 + 1), x) + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a^2*b/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
a^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{atan}\,(cx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3,x)
```

```
[Out] Integral((a + b*atan(c*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b\,\mathrm{atan}(c\,x))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3,x)
```

```
[Out] int((a + b*atan(c*x))^3, x)
```

3.30 $\int \frac{(a+b\mathbf{ArcTan}(cx))^3}{x}\,dx$

Optimal. Leaf size=206

$$2(a+b\text{ArcTan}(cx))^3\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-\frac{3}{2}ib(a+b\text{ArcTan}(cx))^2\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)+\frac{3}{2}ib(a+b\text{ArcT}$$

```
[Out] -2*(a+b*arctan(c*x))^3*arctanh(-1+2/(1+I*c*x))-3/2*I*b*(a+b*arctan(c*x))^2*
polylog(2,1-2/(1+I*c*x))+3/2*I*b*(a+b*arctan(c*x))^2*polylog(2,-1+2/(1+I*c*
x))-3/2*b^2*(a+b*arctan(c*x))*polylog(3,1-2/(1+I*c*x))+3/2*b^2*(a+b*arctan(
c*x))*polylog(3,-1+2/(1+I*c*x))+3/4*I*b^3*polylog(4,1-2/(1+I*c*x))-3/4*I*b^
3*polylog(4,-1+2/(1+I*c*x))
```

Rubi [A]
time = 0.29, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4942, 5108, 5004, 5114, 5118, 6745}

$$-\frac{3}{2}b^2\text{Li}_3\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))+\frac{3}{2}b^2\text{Li}_3\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))-\frac{3}{2}ib\text{Li}_2\left(1-\frac{2}{icx+1}\right)(a+b\text{ArcTan}(cx))^2+\frac{3}{2}ib\text{Li}_2\left(\frac{2}{icx+1}-1\right)(a+b\text{ArcTan}(cx))^2+2\tanh^{-1}\left(1-\frac{2}{1+icx}\right)(a+b\text{ArcTan}(cx))^3+\frac{3}{4}ib^3\text{Li}_4\left(1-\frac{2}{icx+1}\right)-\frac{3}{4}ib^3\text{Li}_4\left(\frac{2}{icx+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/x,x]
```

```
[Out] 2*(a + b*ArcTan[c*x])^3*ArcTanh[1 - 2/(1 + I*c*x)] - ((3*I)/2)*b*(a + b*Arc
Tan[c*x])^2*PolyLog[2, 1 - 2/(1 + I*c*x)] + ((3*I)/2)*b*(a + b*ArcTan[c*x])
^2*PolyLog[2, -1 + 2/(1 + I*c*x)] - (3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, 1
 - 2/(1 + I*c*x)])/2 + (3*b^2*(a + b*ArcTan[c*x])*PolyLog[3, -1 + 2/(1 + I*
c*x)])/2 + ((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 + I*c*x)] - ((3*I)/4)*b^3*Poly
Log[4, -1 + 2/(1 + I*c*x)]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
```

```
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}\,dx &= 2\left(a+b\tan^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-(6bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2\tanh^{-1}}{1+c^2x^2}\\ &= 2\left(a+b\tan^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx}\right)+(3bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2\log\left(\frac{}{1+}\right.}{1+c^2x^2}\\ &= 2\left(a+b\tan^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-\frac{3}{2}ib\left(a+b\tan^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{}{1}\right.\\ &= 2\left(a+b\tan^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-\frac{3}{2}ib\left(a+b\tan^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{}{1}\right.\\ &= 2\left(a+b\tan^{-1}(cx)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx}\right)-\frac{3}{2}ib\left(a+b\tan^{-1}(cx)\right)^2\text{Li}_2\left(1-\frac{}{1}\right.\end{aligned}$$

Mathematica [A]
time = 0.10, size = 212, normalized size = 1.03

$$2(a+b\text{ArcTan}(cx))^3\tanh^{-1}\left(\frac{i+cx}{-i+cx}\right)+\frac{3}{4}ib\left(2(a+b\text{ArcTan}(cx))^2\text{PolyLog}\left(2,\frac{i+cx}{i-cx}\right)-2(a+b\text{ArcTan}(cx))^2\text{PolyLog}\left(2,\frac{i+cx}{-i+cx}\right)+b\left(-2i(a+b\text{ArcTan}(cx))\text{PolyLog}\left(3,\frac{i+cx}{i-cx}\right)+2i(a+b\text{ArcTan}(cx))\text{PolyLog}\left(3,\frac{i+cx}{-i+cx}\right)+b\left(-\text{PolyLog}\left(4,\frac{i+cx}{i-cx}\right)+\text{PolyLog}\left(4,\frac{i+cx}{-i+cx}\right)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/x,x]
```

```
[Out] 2*(a + b*ArcTan[c*x])^3*ArcTanh[(I + c*x)/(-I + c*x)] + ((3*I)/4)*b*(2*(a +
 b*ArcTan[c*x])^2*PolyLog[2, (I + c*x)/(I - c*x)] - 2*(a + b*ArcTan[c*x])^2
*PolyLog[2, (I + c*x)/(-I + c*x)] + b*((-2*I)*(a + b*ArcTan[c*x])*PolyLog[3
, (I + c*x)/(I - c*x)] + (2*I)*(a + b*ArcTan[c*x])*PolyLog[3, (I + c*x)/(-I
 + c*x)] + b*(-PolyLog[4, (I + c*x)/(I - c*x)] + PolyLog[4, (I + c*x)/(-I +
 c*x)])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 4.77, size = 2309, normalized size = 11.21

method	result	size
derivativedivides	Expression too large to display	2309
default	Expression too large to display	2309

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*I*a*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1))
)^3*arctan(c*x)^2+3/2*I*a^2*b*ln(c*x)*ln(1+I*c*x)-3/2*I*a^2*b*ln(c*x)*ln(1-
I*c*x)+1/2*I*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*
x^2+1)))^3*arctan(c*x)^3-1/2*I*b^3*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(
1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^3+1/2*I*b^3*Pi*csgn(((1+I*c*x)^2/(c^
2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^3+3*I*a*b^2*arctan(c
*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-6*I*a*b^2*arctan(c*x)*polylog(2,-(1
+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*I*a*b^2*Pi*arctan(c*x)^2-6*I*a*b^2*arctan(c*
x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+a^3*ln(c*x)+b^3*ln(c*x)*arctan(c*
x)^3+3/2*I*a*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x)
^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2
+1)))*arctan(c*x)^2+3/2*I*b^3*arctan(c*x)^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2
+1))-3/2*I*a*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*
x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*ar
ctan(c*x)^2+3/2*I*a*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^
2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)
))*arctan(c*x)^2-3/2*I*a*b^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*
((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-3/
2*I*a*b^2*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x
^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+1/2*I*b^3*Pi*csgn(I*(
(1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+
I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^3+3*a*b^2*
ln(c*x)*arctan(c*x)^2-3*a*b^2*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+3
*a*b^2*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*a*b^2*arctan(c*x)^
```

```
2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*a^2*b*ln(c*x)*arctan(c*x)+3/2*I*a^2*b
*dilog(1+I*c*x)-3/2*I*a^2*b*dilog(1-I*c*x)+1/2*I*b^3*Pi*arctan(c*x)^3-3*I*b
^3*arctan(c*x)^2*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*b^3*arctan(c*x
)^2*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*a*b^2*polylog(3,-(1+I*c*x)^2
/(c^2*x^2+1))+6*a*b^2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*a*b^2*polylo
g(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-b^3*arctan(c*x)^3*ln((1+I*c*x)^2/(c^2*x^2
+1)-1)+b^3*arctan(c*x)^3*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*b^3*arctan(c*x
)*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^3*arctan(c*x)^3*ln(1+(1+I*c*x)/(
c^2*x^2+1)^(1/2))+6*b^3*arctan(c*x)*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))
-3/2*b^3*arctan(c*x)*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+6*I*b^3*polylog(4,
(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*b^3*polylog(4,-(1+I*c*x)/(c^2*x^2+1)^(1/2)
)-3/4*I*b^3*polylog(4,-(1+I*c*x)^2/(c^2*x^2+1))+3/2*I*a*b^2*Pi*csgn(I*((1+I
*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2-3/2*I*a
*b^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arc
tan(c*x)^2+1/2*I*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(
c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*
arctan(c*x)^3-1/2*I*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^
2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)
))^2*arctan(c*x)^3-1/2*I*b^3*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*
((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^3-1/
2*I*b^3*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2
+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^3
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + 1/32*integrate((28*b^3*arctan(c*x)^3 + 3*b^3*arctan(c*x)*log(c
^2*x^2 + 1)^2 + 96*a*b^2*arctan(c*x)^2 + 96*a^2*b*arctan(c*x))/x, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atan}(cx))^3}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x,x)
```

```
[Out] Integral((a + b*atan(c*x))**3/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{atan}(c\,x))^3}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3/x,x)
```

```
[Out] int((a + b*atan(c*x))^3/x, x)
```

3.31 $\int \frac{(a+b\,\mathbf{ArcTan}(cx))^3}{x^2}\,dx$

Optimal. Leaf size=116

$$-ic(a+b\text{ArcTan}(cx))^3-\frac{(a+b\text{ArcTan}(cx))^3}{x}+3bc(a+b\text{ArcTan}(cx))^2\log\left(2-\frac{2}{1-icx}\right)-3ib^2c(a+b\text{ArcTan}$$

```
[Out] -I*c*(a+b*arctan(c*x))^3-(a+b*arctan(c*x))^3/x+3*b*c*(a+b*arctan(c*x))^2*ln
(2-2/(1-I*c*x))-3*I*b^2*c*(a+b*arctan(c*x))*polylog(2,-1+2/(1-I*c*x))+3/2*b
^3*c*polylog(3,-1+2/(1-I*c*x))
```

Rubi [A]
time = 0.19, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4946, 5044, 4988, 5004, 5112, 6745}

$$-3ib^2c\text{Li}_2\left(\frac{2}{1-icx}-1\right)(a+b\text{ArcTan}(cx))-ic(a+b\text{ArcTan}(cx))^3-\frac{(a+b\text{ArcTan}(cx))^3}{x}+3bc\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))^2+\frac{3}{2}b^3c\text{Li}_3\left(\frac{2}{1-icx}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/x^2,x]
```

```
[Out] (-I)*c*(a + b*ArcTan[c*x])^3 - (a + b*ArcTan[c*x])^3/x + 3*b*c*(a + b*ArcTa
n[c*x])^2*Log[2 - 2/(1 - I*c*x)] - (3*I)*b^2*c*(a + b*ArcTan[c*x])*PolyLog[
2, -1 + 2/(1 - I*c*x)] + (3*b^3*c*PolyLog[3, -1 + 2/(1 - I*c*x)])/2
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^2}\,dx = -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}+(3bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x\left(1+c^2x^2\right)}\,dx$$
$$= -ic\left(a+b\tan^{-1}(cx)\right)^3-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}+(3ibc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x(i+cx)}\,dx$$
$$= -ic\left(a+b\tan^{-1}(cx)\right)^3-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}+3bc\left(a+b\tan^{-1}(cx)\right)^2\log\left(2-\frac{}{1-}\right.$$
$$= -ic\left(a+b\tan^{-1}(cx)\right)^3-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}+3bc\left(a+b\tan^{-1}(cx)\right)^2\log\left(2-\frac{}{1-}\right.$$
$$= -ic\left(a+b\tan^{-1}(cx)\right)^3-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}+3bc\left(a+b\tan^{-1}(cx)\right)^2\log\left(2-\frac{}{1-}\right.$$

Mathematica [A]
time = 0.26, size = 214, normalized size = 1.84

$$-\frac{a^3}{x}-\frac{3a^2b\text{ArcTan}(cx)}{x}+3a^2bc\log(x)-\frac{3}{2}a^2bc\log\left(1+c^2x^2\right)+3ab^2c\left(-\frac{\text{ArcTan}(cx)^2}{cx}+2\text{ArcTan}(cx)\log\left(1-e^{2i\text{ArcTan}(cx)}\right)-i\left(\text{ArcTan}(cx)^2+\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx)}\right)\right)\right)+b^3c\left(-\frac{i\pi^3}{8}+i\text{ArcTan}(cx)^3-\frac{\text{ArcTan}(cx)^3}{cx}+3\text{ArcTan}(cx)^2\log\left(1-e^{-2i\text{ArcTan}(cx)}\right)+3i\text{ArcTan}(cx)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(cx)}\right)+\frac{3}{2}\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(cx)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/x^2,x]
```

```
[Out] -(a^3/x) - (3*a^2*b*ArcTan[c*x])/x + 3*a^2*b*c*Log[x] - (3*a^2*b*c*Log[1 +
c^2*x^2])/2 + 3*a*b^2*c*(-(ArcTan[c*x]^2/(c*x)) + 2*ArcTan[c*x]*Log[1 - E^(
(2*I)*ArcTan[c*x])] - I*(ArcTan[c*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c*x])])
) + b^3*c*((-1/8*I)*Pi^3 + I*ArcTan[c*x]^3 - ArcTan[c*x]^3/(c*x) + 3*ArcTan
[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (3*I)*ArcTan[c*x]*PolyLog[2, E^((
-2*I)*ArcTan[c*x])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c*x])])/2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.30, size = 2126, normalized size = 18.33

method	result	size
derivativedivides	Expression too large to display	2126
default	Expression too large to display	2126

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-I*b^3*arctan(c*x)^3+3/4*I*b^3*arctan(c*x)^2*Pi*csgn(I/(1+(1+I*c*x)^2/(c
^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)
^2-3/2*I*a*b^2*ln(c*x-I)*ln(c^2*x^2+1)+3/2*I*a*b^2*ln(c*x-I)*ln(-1/2*I*(c*x
+I))+3/2*I*a*b^2*ln(c*x+I)*ln(c^2*x^2+1)-3/2*I*a*b^2*ln(c*x+I)*ln(1/2*I*(c*
x-I))+3*I*a*b^2*ln(c*x)*ln(1+I*c*x)-3*I*a*b^2*ln(c*x)*ln(1-I*c*x)-3*a*b^2/c
/x*arctan(c*x)^2-3*a^2*b/c/x*arctan(c*x)-3/4*I*b^3*arctan(c*x)^2*Pi*csgn(I*
(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3+3/2*I*b^3*arctan(c
*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3-
3/4*I*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3-3/2*I*b^3*arct
an(c*x)^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^
2+3/2*I*b^3*arctan(c*x)^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^
2/(c^2*x^2+1)))^3+3/4*I*b^3*arctan(c*x)^2*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2
+1))^2)^3-a^3/c/x-3/2*a^2*b*ln(c^2*x^2+1)-3*b^3*arctan(c*x)^2*ln((1+I*c*x)^
2/(c^2*x^2+1)-1)+3*b^3*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*b^
3*ln(c*x)*arctan(c*x)^2+3*b^3*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2
))+3*a^2*b*ln(c*x)+3/2*I*a*b^2*dilog(-1/2*I*(c*x+I))+3/4*I*a*b^2*ln(c*x-I)^
2-3/2*I*a*b^2*dilog(1/2*I*(c*x-I))-3/4*I*a*b^2*ln(c*x+I)^2+3*I*a*b^2*dilog(
1+I*c*x)-3*I*a*b^2*dilog(1-I*c*x)+6*a*b^2*ln(c*x)*arctan(c*x)+3/2*I*b^3*arc
tan(c*x)^2*Pi-6*I*b^3*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-6
*I*b^3*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*I*b^3*arctan(
c*x)^2*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1
))^2-3/2*I*b^3*arctan(c*x)^2*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*
(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2+3/2*I*b^3*arctan(c*x)^2*Pi*csgn(I*((1+I*c*
x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2
+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))+3/4*I*b^3*arctan(c*x)^2*Pi*csgn(I*(1+I*
c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+
1))^2)^2-3/2*I*b^3*arctan(c*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(
1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c
```

```
^2*x^2+1)))^2-3/2*I*b^3*arctan(c*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)
)*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2-3/2*I*b
^3*arctan(c*x)^2*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2
/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2-3/4*I*b^3*arctan(c*x)^2*Pi*c
sgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))+3/4*I*
b^3*arctan(c*x)^2*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I*(1+(1+I*c
*x)^2/(c^2*x^2+1))^2)+3*b^3*arctan(c*x)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-3
/2*b^3*arctan(c*x)^2*ln(c^2*x^2+1)+3*b^3*ln(2)*arctan(c*x)^2-3*a*b^2*arctan
(c*x)*ln(c^2*x^2+1)+3/2*I*b^3*arctan(c*x)^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2
+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)
-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))-3/4*I*b^3*arctan(c*x)^2*Pi*csgn(I/(1+(1+I*
c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(
c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)-b^3/c/x*arctan(c*x)^3+6*b^3*polyl
og(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*b^3*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1
/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^2,x, algorithm="maxima")
```

```
[Out] -3/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a^2*b - a^3/x - 1/
32*(4*b^3*arctan(c*x)^3 - 3*b^3*arctan(c*x)*log(c^2*x^2 + 1)^2 - (7*b^3*c*a
rctan(c*x)^4 + 32*a*b^2*c*arctan(c*x)^3 + 96*b^3*c^2*integrate(1/32*x^2*arc
tan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x) - 384*b^3*c^2*integrate(1/3
2*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) + 384*b^3*c*integrat
e(1/32*x*arctan(c*x)^2/(c^2*x^4 + x^2), x) - 96*b^3*c*integrate(1/32*x*log(
c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x) + 896*b^3*integrate(1/32*arctan(c*x)^3/(
c^2*x^4 + x^2), x) + 96*b^3*integrate(1/32*arctan(c*x)*log(c^2*x^2 + 1)^2/(
c^2*x^4 + x^2), x) + 3072*a*b^2*integrate(1/32*arctan(c*x)^2/(c^2*x^4 + x^2
), x))*x)/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atan}\,(cx))^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x**2,x)
```

```
[Out] Integral((a + b*atan(c*x))**3/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atan}(c\,x))^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3/x^2,x)
```

```
[Out] int((a + b*atan(c*x))^3/x^2, x)
```

3.32 $\int \frac{(a+b\mathbf{ArcTan}(cx))^3}{x^3}\,dx$

Optimal. Leaf size=133

$$-\frac{3}{2}ibc^2(a+b\text{ArcTan}(cx))^2-\frac{3bc(a+b\text{ArcTan}(cx))^2}{2x}-\frac{1}{2}c^2(a+b\text{ArcTan}(cx))^3-\frac{(a+b\text{ArcTan}(cx))^3}{2x^2}+3b^2c^2(a+$$

```
[Out] -3/2*I*b*c^2*(a+b*arctan(c*x))^2-3/2*b*c*(a+b*arctan(c*x))^2/x-1/2*c^2*(a+b
*arctan(c*x))^3-1/2*(a+b*arctan(c*x))^3/x^2+3*b^2*c^2*(a+b*arctan(c*x))*ln(
2-2/(1-I*c*x))-3/2*I*b^3*c^2*polylog(2,-1+2/(1-I*c*x))
```

Rubi [A]
time = 0.20, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4946, 5038, 5044, 4988, 2497, 5004}

$$3b^2c^2\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))-\frac{3}{2}ibc^2(a+b\text{ArcTan}(cx))^2-\frac{1}{2}c^2(a+b\text{ArcTan}(cx))^3-\frac{(a+b\text{ArcTan}(cx))^3}{2x^2}-\frac{3bc(a+b\text{ArcTan}(cx))^2}{2x}-\frac{3}{2}ib^3c^2\text{Li}_2\left(\frac{2}{1-icx}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/x^3,x]
```

```
[Out] ((-3*I)/2)*b*c^2*(a + b*ArcTan[c*x])^2 - (3*b*c*(a + b*ArcTan[c*x])^2)/(2*x
) - (c^2*(a + b*ArcTan[c*x])^3)/2 - (a + b*ArcTan[c*x])^3/(2*x^2) + 3*b^2*c
^2*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - ((3*I)/2)*b^3*c^2*PolyLog[2
, -1 + 2/(1 - I*c*x)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
```

```
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x^3}\,dx &= -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2x^2}+\frac{1}{2}(3bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2\left(1+c^2x^2\right)}\,dx\\
&= -\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2x^2}+\frac{1}{2}(3bc)\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x^2}\,dx-\frac{1}{2}\left(3bc^3\right)\int \frac{(a+b\tan}{1+c}\\
&= -\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^3-\frac{\left(a+b\tan^{-1}(cx)\right)^3}{2x^2}+\left(3b^2c^2\right.\\
&= -\frac{3}{2}ibc^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^3-\\
&= -\frac{3}{2}ibc^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^3-\\
&= -\frac{3}{2}ibc^2\left(a+b\tan^{-1}(cx)\right)^2-\frac{3bc\left(a+b\tan^{-1}(cx)\right)^2}{2x}-\frac{1}{2}c^2\left(a+b\tan^{-1}(cx)\right)^3-
\end{aligned}$$

Mathematica [A]
time = 0.22, size = 176, normalized size = 1.32

$$-\frac{3b^2\left(a+ac^2x^2+bcx(1+icx)\right)\text{ArcTan}(cx)^2+b^3\left(1+c^2x^2\right)\text{ArcTan}(cx)^3+3b\text{ArcTan}(cx)\left(a\left(a+2bcx+ac^2x^2\right)-2b^2c^2x^2\log\left(1-e^{2i\text{ArcTan}(cx)}\right)\right)+a\left(a(a+3bcx)-6b^2c^2x^2\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)\right)+3ib^3c^2x^2\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx)}\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/x^3,x]
```

```
[Out] -1/2*(3*b^2*(a + a*c^2*x^2 + b*c*x*(1 + I*c*x))*ArcTan[c*x]^2 + b^3*(1 + c^
2*x^2)*ArcTan[c*x]^3 + 3*b*ArcTan[c*x]*(a*(a + 2*b*c*x + a*c^2*x^2) - 2*b^2
*c^2*x^2*Log[1 - E^((2*I)*ArcTan[c*x])]) + a*(a*(a + 3*b*c*x) - 6*b^2*c^2*x
^2*Log[(c*x)/Sqrt[1 + c^2*x^2]]) + (3*I)*b^3*c^2*x^2*PolyLog[2, E^((2*I)*Ar
cTan[c*x])])/x^2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(119) = 238$.
time = 1.06, size = 422, normalized size = 3.17

method	result
derivativedivides	$c^2\left(-\frac{3a^2b\arctan(cx)}{2c^2x^2}-\frac{3a\,b^2\arctan(cx)^2}{2c^2x^2}-\frac{3a\,b^2\arctan(cx)}{cx}-\frac{3b^3\arctan(cx)\ln(c^2x^2+1)}{2}-\frac{3a\,b^2\arctan(cx)^2}{2}\right.$
default	$c^2\left(-\frac{3a^2b\arctan(cx)}{2c^2x^2}-\frac{3a\,b^2\arctan(cx)^2}{2c^2x^2}-\frac{3a\,b^2\arctan(cx)}{cx}-\frac{3b^3\arctan(cx)\ln(c^2x^2+1)}{2}-\frac{3a\,b^2\arctan(cx)^2}{2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-3/8*I*b^3*ln(c*x+I)^2-3/2*a^2*b/c^2/x^2*arctan(c*x)-3/2*a*b^2/c^2/x^2
*arctan(c*x)^2-3*a*b^2/c/x*arctan(c*x)-3/2*b^3*arctan(c*x)*ln(c^2*x^2+1)-3/
2*a*b^2*arctan(c*x)^2-3/2*a*b^2*ln(c^2*x^2+1)-3/2*a^2*b*arctan(c*x)+3*b^3*l
n(c*x)*arctan(c*x)+3/2*I*b^3*dilog(1+I*c*x)-3/2*I*b^3*dilog(1-I*c*x)+3/4*I*
b^3*dilog(-1/2*I*(c*x+I))+3*a*b^2*ln(c*x)-3/4*I*b^3*dilog(1/2*I*(c*x-I))+3/
8*I*b^3*ln(c*x-I)^2-1/2*a^3/c^2/x^2-3/4*I*b^3*ln(c*x-I)*ln(c^2*x^2+1)-3/4*I
*b^3*ln(c*x+I)*ln(1/2*I*(c*x-I))+3/4*I*b^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))+3/4
*I*b^3*ln(c*x+I)*ln(c^2*x^2+1)-1/2*b^3*arctan(c*x)^3-1/2*b^3/c^2/x^2*arctan
(c*x)^3-3/2*a^2*b/c/x-3/2*b^3*arctan(c*x)^2/c/x+3/2*I*b^3*ln(c*x)*ln(1+I*c*
x)-3/2*I*b^3*ln(c*x)*ln(1-I*c*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^3,x, algorithm="maxima")
```

```
[Out] -3/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a^2*b + 3/2*((arctan(c*x)^
2 - log(c^2*x^2 + 1) + 2*log(x))*c^2 - 2*(c*arctan(c*x) + 1/x)*c*arctan(c*x
))*a*b^2 - 3/2*a*b^2*arctan(c*x)^2/x^2 - 1/32*(12*c*x*arctan(c*x)^2 + 8*(c^
```

```
2*x^2 + 1)*arctan(c*x)^3 - 3*c*x*log(c^2*x^2 + 1)^2 - 4*(c^2*arctan(c*x)^3
+ 24*c^3*integrate(1/32*x^3*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) - 96*c^3
*integrate(1/32*x^3*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + 128*c^2*integrat
e(1/32*x^2*arctan(c*x)^3/(c^2*x^5 + x^3), x) + 192*c^2*integrate(1/32*x^2*a
rctan(c*x)/(c^2*x^5 + x^3), x) + 96*c*integrate(1/32*x*arctan(c*x)^2/(c^2*x
^5 + x^3), x) + 24*c*integrate(1/32*x*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x
) + 128*integrate(1/32*arctan(c*x)^3/(c^2*x^5 + x^3), x))*x^2)*b^3/x^2 - 1/
2*a^3/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^3,x, algorithm="fricas")
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{atan}(cx))^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x**3,x)
[Out] Integral((a + b*atan(c*x))**3/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^3,x, algorithm="giac")
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\operatorname{atan}(c\,x))^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3/x^3,x)
[Out] int((a + b*atan(c*x))^3/x^3, x)
```

3.33 $\int \frac{(a+b\mathbf{ArcTan}(cx))^3}{x^4}\,dx$

Optimal. Leaf size=213

$$-\frac{b^2c^2(a+b\text{ArcTan}(cx))}{x}-\frac{1}{2}bc^3(a+b\text{ArcTan}(cx))^2-\frac{bc(a+b\text{ArcTan}(cx))^2}{2x^2}+\frac{1}{3}ic^3(a+b\text{ArcTan}(cx))^3-\frac{(a+bA}{}$$

```
[Out] -b^2*c^2*(a+b*arctan(c*x))/x-1/2*b*c^3*(a+b*arctan(c*x))^2-1/2*b*c*(a+b*arc
tan(c*x))^2/x^2+1/3*I*c^3*(a+b*arctan(c*x))^3-1/3*(a+b*arctan(c*x))^3/x^3+b
^3*c^3*ln(x)-1/2*b^3*c^3*ln(c^2*x^2+1)-b*c^3*(a+b*arctan(c*x))^2*ln(2-2/(1-
I*c*x))+I*b^2*c^3*(a+b*arctan(c*x))*polylog(2,-1+2/(1-I*c*x))-1/2*b^3*c^3*p
olylog(3,-1+2/(1-I*c*x))
```

Rubi [A]
time = 0.33, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {4946, 5038, 272, 36, 29, 31, 5004, 5044, 4988, 5112, 6745}

$$ib^2c^3\text{Li}_2\left(\frac{2}{1-icx}-1\right)(a+b\text{ArcTan}(cx))-\frac{b^2c^2(a+b\text{ArcTan}(cx))}{x}+\frac{1}{3}ic^3(a+b\text{ArcTan}(cx))^3-\frac{1}{2}bc^3(a+b\text{ArcTan}(cx))^2-bc^3\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))^2-\frac{(a+b\text{ArcTan}(cx))^3}{3x^3}-\frac{bc(a+b\text{ArcTan}(cx))^2}{2x^2}-\frac{1}{2}b^3c^3\text{Li}_3\left(\frac{2}{1-icx}-1\right)+b^3c^3\log(x)-\frac{1}{2}b^3c^3\log\left(c^2x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/x^4,x]
```

```
[Out] -((b^2*c^2*(a + b*ArcTan[c*x]))/x) - (b*c^3*(a + b*ArcTan[c*x])^2)/2 - (b*c
*(a + b*ArcTan[c*x])^2)/(2*x^2) + (I/3)*c^3*(a + b*ArcTan[c*x])^3 - (a + b*
ArcTan[c*x])^3/(3*x^3) + b^3*c^3*Log[x] - (b^3*c^3*Log[1 + c^2*x^2])/2 - b*
c^3*(a + b*ArcTan[c*x])^2*Log[2 - 2/(1 - I*c*x)] + I*b^2*c^3*(a + b*ArcTan[
c*x])*PolyLog[2, -1 + 2/(1 - I*c*x)] - (b^3*c^3*PolyLog[3, -1 + 2/(1 - I*c*
x)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
```

```
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\tan^{-1}(cx))^3}{x^4}\,dx &= -\frac{(a+b\tan^{-1}(cx))^3}{3x^3}+(bc)\int \frac{(a+b\tan^{-1}(cx))^2}{x^3(1+c^2x^2)}\,dx \\
&= -\frac{(a+b\tan^{-1}(cx))^3}{3x^3}+(bc)\int \frac{(a+b\tan^{-1}(cx))^2}{x^3}\,dx-\left(bc^3\right)\int \frac{(a+b\tan^{-1}(cx))}{x(1+c^2x^2)} \\
&= -\frac{bc(a+b\tan^{-1}(cx))^2}{2x^2}+\frac{1}{3}ic^3(a+b\tan^{-1}(cx))^3-\frac{(a+b\tan^{-1}(cx))^3}{3x^3}+\left(b^2c^2\right)\int \\
&= -\frac{bc(a+b\tan^{-1}(cx))^2}{2x^2}+\frac{1}{3}ic^3(a+b\tan^{-1}(cx))^3-\frac{(a+b\tan^{-1}(cx))^3}{3x^3}-bc^3(a+ \\
&= -\frac{b^2c^2(a+b\tan^{-1}(cx))}{x}-\frac{1}{2}bc^3(a+b\tan^{-1}(cx))^2-\frac{bc(a+b\tan^{-1}(cx))^2}{2x^2}+\frac{1}{3}ic^3 \\
&= -\frac{b^2c^2(a+b\tan^{-1}(cx))}{x}-\frac{1}{2}bc^3(a+b\tan^{-1}(cx))^2-\frac{bc(a+b\tan^{-1}(cx))^2}{2x^2}+\frac{1}{3}ic^3 \\
&= -\frac{b^2c^2(a+b\tan^{-1}(cx))}{x}-\frac{1}{2}bc^3(a+b\tan^{-1}(cx))^2-\frac{bc(a+b\tan^{-1}(cx))^2}{2x^2}+\frac{1}{3}ic^3 \\
&= -\frac{b^2c^2(a+b\tan^{-1}(cx))}{x}-\frac{1}{2}bc^3(a+b\tan^{-1}(cx))^2-\frac{bc(a+b\tan^{-1}(cx))^2}{2x^2}+\frac{1}{3}ic^3
\end{aligned}$$

Mathematica [A]

time = 0.62, size = 321, normalized size = 1.51

$$-\frac{a^3}{3x^3}-\frac{a^2bc}{2x^2}-\frac{a^2b\text{ArcTan}(cx)}{x^3}-a^2bc^3\log(x)+\frac{1}{2}a^2bc^3\log\left(1+c^2x^2\right)+\frac{iab^2\left(ic^2x^2+\left(i+c^3x^3\right)\text{ArcTan}(cx)^2+icx\text{ArcTan}(cx)\left(1+c^2x^2+2c^2x^2\log\left(1-e^{2i\text{ArcTan}(cx)}\right)\right)+c^3x^3\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx)}\right)\right)}{x^3}+\frac{1}{24}b^3c^3\left(i\pi^3-\frac{24\text{ArcTan}(cx)}{cx}-12\text{ArcTan}(cx)^2-\frac{12\text{ArcTan}(cx)^2}{c^2x^2}-8i\text{ArcTan}(cx)^3-\frac{8\text{ArcTan}(cx)^3}{c^3x^3}-24\text{ArcTan}(cx)^2\log\left(1-e^{-2i\text{ArcTan}(cx)}\right)+24\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)-24i\text{ArcTan}(cx)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}(cx)}\right)-12\text{PolyLog}\left(3,e^{-2i\text{ArcTan}(cx)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 - (a^2*b*c)/(2*x^2) - (a^2*b*ArcTan[c*x])/x^3 - a^2*b*c^3*Log[
x] + (a^2*b*c^3*Log[1 + c^2*x^2])/2 + (I*a*b^2*(I*c^2*x^2 + (I + c^3*x^3)*A
rcTan[c*x]^2 + I*c*x*ArcTan[c*x]*(1 + c^2*x^2 + 2*c^2*x^2*Log[1 - E^((2*I)*
ArcTan[c*x])]) + c^3*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/x^3 + (b^3*c^3
*(I*Pi^3 - (24*ArcTan[c*x])/(c*x) - 12*ArcTan[c*x]^2 - (12*ArcTan[c*x]^2)/(
```

```
c^2*x^2) - (8*I)*ArcTan[c*x]^3 - (8*ArcTan[c*x]^3)/(c^3*x^3) - 24*ArcTan[c*
x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + 24*Log[(c*x)/Sqrt[1 + c^2*x^2]] - (2
4*I)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] - 12*PolyLog[3, E^((-2*
I)*ArcTan[c*x])]))/24
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 19.82, size = 5807, normalized size = 27.26

method	result	size
derivativedivides	Expression too large to display	5807
default	Expression too large to display	5807

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*a
^2*b - 1/3*a^3/x^3 - 1/96*(4*b^3*arctan(c*x)^3 - 3*b^3*arctan(c*x)*log(c^2*
x^2 + 1)^2 - 96*x^3*integrate(-1/32*(4*b^3*c^2*x^2*arctan(c*x)*log(c^2*x^2
+ 1) - 28*(b^3*c^2*x^2 + b^3)*arctan(c*x)^3 - 4*(24*a*b^2*c^2*x^2 + b^3*c*x
 + 24*a*b^2)*arctan(c*x)^2 + (b^3*c*x - 3*(b^3*c^2*x^2 + b^3)*arctan(c*x))*
log(c^2*x^2 + 1)^2)/(c^2*x^6 + x^4), x))/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{atan}(cx))^3}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x**4,x)
```

```
[Out] Integral((a + b*atan(c*x))**3/x**4, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^4,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3/x^4,x)
```

```
[Out] int((a + b*atan(c*x))^3/x^4, x)
```

3.34 $\int \frac{(a+b\mathbf{ArcTan}(cx))^3}{x^5}\,dx$

Optimal. Leaf size=198

$$-\frac{b^3c^3}{4x}-\frac{1}{4}b^3c^4\text{ArcTan}(cx)-\frac{b^2c^2(a+b\text{ArcTan}(cx))}{4x^2}+ibc^4(a+b\text{ArcTan}(cx))^2-\frac{bc(a+b\text{ArcTan}(cx))^2}{4x^3}+\frac{3bc^3(a$$

```
[Out] -1/4*b^3*c^3/x-1/4*b^3*c^4*arctan(c*x)-1/4*b^2*c^2*(a+b*arctan(c*x))/x^2+I*
b*c^4*(a+b*arctan(c*x))^2-1/4*b*c*(a+b*arctan(c*x))^2/x^3+3/4*b*c^3*(a+b*ar
ctan(c*x))^2/x+1/4*c^4*(a+b*arctan(c*x))^3-1/4*(a+b*arctan(c*x))^3/x^4-2*b^
2*c^4*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))+I*b^3*c^4*polylog(2,-1+2/(1-I*c*x
))
```

Rubi [A]

time = 0.41, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4946, 5038, 331, 209, 5044, 4988, 2497, 5004}

$$-2b^2c^4\log\left(2-\frac{2}{1-icx}\right)(a+b\text{ArcTan}(cx))-\frac{b^2c^2(a+b\text{ArcTan}(cx))}{4x^2}+\frac{1}{4}c^4(a+b\text{ArcTan}(cx))^3+ibc^4(a+b\text{ArcTan}(cx))^2+\frac{3bc^3(a+b\text{ArcTan}(cx))^2}{4x}-\frac{(a+b\text{ArcTan}(cx))^3}{4x^4}-\frac{bc(a+b\text{ArcTan}(cx))^2}{4x^3}-\frac{1}{4}b^3c^4\text{ArcTan}(cx)+ib^3c^4\text{Li}_2\left(\frac{2}{1-icx}-1\right)-\frac{b^3c^3}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x])^3/x^5,x]
```

```
[Out] -1/4*(b^3*c^3)/x - (b^3*c^4*ArcTan[c*x])/4 - (b^2*c^2*(a + b*ArcTan[c*x]))/
(4*x^2) + I*b*c^4*(a + b*ArcTan[c*x])^2 - (b*c*(a + b*ArcTan[c*x])^2)/(4*x^
3) + (3*b*c^3*(a + b*ArcTan[c*x])^2)/(4*x) + (c^4*(a + b*ArcTan[c*x])^3)/4
- (a + b*ArcTan[c*x])^3/(4*x^4) - 2*b^2*c^4*(a + b*ArcTan[c*x])*Log[2 - 2/(
1 - I*c*x)] + I*b^3*c^4*PolyLog[2, -1 + 2/(1 - I*c*x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\tan^{-1}(cx))^3}{x^5}\,dx &= -\frac{(a+b\tan^{-1}(cx))^3}{4x^4}+\frac{1}{4}(3bc)\int \frac{(a+b\tan^{-1}(cx))^2}{x^4\,(1+c^2x^2)}\,dx \\
&= -\frac{(a+b\tan^{-1}(cx))^3}{4x^4}+\frac{1}{4}(3bc)\int \frac{(a+b\tan^{-1}(cx))^2}{x^4}\,dx-\frac{1}{4}\left(3bc^3\right)\int \frac{(a+b\tan^{-1}}{x^2\,(1+} \\
&= -\frac{bc(a+b\tan^{-1}(cx))^2}{4x^3}-\frac{(a+b\tan^{-1}(cx))^3}{4x^4}+\frac{1}{2}\left(b^2c^2\right)\int \frac{a+b\tan^{-1}(cx)}{x^3\,(1+c^2x^2)}\,dx- \\
&= -\frac{bc(a+b\tan^{-1}(cx))^2}{4x^3}+\frac{3bc^3(a+b\tan^{-1}(cx))^2}{4x}+\frac{1}{4}c^4\left(a+b\tan^{-1}(cx)\right)^3-\frac{(a+}{} \\
&= -\frac{b^2c^2(a+b\tan^{-1}(cx))}{4x^2}+ibc^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{bc(a+b\tan^{-1}(cx))^2}{4x^3}+\frac{3bc^3}{} \\
&= -\frac{b^3c^3}{4x}-\frac{b^2c^2(a+b\tan^{-1}(cx))}{4x^2}+ibc^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{bc(a+b\tan^{-1}(cx))^2}{4x^3} \\
&= -\frac{b^3c^3}{4x}-\frac{1}{4}b^3c^4\tan^{-1}(cx)-\frac{b^2c^2(a+b\tan^{-1}(cx))}{4x^2}+ibc^4\left(a+b\tan^{-1}(cx)\right)^2-\frac{bc}{}
\end{aligned}
$$

Mathematica [A]
time = 0.49, size = 265, normalized size = 1.34

$$
-\frac{a^3+a^2bcx+ab^2c^2x^2-3a^2bc^3x^3+b^3c^3x^3+ab^2c^4x^4+b^2(bcx(1-3c^2x^2-4ic^3x^3)+a(3-3c^4x^4))\operatorname{ArcTan}(cx)^2-b^3(-1+c^4x^4)\operatorname{ArcTan}(cx)^3+b\operatorname{ArcTan}(cx)\left(b^2c^2x^2(1+c^2x^2)+ab(2cx-6c^3x^3)+a^2(3-3c^4x^4)+8b^2c^4x^4\log\left(1-e^{2i\operatorname{ArcTan}(cx)}\right)\right)+8ab^2c^4x^4\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)-4ib^3c^4x^4\operatorname{PolyLog}\left(2,e^{2i\operatorname{ArcTan}(cx)}\right)}{4x^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x])^3/x^5,x]
```

```
[Out] -1/4*(a^3 + a^2*b*c*x + a*b^2*c^2*x^2 - 3*a^2*b*c^3*x^3 + b^3*c^3*x^3 + a*b
^2*c^4*x^4 + b^2*(b*c*x*(1 - 3*c^2*x^2 - (4*I)*c^3*x^3) + a*(3 - 3*c^4*x^4)
)*ArcTan[c*x]^2 - b^3*(-1 + c^4*x^4)*ArcTan[c*x]^3 + b*ArcTan[c*x]*(b^2*c^2
*x^2*(1 + c^2*x^2) + a*b*(2*c*x - 6*c^3*x^3) + a^2*(3 - 3*c^4*x^4) + 8*b^2*
c^4*x^4*Log[1 - E^((2*I)*ArcTan[c*x])]) + 8*a*b^2*c^4*x^4*Log[(c*x)/Sqrt[1
+ c^2*x^2]] - (4*I)*b^3*c^4*x^4*PolyLog[2, E^((2*I)*ArcTan[c*x])])/x^4
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(180) = 360.
time = 1.58, size = 512, normalized size = 2.59

method	result
derivativedivides	$c^4\left(-\frac{b^3}{4cx}-\frac{a^2b}{4c^3x^3}-\frac{b^3\arctan(cx)^2}{4c^3x^3}-\frac{b^3\arctan(cx)}{4c^2x^2}-\frac{b^3\arctan(cx)^3}{4c^4x^4}-\frac{a\,b^2}{4c^2x^2}+\frac{3a\,b^2\arctan(cx)}{2cx}+b^3\,\text{a}\right.$

default	$c^4\left(-\frac{b^3}{4cx}-\frac{a^2b}{4c^3x^3}-\frac{b^3\arctan(cx)^2}{4c^3x^3}-\frac{b^3\arctan(cx)}{4c^2x^2}-\frac{b^3\arctan(cx)^3}{4c^4x^4}-\frac{a\,b^2}{4c^2x^2}+\frac{3a\,b^2\arctan(cx)}{2cx}+b^3\,\mathrm{arc}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^3/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(-1/4*b^3/c/x-1/2*I*b^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))-I*b^3*ln(c*x)*ln(1
+I*c*x)-1/2*I*b^3*ln(c*x+I)*ln(c^2*x^2+1)+1/2*I*b^3*ln(c*x-I)*ln(c^2*x^2+1)
+1/2*I*b^3*ln(c*x+I)*ln(1/2*I*(c*x-I))+I*b^3*ln(c*x)*ln(1-I*c*x)-1/4*a^2*b/
c^3/x^3-1/4*b^3*arctan(c*x)^2/c^3/x^3-1/4*b^3*arctan(c*x)/c^2/x^2-1/4*b^3/c
^4/x^4*arctan(c*x)^3-1/4*a*b^2/c^2/x^2+3/2*a*b^2/c/x*arctan(c*x)+b^3*arctan
(c*x)*ln(c^2*x^2+1)+3/4*a*b^2*arctan(c*x)^2+a*b^2*ln(c^2*x^2+1)+3/4*a^2*b*a
rctan(c*x)-1/4*a^3/c^4/x^4-1/4*I*b^3*ln(c*x-I)^2+I*b^3*dilog(1-I*c*x)-I*b^3
*dilog(1+I*c*x)-1/2*I*b^3*dilog(-1/2*I*(c*x+I))+1/4*I*b^3*ln(c*x+I)^2+1/2*I
*b^3*dilog(1/2*I*(c*x-I))-2*b^3*ln(c*x)*arctan(c*x)-2*a*b^2*ln(c*x)+1/4*b^3
*arctan(c*x)^3-1/4*b^3*arctan(c*x)-1/2*a*b^2*arctan(c*x)/c^3/x^3-3/4*a*b^2/
c^4/x^4*arctan(c*x)^2-3/4*a^2*b/c^4/x^4*arctan(c*x)+3/4*a^2*b/c/x+3/4*b^3*a
rctan(c*x)^2/c/x)
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^5,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^5,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{atan}\,(cx))^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**3/x**5,x)
```

```
[Out] Integral((a + b*atan(c*x))**3/x**5, x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^3/x^5,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atan}(c\,x))^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3/x^5,x)
```

```
[Out] int((a + b*atan(c*x))^3/x^5, x)
```

3.35 $\int \frac{x}{\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=11

$$\mathrm{Int}\left(\frac{x}{\mathrm{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x/arctan(a*x),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/ArcTan[a*x],x]
```

```
[Out] Defer[Int][x/ArcTan[a*x], x]
```

Rubi steps

$$\int \frac{x}{\tan^{-1}(ax)}\,dx = \int \frac{x}{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{x}{\mathrm{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/ArcTan[a*x],x]
```

```
[Out] Integrate[x/ArcTan[a*x], x]
```

Maple [A]
time = 1.81, size = 0, normalized size = 0.00

$$\int \frac{x}{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x),x)
```

```
[Out] int(x/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(x/arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(x/arctan(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x),x)
```

```
[Out] Integral(x/atan(a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{x}{\operatorname{atan}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/atan(a*x),x)
```

```
[Out] int(x/atan(a*x), x)
```

3.36 $\int \frac{1}{\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=9

$$\text{Int}\left(\frac{1}{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(1/arctan(a*x),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(-1),x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(-1), x]
```

Rubi steps

$$\int \frac{1}{\tan^{-1}(ax)}\,dx = \int \frac{1}{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(-1),x]
```

```
[Out] Integrate[ArcTan[a*x]^(-1), x]
```

Maple [A]
time = 1.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x),x)
```

```
[Out] int(1/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/arctan(a*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/arctan(a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x),x)
```

```
[Out] Integral(1/atan(a*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{\operatorname{atan}(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/atan(a*x),x)
```

```
[Out] int(1/atan(a*x), x)
```

3.37 $\int \frac{1}{x\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(1/x/arctan(a*x),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*ArcTan[a*x]),x]
```

```
[Out] Defer[Int][1/(x*ArcTan[a*x]), x]
```

Rubi steps

$$\int \frac{1}{x\tan^{-1}(ax)}\,dx = \int \frac{1}{x\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{1}{x\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*ArcTan[a*x]),x]
```

```
[Out] Integrate[1/(x*ArcTan[a*x]), x]
```

Maple [A]
time = 1.34, size = 0, normalized size = 0.00

$$\int \frac{1}{x\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x),x)
```

```
[Out] int(1/x/arctan(a*x),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/(x*arctan(a*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x),x, algorithm="fricas")
```

```
[Out] integral(1/(x*arctan(a*x)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{atan}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x),x)
```

```
[Out] Integral(1/(x*atan(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x \operatorname{atan}(a\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)),x)
```

```
[Out] int(1/(x*atan(a*x)), x)
```

3.38 $\int \frac{x}{\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=11

$$\text{Int}\left(\frac{x}{\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable(x/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/ArcTan[a*x]^2,x]
```

```
[Out] Defer[Int][x/ArcTan[a*x]^2, x]
```

Rubi steps

$$\int \frac{x}{\tan^{-1}(ax)^2}\,dx = \int \frac{x}{\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x}{\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/ArcTan[a*x]^2,x]
```

```
[Out] Integrate[x/ArcTan[a*x]^2, x]
```

Maple [A]
time = 2.02, size = 0, normalized size = 0.00

$$\int \frac{x}{\arctan\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x)^2,x)
```

```
[Out] int(x/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^2*x^3 - arctan(a*x)*integrate((3*a^2*x^2 + 1)/arctan(a*x), x) + x)/(a*a
rctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x/arctan(a*x)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{atan}^2(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x)**2,x)
```

```
[Out] Integral(x/atan(a*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{x}{\operatorname{atan}(a\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/atan(a*x)^2,x)
```

```
[Out] int(x/atan(a*x)^2, x)
```

3.39 $\int \frac{1}{\mathbf{ArcTan}(ax)^2} dx$

Optimal. Leaf size=9

$$\mathrm{Int}\left(\frac{1}{\mathrm{ArcTan}(ax)^2}, x\right)$$

```
[Out] Unintegrable(1/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\mathrm{ArcTan}(ax)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(-2),x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(-2), x]
```

Rubi steps

$$\int \frac{1}{\tan^{-1}(ax)^2} dx = \int \frac{1}{\tan^{-1}(ax)^2} dx$$

Mathematica [A]
time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{1}{\mathrm{ArcTan}(ax)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(-2),x]
```

```
[Out] Integrate[ArcTan[a*x]^(-2), x]
```

Maple [A]
time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{\arctan(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x)^2,x)
```

```
[Out] int(1/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^2*x^2 - 2*a^2*arctan(a*x)*integrate(x/arctan(a*x), x) + 1)/(a*arctan(a*
x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x)^(-2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{atan}^2(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x)**2,x)
```

```
[Out] Integral(atan(a*x)**(-2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.11

$$\int \frac{1}{\operatorname{atan}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/atan(a*x)^2,x)
```

```
[Out] int(1/atan(a*x)^2, x)
```

3.40 $\int \frac{1}{x\mathbf{ArcTan}(ax)^2}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{1}{x\text{ArcTan}(ax)^2},x\right)$$

```
[Out] Unintegrable(1/x/arctan(a*x)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*ArcTan[a*x]^2),x]
```

```
[Out] Defer[Int][1/(x*ArcTan[a*x]^2), x]
```

Rubi steps

$$\int \frac{1}{x\tan^{-1}(ax)^2}\,dx = \int \frac{1}{x\tan^{-1}(ax)^2}\,dx$$

Mathematica [A]
time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{1}{x\text{ArcTan}(ax)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*ArcTan[a*x]^2),x]
```

```
[Out] Integrate[1/(x*ArcTan[a*x]^2), x]
```

Maple [A]
time = 1.38, size = 0, normalized size = 0.00

$$\int \frac{1}{x\arctan\left(ax\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x)^2,x)
```

```
[Out] int(1/x/arctan(a*x)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^2*x^2 - x*arctan(a*x)*integrate((a^2*x^2 - 1)/(x^2*arctan(a*x)), x) + 1
)/(a*x*arctan(a*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(x*arctan(a*x)^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \operatorname{atan}^2(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x)**2,x)
```

```
[Out] Integral(1/(x*atan(a*x)**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{x \operatorname{atan}(a x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^2),x)
```

```
[Out] int(1/(x*atan(a*x)^2), x)
```

3.41 $\int x\sqrt{\text{ArcTan}(ax)}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(x\sqrt{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(x*arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][x*Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int x\sqrt{\tan^{-1}(ax)}\,dx = \int x\sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 1.51, size = 0, normalized size = 0.00

$$\int x\sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*Sqrt[ArcTan[a*x]],x]
[Out] Integrate[x*Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.30, size = 0, normalized size = 0.00

$$\int x\sqrt{\arctan\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(1/2),x)
```

```
[Out] int(x*arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\operatorname{atan}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(atan(a*x)), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int x\sqrt{\operatorname{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(1/2),x)
```

```
[Out] int(x*atan(a*x)^(1/2), x)
```

3.42 $\int \sqrt{\mathbf{ArcTan}(ax)}\,dx$

Optimal. Leaf size=11

$$\text{Int}\left(\sqrt{\text{ArcTan}(ax)},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]],x]
[Out] Defer[Int][Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \sqrt{\tan^{-1}(ax)}\,dx = \int \sqrt{\tan^{-1}(ax)}\,dx$$

Mathematica [A]
time = 1.33, size = 0, normalized size = 0.00

$$\int \sqrt{\text{ArcTan}(ax)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]],x]
[Out] Integrate[Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 0.69, size = 0, normalized size = 0.00

$$\int \sqrt{\arctan(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2),x)
```

```
[Out] int(arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\operatorname{atan}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(atan(a*x)), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.09

$$\int \sqrt{\operatorname{atan}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2),x)
```

```
[Out] int(atan(a*x)^(1/2), x)
```

3.43 $\int \frac{\sqrt{\text{ArcTan}(ax)}}{x} dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{\sqrt{\text{ArcTan}(ax)}}{x},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(1/2)/x,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x} dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[ArcTan[a*x]]/x,x]
[Out] Defer[Int][Sqrt[ArcTan[a*x]]/x, x]
```

Rubi steps

$$\int \frac{\sqrt{\tan^{-1}(ax)}}{x} dx = \int \frac{\sqrt{\tan^{-1}(ax)}}{x} dx$$

Mathematica [A]
time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\text{ArcTan}(ax)}}{x} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[ArcTan[a*x]]/x,x]
[Out] Integrate[Sqrt[ArcTan[a*x]]/x, x]
```

Maple [A]
time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\arctan(ax)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(1/2)/x,x)
```

```
[Out] int(arctan(a*x)^(1/2)/x,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{atan}\left(ax\right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(atan(a*x))/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\sqrt{\operatorname{atan}(a\,x)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(1/2)/x,x)
```

```
[Out] int(atan(a*x)^(1/2)/x, x)
```

3.44 $\int x\text{ArcTan}(ax)^{3/2}\,dx$

Optimal. Leaf size=13

$$\text{Int}\left(x\text{ArcTan}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable(x*arctan(a*x)^(3/2),x)
```

Rubi [A]

time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x\text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x*ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][x*ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int x\tan^{-1}(ax)^{3/2}\,dx = \int x\tan^{-1}(ax)^{3/2}\,dx$$

Mathematica [A]

time = 0.71, size = 0, normalized size = 0.00

$$\int x\text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x*ArcTan[a*x]^(3/2),x]
[Out] Integrate[x*ArcTan[a*x]^(3/2), x]
```

Maple [A]

time = 1.24, size = 0, normalized size = 0.00

$$\int x\arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(a*x)^(3/2),x)
```

```
[Out] int(x*arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x \operatorname{atan}^{\frac{3}{2}}(ax)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(a*x)**(3/2),x)
```

```
[Out] Integral(x*atan(a*x)**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int x \operatorname{atan}(a\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(a*x)^(3/2),x)
```

```
[Out] int(x*atan(a*x)^(3/2), x)
```

3.45 $\int \mathbf{ArcTan}(ax)^{3/2}\,dx$

Optimal. Leaf size=11

$$\text{Int}\left(\text{ArcTan}(ax)^{3/2},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2),x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \tan^{-1}(ax)^{3/2}\,dx = \int \tan^{-1}(ax)^{3/2}\,dx$$

Mathematica [A]
time = 1.61, size = 0, normalized size = 0.00

$$\int \text{ArcTan}(ax)^{3/2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2),x]
```

```
[Out] Integrate[ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 0.68, size = 0, normalized size = 0.00

$$\int \arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2),x)
```

```
[Out] int(arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \operatorname{atan}^{\frac{3}{2}}(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**(3/2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.09

$$\int \operatorname{atan}(a\,x)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2),x)
```

```
[Out] int(atan(a*x)^(3/2), x)
```

3.46 $\int \frac{\mathbf{ArcTan}(ax)^{3/2}}{x}\,dx$

Optimal. Leaf size=15

$$\operatorname{Int}\left(\frac{\operatorname{ArcTan}(ax)^{3/2}}{x},x\right)$$

```
[Out] Unintegrable(arctan(a*x)^(3/2)/x,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{ArcTan}(ax)^{3/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(3/2)/x,x]
[Out] Defer[Int][ArcTan[a*x]^(3/2)/x, x]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax)^{3/2}}{x}\,dx = \int \frac{\tan^{-1}(ax)^{3/2}}{x}\,dx$$

Mathematica [A]
time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{ArcTan}(ax)^{3/2}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(3/2)/x,x]
[Out] Integrate[ArcTan[a*x]^(3/2)/x, x]
```

Maple [A]
time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x)^(3/2)/x,x)
```

```
[Out] int(arctan(a*x)^(3/2)/x,x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x)**(3/2)/x,x)
```

```
[Out] Integral(atan(a*x)**(3/2)/x, x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\operatorname{atan}(a\,x)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x)^(3/2)/x,x)
```

```
[Out] int(atan(a*x)^(3/2)/x, x)
```

3.47 $\int \frac{x}{\sqrt{\text{ArcTan}(ax)}} \, dx$

Optimal. Leaf size=13

$$\text{Int}\left(\frac{x}{\sqrt{\text{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(x/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{\sqrt{\text{ArcTan}(ax)}} \, dx$$

Verification is not applicable to the result.

```
[In] Int[x/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Defer[Int][x/Sqrt[ArcTan[a*x]], x]
```

Rubi steps

$$\int \frac{x}{\sqrt{\tan^{-1}(ax)}} \, dx = \int \frac{x}{\sqrt{\tan^{-1}(ax)}} \, dx$$

Mathematica [A]
time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\text{ArcTan}(ax)}} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/Sqrt[ArcTan[a*x]],x]
```

```
[Out] Integrate[x/Sqrt[ArcTan[a*x]], x]
```

Maple [A]
time = 1.36, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\arctan(ax)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x)^(1/2),x)
```

```
[Out] int(x/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\operatorname{atan}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(atan(a*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{x}{\sqrt{\operatorname{atan}(a\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/atan(a*x)^(1/2),x)
[Out] int(x/atan(a*x)^(1/2), x)
```

3.48 $\int \frac{1}{\sqrt{\textbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=11

$$\text{Int}\left(\frac{1}{\sqrt{\text{ArcTan}(ax)}},x\right)$$

`[Out] Unintegrable(1/arctan(a*x)^(1/2),x)`

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

`[In] Int[1/Sqrt[ArcTan[a*x]],x]`

`[Out] Defer[Int][1/Sqrt[ArcTan[a*x]], x]`

Rubi steps

$$\int \frac{1}{\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{1}{\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\text{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[1/Sqrt[ArcTan[a*x]],x]`

`[Out] Integrate[1/Sqrt[ArcTan[a*x]], x]`

Maple [A]
time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\arctan\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/arctan(a*x)^(1/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\operatorname{atan}{\left(ax \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(atan(a*x)), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(1/2),x, algorithm="giac")
```

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{1}{\sqrt{\operatorname{atan}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/atan(a*x)^(1/2),x)
```

```
[Out] int(1/atan(a*x)^(1/2), x)
```

3.49 $\int \frac{1}{x\sqrt{\mathbf{ArcTan}(ax)}}\,dx$

Optimal. Leaf size=15

$$\operatorname{Int}\left(\frac{1}{x\sqrt{\operatorname{ArcTan}(ax)}},x\right)$$

```
[Out] Unintegrable(1/x/arctan(a*x)^(1/2),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{\operatorname{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Defer[Int][1/(x*Sqrt[ArcTan[a*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{\tan^{-1}(ax)}}\,dx = \int \frac{1}{x\sqrt{\tan^{-1}(ax)}}\,dx$$

Mathematica [A]
time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\operatorname{ArcTan}(ax)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[ArcTan[a*x]]),x]
```

```
[Out] Integrate[1/(x*Sqrt[ArcTan[a*x]]), x]
```

Maple [A]
time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\arctan(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x)^(1/2),x)
```

```
[Out] int(1/x/arctan(a*x)^(1/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{\operatorname{atan}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(atan(a*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

[Out] sage0*x

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x\sqrt{\operatorname{atan}(a\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(1/2)),x)
```

```
[Out] int(1/(x*atan(a*x)^(1/2)), x)
```

3.50 $\int \frac{x}{\textbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=13

$$\mathrm{Int}\left(\frac{x}{\mathrm{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(x/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/ArcTan[a*x]^(3/2),x]
[Out] Defer[Int][x/ArcTan[a*x]^(3/2), x]
```

Rubi steps

$$\int \frac{x}{\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{x}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 1.10, size = 0, normalized size = 0.00

$$\int \frac{x}{\mathrm{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/ArcTan[a*x]^(3/2),x]
[Out] Integrate[x/ArcTan[a*x]^(3/2), x]
```

Maple [A]
time = 1.34, size = 0, normalized size = 0.00

$$\int \frac{x}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arctan(a*x)^(3/2),x)
```

```
[Out] int(x/arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{atan}^{\frac{3}{2}}(ax)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/atan(a*x)**(3/2),x)
```

```
[Out] Integral(x/atan(a*x)**(3/2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{x}{\operatorname{atan}\left(a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/atan(a*x)^(3/2),x)
```

```
[Out] int(x/atan(a*x)^(3/2), x)
```

3.51 $\int \frac{1}{\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=11

$$\text{Int}\left(\frac{1}{\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(1/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[ArcTan[a*x]^(-3/2),x]
```

```
[Out] Defer[Int][ArcTan[a*x]^(-3/2), x]
```

Rubi steps

$$\int \frac{1}{\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{1}{\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 1.91, size = 0, normalized size = 0.00

$$\int \frac{1}{\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[ArcTan[a*x]^(-3/2),x]
```

```
[Out] Integrate[ArcTan[a*x]^(-3/2), x]
```

Maple [A]
time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{1}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/arctan(a*x)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\operatorname{atan}^{\frac{3}{2}}{\left(ax \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/atan(a*x)**(3/2),x)
```

```
[Out] Integral(atan(a*x)**(-3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.09

$$\int \frac{1}{\operatorname{atan}\left(a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/atan(a*x)^(3/2),x)
```

```
[Out] int(1/atan(a*x)^(3/2), x)
```

3.52 $\int \frac{1}{x\mathbf{ArcTan}(ax)^{3/2}}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{1}{x\text{ArcTan}(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable(1/x/arctan(a*x)^(3/2),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*ArcTan[a*x]^(3/2)),x]
[Out] Defer[Int][1/(x*ArcTan[a*x]^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x\tan^{-1}(ax)^{3/2}}\,dx = \int \frac{1}{x\tan^{-1}(ax)^{3/2}}\,dx$$

Mathematica [A]
time = 2.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x\text{ArcTan}(ax)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*ArcTan[a*x]^(3/2)),x]
[Out] Integrate[1/(x*ArcTan[a*x]^(3/2)), x]
```

Maple [A]
time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{1}{x\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arctan(a*x)^(3/2),x)
```

```
[Out] int(1/x/arctan(a*x)^(3/2),x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/atan(a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*atan(a*x)**(3/2)), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{1}{x \operatorname{atan}(a x)^{3/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*atan(a*x)^(3/2)),x)
```

```
[Out] int(1/(x*atan(a*x)^(3/2)), x)
```

3.53 $\int \sqrt{x} \operatorname{ArcTan}(x) \, dx$

Optimal. Leaf size=117

$$-\frac{4\sqrt{x}}{3}-\frac{1}{3}\sqrt{2}\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)+\frac{1}{3}\sqrt{2}\operatorname{ArcTan}\left(1+\sqrt{2}\sqrt{x}\right)+\frac{2}{3}x^{3/2}\operatorname{ArcTan}(x)-\frac{\log\left(1-\sqrt{2}\sqrt{x}\right.}{3\sqrt{2}}$$

```
[Out] 2/3*x^(3/2)*arctan(x)-1/6*ln(1+x-2^(1/2)*x^(1/2))*2^(1/2)+1/6*ln(1+x+2^(1/2
)*x^(1/2))*2^(1/2)+1/3*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/3*arctan(1+2^(1
/2)*x^(1/2))*2^(1/2)-4/3*x^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.125$, Rules used = {4946, 327, 335, 217, 1179, 642, 1176, 631, 210}

$$\frac{2}{3}x^{3/2}\operatorname{ArcTan}(x)-\frac{1}{3}\sqrt{2}\operatorname{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right)+\frac{1}{3}\sqrt{2}\operatorname{ArcTan}\left(\sqrt{2}\sqrt{x}+1\right)-\frac{4\sqrt{x}}{3}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{3\sqrt{2}}+\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTan[x],x]
```

```
[Out] (-4*Sqrt[x])/3 - (Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]])/3 + (Sqrt[2]*ArcTan[
1 + Sqrt[2]*Sqrt[x]])/3 + (2*x^(3/2)*ArcTan[x])/3 - Log[1 - Sqrt[2]*Sqrt[x]
 + x]/(3*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[x] + x]/(3*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{x}\tan^{-1}(x)\,dx = \frac{2}{3}x^{3/2}\tan^{-1}(x) - \frac{2}{3}\int \frac{x^{3/2}}{1+x^2}\,dx$$
$$= -\frac{4\sqrt{x}}{3} + \frac{2}{3}x^{3/2}\tan^{-1}(x) + \frac{2}{3}\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx$$
$$= -\frac{4\sqrt{x}}{3} + \frac{2}{3}x^{3/2}\tan^{-1}(x) + \frac{4}{3}\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{4\sqrt{x}}{3} + \frac{2}{3}x^{3/2}\tan^{-1}(x) + \frac{2}{3}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{x}\right) + \frac{2}{3}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, \right.$$
$$= -\frac{4\sqrt{x}}{3} + \frac{2}{3}x^{3/2}\tan^{-1}(x) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}\,x+x^2}\,dx, x, \sqrt{x}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{}{1-}\right.$$
$$= -\frac{4\sqrt{x}}{3} + \frac{2}{3}x^{3/2}\tan^{-1}(x) - \frac{\log\left(1-\sqrt{2}\sqrt{x}+x\right)}{3\sqrt{2}} + \frac{\log\left(1+\sqrt{2}\sqrt{x}+x\right)}{3\sqrt{2}} + \frac{1}{3}\sqrt{2}$$
$$= -\frac{4\sqrt{x}}{3} - \frac{1}{3}\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{x}\right) + \frac{1}{3}\sqrt{2}\tan^{-1}\left(1+\sqrt{2}\sqrt{x}\right) + \frac{2}{3}x^{3/2}\tan^{-1}(x$$

Mathematica [A]
time = 0.02, size = 108, normalized size = 0.92

$$\frac{1}{6}\left(-8\sqrt{x} - 2\sqrt{2}\,\text{ArcTan}\left(1-\sqrt{2}\sqrt{x}\right) + 2\sqrt{2}\,\text{ArcTan}\left(1+\sqrt{2}\sqrt{x}\right) + 4x^{3/2}\text{ArcTan}(x) - \sqrt{2}\log\left(1-\sqrt{2}\sqrt{x}+x\right) + \sqrt{2}\log\left(1+\sqrt{2}\sqrt{x}+x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTan[x],x]
```

```
[Out] (-8*Sqrt[x] - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] + 2*Sqrt[2]*ArcTan[1 +
Sqrt[2]*Sqrt[x]] + 4*x^(3/2)*ArcTan[x] - Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[x] +
x] + Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[x] + x])/6
```

Maple [A]
time = 0.09, size = 69, normalized size = 0.59

method	result
derivativedivides	$\frac{2x^{\frac{3}{2}}\arctan(x)}{3} - \frac{4\sqrt{x}}{3} + \frac{\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right.}{6}$
default	$\frac{2x^{\frac{3}{2}}\arctan(x)}{3} - \frac{4\sqrt{x}}{3} + \frac{\sqrt{2}\left(\ln\left(\frac{1+x+\sqrt{2}\sqrt{x}}{1+x-\sqrt{2}\sqrt{x}}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right.}{6}$

meijerg	$-\frac{4\sqrt{x}}{3}+\frac{\sqrt{x}\left(-\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^2)^{\frac{1}{4}}\right.}{2(x^2)^{\frac{1}{4}}}\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)*x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*x^(3/2)*arctan(x)-4/3*x^(1/2)+1/6*2^(1/2)*(ln((1+x+2^(1/2)*x^(1/2))/(1+
x-2^(1/2)*x^(1/2)))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)
))
```

Maxima [A]
time = 0.49, size = 86, normalized size = 0.74

$$\frac{2}{3}x^{\frac{3}{2}}\arctan(x)+\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)+\frac{1}{3}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{1}{6}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{6}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{4}{3}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)*arctan(x) + 1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x)
)) + 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/6*sqrt(2)*l
og(sqrt(2)*sqrt(x) + x + 1) - 1/6*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 4
/3*sqrt(x)
```

Fricas [A]
time = 5.00, size = 118, normalized size = 1.01

$$\frac{2}{3}\left(x\arctan(x)-2\right)\sqrt{x}-\frac{2}{3}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\sqrt{x}+x+1}-\sqrt{2}\sqrt{x}-1\right)-\frac{2}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\sqrt{x}+4x+4}-\sqrt{2}\sqrt{x}+1\right)+\frac{1}{6}\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)-\frac{1}{6}\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(x*arctan(x) - 2)*sqrt(x) - 2/3*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*sqr
t(x) + x + 1) - sqrt(2)*sqrt(x) - 1) - 2/3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(
-4*sqrt(2)*sqrt(x) + 4*x + 4) - sqrt(2)*sqrt(x) + 1) + 1/6*sqrt(2)*log(4*sq
rt(2)*sqrt(x) + 4*x + 4) - 1/6*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
```

Sympy [A]
time = 1.31, size = 104, normalized size = 0.89

$$\frac{2x^{\frac{3}{2}}\operatorname{atan}(x)}{3}-\frac{4\sqrt{x}}{3}-\frac{\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)}{6}+\frac{\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)}{6}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{3}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)*x**(1/2),x)
```

```
[Out] 2*x**(3/2)*atan(x)/3 - 4*sqrt(x)/3 - sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)/
6 + sqrt(2)*log(sqrt(2)*sqrt(x) + x + 1)/6 + sqrt(2)*atan(sqrt(2)*sqrt(x) -
 1)/3 + sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/3
```

Giac [**A**]
time = 0.46, size = 86, normalized size = 0.74

$$\frac{2}{3}\,x^{\frac{3}{2}}\arctan\left(x\right)+\frac{1}{3}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{x}\right)\right)+\frac{1}{3}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{x}\right)\right)+\frac{1}{6}\,\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{6}\,\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{4}{3}\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)*arctan(x) + 1/3*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x)
)) + 1/3*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/6*sqrt(2)*l
og(sqrt(2)*sqrt(x) + x + 1) - 1/6*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 4
/3*sqrt(x)
```

Mupad [**B**]
time = 0.32, size = 49, normalized size = 0.42

$$\frac{2\,x^{3/2}\,\mathrm{atan}(x)}{3}-\frac{4\,\sqrt{x}}{3}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atan(x),x)
```

```
[Out] (2*x^(3/2)*atan(x))/3 + 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/3 + 1
i/3) + 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/3 - 1i/3) - (4*x^(1/2)
)/3
```

3.54 $\int (dx)^m(a+b\mathbf{ArcTan}(cx))^3\,dx$

Optimal. Leaf size=19

$$\text{Int}\left((dx)^m(a+b\text{ArcTan}(cx))^3,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x))^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m(a+b\text{ArcTan}(cx))^3\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x])^3, x]
```

Rubi steps

$$\int (dx)^m\left(a+b\tan^{-1}(cx)\right)^3\,dx=\int (dx)^m\left(a+b\tan^{-1}(cx)\right)^3\,dx$$

Mathematica [A]
time = 2.71, size = 0, normalized size = 0.00

$$\int (dx)^m(a+b\text{ArcTan}(cx))^3\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^3,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^3, x]
```

Maple [A]
time = 1.53, size = 0, normalized size = 0.00

$$\int (dx)^m\left(a+b\arctan\left(cx\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/32*(4*b^3*d^m*x*x^m*arctan(c*x)^3 - 3*b^3
*d^m*x*x^m*arctan(c*x)*log(c^2*x^2 + 1)^2 + 32*(m + 1)*integrate(1/32*(12*b
^3*c^2*d^m*x^2*x^m*arctan(c*x)*log(c^2*x^2 + 1) + 28*(b^3*d^m*m + b^3*d^m +
 (b^3*c^2*d^m*m + b^3*c^2*d^m)*x^2)*x^m*arctan(c*x)^3 - 12*(b^3*c*d^m*x - 8
*a*b^2*d^m*m - 8*a*b^2*d^m - 8*(a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^2)*x^m*a
rctan(c*x)^2 + 96*(a^2*b*d^m*m + a^2*b*d^m + (a^2*b*c^2*d^m*m + a^2*b*c^2*d
^m)*x^2)*x^m*arctan(c*x) + 3*(b^3*c*d^m*x*x^m + (b^3*d^m*m + b^3*d^m + (b^3
*c^2*d^m*m + b^3*c^2*d^m)*x^2)*x^m*arctan(c*x))*log(c^2*x^2 + 1)^2)/((c^2*m
 + c^2)*x^2 + m + 1), x))/(m + 1)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x)^3 + 3*a*b^2*arctan(c*x)^2 + 3*a^2*b*arctan(c*x) +
 a^3)*(d*x)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{atan}(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x))**3,x)
```

```
[Out] Integral((d*x)**m*(a + b*atan(c*x))**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (a+b\,\mathrm{atan}(c\,x))^3\,(d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^3*(d*x)^m,x)
```

```
[Out] int((a + b*atan(c*x))^3*(d*x)^m, x)
```

3.55 $\int (dx)^m(a+b\mathbf{ArcTan}(cx))^2\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left((dx)^m(a+b\mathrm{ArcTan}(cx))^2,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x))^2,x)
```

Rubi [A]

time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m(a+b\mathrm{ArcTan}(cx))^2\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x])^2,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x])^2, x]
```

Rubi steps

$$\int (dx)^m\left(a+b\tan^{-1}(cx)\right)^2\,dx=\int (dx)^m\left(a+b\tan^{-1}(cx)\right)^2\,dx$$

Mathematica [A]

time = 1.78, size = 0, normalized size = 0.00

$$\int (dx)^m(a+b\mathrm{ArcTan}(cx))^2\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^2, x]
```

Maple [A]

time = 1.78, size = 0, normalized size = 0.00

$$\int (dx)^m\left(a+b\arctan\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/16*(4*b^2*d^m*x*x^m*arctan(c*x)^2 - b^2*d
^m*x*x^m*log(c^2*x^2 + 1)^2 + 16*(m + 1)*integrate(1/16*(4*b^2*c^2*d^m*x^2*
x^m*log(c^2*x^2 + 1) + 12*(b^2*d^m*m + b^2*d^m + (b^2*c^2*d^m*m + b^2*c^2*d
^m)*x^2)*x^m*arctan(c*x)^2 + (b^2*d^m*m + b^2*d^m + (b^2*c^2*d^m*m + b^2*c^
2*d^m)*x^2)*x^m*log(c^2*x^2 + 1)^2 - 8*(b^2*c*d^m*x - 4*a*b*d^m*m - 4*a*b*d
^m - 4*(a*b*c^2*d^m*m + a*b*c^2*d^m)*x^2)*x^m*arctan(c*x))/((c^2*m + c^2)*x
^2 + m + 1), x))/(m + 1)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)*(d*x)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b \operatorname{atan}{\left(cx \right)}\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x))**2,x)
```

```
[Out] Integral((d*x)**m*(a + b*atan(c*x))**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (a + b\,\mathrm{atan}(c\,x))^2\,(d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^2*(d*x)^m,x)
```

```
[Out] int((a + b*atan(c*x))^2*(d*x)^m, x)
```

3.56 $\int (dx)^m (a + b\textbf{ArcTan}(cx))\, dx$

Optimal. Leaf size=73

$$\frac{(dx)^{1+m}(a+b\text{ArcTan}(cx))}{d(1+m)}-\frac{bc(dx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-c^2x^2\right)}{d^2(1+m)(2+m)}$$

```
[Out] (d*x)^(1+m)*(a+b*arctan(c*x))/d/(1+m)-b*c*(d*x)^(2+m)*hypergeom([1, 1+1/2*m
],[2+1/2*m],-c^2*x^2)/d^2/(1+m)/(2+m)
```

Rubi [A]

time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4958, 371}

$$\frac{(dx)^{m+1}(a+b\text{ArcTan}(cx))}{d(m+1)}-\frac{bc(dx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-c^2x^2\right)}{d^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTan[c*x]))/(d*(1 + m)) - (b*c*(d*x)^(2 + m)*Hyperg
eometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(d^2*(1 + m)*(2 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :>
 Simp[(d*x)^(m + 1)*((a + b*ArcTan[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n/(
d^n*(m + 1))), Int[(d*x)^(m + n)/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{a, b,
c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a+b\tan^{-1}(cx)\right)\,dx &= \frac{(dx)^{1+m}\left(a+b\tan^{-1}(cx)\right)}{d(1+m)}-\frac{(bc)\int\frac{(dx)^{1+m}}{1+c^2x^2}\,dx}{d(1+m)}\\ &= \frac{(dx)^{1+m}\left(a+b\tan^{-1}(cx)\right)}{d(1+m)}-\frac{bc(dx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-c^2x^2\right)}{d^2(1+m)(2+m)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 60, normalized size = 0.82

$$-\frac{x(dx)^m\left(-((2+m)(a+b\text{ArcTan}(cx)))+bcx\,{}_2F_1\left(1,1+\frac{m}{2};2+\frac{m}{2};-c^2x^2\right)\right)}{(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x]),x]
[Out] -((x*(d*x)^m*(-((2 + m)*(a + b*ArcTan[c*x])) + b*c*x*Hypergeometric2F1[1, 1
 + m/2, 2 + m/2, -(c^2*x^2)]))/((1 + m)*(2 + m)))
```

Maple [F]
time = 1.71, size = 0, normalized size = 0.00

$$\int (dx)^m (a+b\arctan(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x)),x)
[Out] int((d*x)^m*(a+b*arctan(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] (d^m*x*x^m*arctan(c*x) - (c*d^m*m + c*d^m)*integrate(x*x^m/((c^2*m + c^2)*x
^2 + m + 1), x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x)),x, algorithm="fricas")
[Out] integral((b*arctan(c*x) + a)*(d*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a+b\,\text{atan}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x)),x)
```

```
[Out] Integral((d*x)**m*(a + b*atan(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x)),x, algorithm="giac")
```

```
[Out] sage0*x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{atan}(c\,x))\,\left(d\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))*(d*x)^m,x)
```

```
[Out] int((a + b*atan(c*x))*(d*x)^m, x)
```

3.57 $\int \frac{(dx)^m}{a+b\mathbf{ArcTan}(cx)}\,dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{(dx)^m}{a+b\operatorname{ArcTan}(cx)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctan(c*x)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(dx)^m}{a+b\operatorname{ArcTan}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTan[c*x]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTan[c*x]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tan^{-1}(cx)}\,dx = \int \frac{(dx)^m}{a+b\tan^{-1}(cx)}\,dx$$

Mathematica [A]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\operatorname{ArcTan}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTan[c*x]),x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTan[c*x]), x]
```

Maple [A]
time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\arctan(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctan(c*x)),x)
[Out] int((d*x)^m/(a+b*arctan(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arctan(c*x) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arctan(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b\operatorname{atan}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atan(c*x)),x)
[Out] Integral((d*x)**m/(a + b*atan(c*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x)),x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{a + b\operatorname{atan}(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atan(c*x)),x)
[Out] int((d*x)^m/(a + b*atan(c*x)), x)
```

3.58 $\int (a+b\mathbf{ArcTan}(cx))^p\,dx$

Optimal. Leaf size=13

$$\text{Int}((a+b\text{ArcTan}(cx))^p,x)$$

```
[Out] Unintegrable((a+b*arctan(c*x))^p,x)
```

Rubi [A]

time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\text{ArcTan}(cx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcTan[c*x])^p,x]
[Out] Defer[Int][(a + b*ArcTan[c*x])^p, x]
```

Rubi steps

$$\int \left(a+b\tan^{-1}(cx)\right)^p\,dx = \int \left(a+b\tan^{-1}(cx)\right)^p\,dx$$

Mathematica [A]

time = 0.34, size = 0, normalized size = 0.00

$$\int (a+b\text{ArcTan}(cx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x])^p,x]
[Out] Integrate[(a + b*ArcTan[c*x])^p, x]
```

Maple [A]

time = 0.49, size = 0, normalized size = 0.00

$$\int (a+b\arctan(cx))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x))^p,x)
```

```
[Out] int((a+b*arctan(c*x))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^p,x, algorithm="maxima")
[Out] integrate((b*arctan(c*x) + a)^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^p,x, algorithm="fricas")
[Out] integral((b*arctan(c*x) + a)^p, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{atan}(cx))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x))**p,x)
[Out] Integral((a + b*atan(c*x))**p, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x))^p,x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int (a + b \operatorname{atan}(c\,x))^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^p,x)
[Out] int((a + b*atan(c*x))^p, x)
```

3.59 $\int (dx)^m(a+b\mathbf{ArcTan}(cx))^p\,dx$

Optimal. Leaf size=19

$$\text{Int}((dx)^m(a+b\text{ArcTan}(cx))^p,x)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x))^p,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m(a+b\text{ArcTan}(cx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x])^p,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x])^p, x]
```

Rubi steps

$$\int (dx)^m\left(a+b\tan^{-1}(cx)\right)^p\,dx = \int (dx)^m\left(a+b\tan^{-1}(cx)\right)^p\,dx$$

Mathematica [A]
time = 0.25, size = 0, normalized size = 0.00

$$\int (dx)^m(a+b\text{ArcTan}(cx))^p\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^p,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x])^p, x]
```

Maple [A]
time = 1.22, size = 0, normalized size = 0.00

$$\int (dx)^m\left(a+b\arctan(cx)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x))^p,x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^p,x, algorithm="maxima")
[Out] integrate((d*x)^m*(b*arctan(c*x) + a)^p, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^p,x, algorithm="fricas")
[Out] integral((d*x)^m*(b*arctan(c*x) + a)^p, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{atan}(cx))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x))**p,x)
[Out] Integral((d*x)**m*(a + b*atan(c*x))**p, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x))^p,x, algorithm="giac")
[Out] sage0*x
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (a + b \operatorname{atan}(c\,x))^p \,(d\,x)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x))^p*(d*x)^m,x)
[Out] int((a + b*atan(c*x))^p*(d*x)^m, x)
```

3.60 $\int x^7(a+b\mathbf{ArcTan}(cx^2))\,dx$

Optimal. Leaf size=54

$$\frac{bx^2}{8c^3}-\frac{bx^6}{24c}-\frac{b\text{ArcTan}(cx^2)}{8c^4}+\frac{1}{8}x^8\left(a+b\text{ArcTan}\left(cx^2\right)\right)$$

```
[Out] 1/8*b*x^2/c^3-1/24*b*x^6/c-1/8*b*arctan(c*x^2)/c^4+1/8*x^8*(a+b*arctan(c*x^
2))
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {4946, 281, 308, 209}

$$\frac{1}{8}x^8\left(a+b\text{ArcTan}\left(cx^2\right)\right)-\frac{b\text{ArcTan}(cx^2)}{8c^4}+\frac{bx^2}{8c^3}-\frac{bx^6}{24c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (b*x^2)/(8*c^3) - (b*x^6)/(24*c) - (b*ArcTan[c*x^2])/(8*c^4) + (x^8*(a + b*
ArcTan[c*x^2]))/8
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
```

```
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^7\left(a+b\tan^{-1}\left(cx^2\right)\right)\,dx &= \frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{4}(bc)\int\frac{x^9}{1+c^2x^4}\,dx\\
&= \frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{8}(bc)\text{Subst}\left(\int\frac{x^4}{1+c^2x^2}\,dx,x,x^2\right)\\
&= \frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{8}(bc)\text{Subst}\left(\int\left(-\frac{1}{c^4}+\frac{x^2}{c^2}+\frac{1}{c^4\left(1+c^2x^2\right)}\right)\,dx,x,\right.\\
&= \frac{bx^2}{8c^3}-\frac{bx^6}{24c}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\text{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,x^2\right)}{8c^3}\\
&= \frac{bx^2}{8c^3}-\frac{bx^6}{24c}-\frac{b\tan^{-1}\left(cx^2\right)}{8c^4}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^2\right)\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 1.09

$$\frac{bx^2}{8c^3}-\frac{bx^6}{24c}+\frac{ax^8}{8}-\frac{b\text{ArcTan}\left(cx^2\right)}{8c^4}+\frac{1}{8}bx^8\text{ArcTan}\left(cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (b*x^2)/(8*c^3) - (b*x^6)/(24*c) + (a*x^8)/8 - (b*ArcTan[c*x^2])/(8*c^4) +
(b*x^8*ArcTan[c*x^2])/8
```

Maple [A]
time = 0.21, size = 50, normalized size = 0.93

method	result	size
default	$\frac{x^8 a}{8}+\frac{b\,x^8\arctan\left(c\,x^2\right)}{8}-\frac{b\,x^6}{24c}+\frac{b\,x^2}{8c^3}-\frac{b\arctan\left(c\,x^2\right)}{8c^4}$	50
risch	$-\frac{i x^8 b\ln\left(i c\,x^2+1\right)}{16}+\frac{i x^8 b\ln\left(-i c\,x^2+1\right)}{16}+\frac{x^8 a}{8}-\frac{b\,x^6}{24c}+\frac{b\,x^2}{8c^3}-\frac{b\arctan\left(c\,x^2\right)}{8c^4}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^8*a+1/8*b*x^8*arctan(c*x^2)-1/24*b*x^6/c+1/8*b*x^2/c^3-1/8*b*arctan(c
*x^2)/c^4
```

Maxima [A]
time = 0.47, size = 54, normalized size = 1.00

$$\frac{1}{8}\,ax^8 + \frac{1}{24}\left(3\,x^8\arctan\left(cx^2\right) - c\left(\frac{c^2x^6 - 3\,x^2}{c^4} + \frac{3\,\arctan\left(cx^2\right)}{c^5}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8 + 1/24*(3*x^8*arctan(c*x^2) - c*((c^2*x^6 - 3*x^2)/c^4 + 3*arctan
(c*x^2)/c^5))*b
```

Fricas [A]
time = 2.64, size = 51, normalized size = 0.94

$$\frac{3\,ac^4x^8 - bc^3x^6 + 3\,bcx^2 + 3\,(bc^4x^8 - b)\arctan\left(cx^2\right)}{24\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/24*(3*a*c^4*x^8 - b*c^3*x^6 + 3*b*c*x^2 + 3*(b*c^4*x^8 - b)*arctan(c*x^2)
)/c^4
```

Sympy [A]
time = 28.52, size = 58, normalized size = 1.07

$$\begin{cases} \frac{ax^8}{8} + \frac{bx^8\operatorname{atan}\left(cx^2\right)}{8} - \frac{bx^6}{24c} + \frac{bx^2}{8c^3} - \frac{b\operatorname{atan}\left(cx^2\right)}{8c^4} & \text{for}\: c \neq 0 \\ \frac{ax^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*x**8/8 + b*x**8*atan(c*x**2)/8 - b*x**6/(24*c) + b*x**2/(8*c**
3) - b*atan(c*x**2)/(8*c**4), Ne(c, 0)), (a*x**8/8, True))
```

Giac [A]
time = 0.43, size = 60, normalized size = 1.11

$$\frac{3\,acx^8 + \left(3\,cx^8\arctan\left(cx^2\right) - \frac{3\,\arctan\left(cx^2\right)}{c^3} - \frac{c^9x^6 - 3\,c^7x^2}{c^9}\right)b}{24\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/24*(3*a*c*x^8 + (3*c*x^8*arctan(c*x^2) - 3*arctan(c*x^2)/c^3 - (c^9*x^6 -
 3*c^7*x^2)/c^9)*b)/c
```

Mupad [B]
time = 0.36, size = 49, normalized size = 0.91

$$\frac{a\,x^8}{8}+\frac{b\,x^2}{8\,c^3}-\frac{b\,x^6}{24\,c}-\frac{b\,\mathrm{atan}\left(c\,x^2\right)}{8\,c^4}+\frac{b\,x^8\,\mathrm{atan}\left(c\,x^2\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*atan(c*x^2)),x)
```

```
[Out] (a*x^8)/8 + (b*x^2)/(8*c^3) - (b*x^6)/(24*c) - (b*atan(c*x^2))/(8*c^4) + (b
*x^8*atan(c*x^2))/8
```

3.61 $\int x^5(a + b\mathbf{ArcTan}(cx^2))\,dx$

Optimal. Leaf size=47

$$-\frac{bx^4}{12c} + \frac{1}{6}x^6\left(a + b\text{ArcTan}\left(cx^2\right)\right) + \frac{b\log\left(1 + c^2x^4\right)}{12c^3}$$

```
[Out] -1/12*b*x^4/c+1/6*x^6*(a+b*arctan(c*x^2))+1/12*b*ln(c^2*x^4+1)/c^3
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4946, 272, 45}

$$\frac{1}{6}x^6\left(a + b\text{ArcTan}\left(cx^2\right)\right) + \frac{b\log\left(c^2x^4 + 1\right)}{12c^3} - \frac{bx^4}{12c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] -1/12*(b*x^4)/c + (x^6*(a + b*ArcTan[c*x^2]))/6 + (b*Log[1 + c^2*x^4])/(12*
c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)\,dx=\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{3}(bc)\int\frac{x^7}{1+c^2x^4}\,dx$$
$$=\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{12}(bc)\mathrm{Subst}\left(\int\frac{x}{1+c^2x}\,dx,x,x^4\right)$$
$$=\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{12}(bc)\mathrm{Subst}\left(\int\left(\frac{1}{c^2}-\frac{1}{c^2\left(1+c^2x\right)}\right)\,dx,x,x^4\right)$$
$$=-\frac{bx^4}{12c}+\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\log\left(1+c^2x^4\right)}{12c^3}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 1.11

$$-\frac{bx^4}{12c}+\frac{ax^6}{6}+\frac{1}{6}bx^6\mathrm{ArcTan}\left(cx^2\right)+\frac{b\log\left(1+c^2x^4\right)}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x^2]),x]
[Out] -1/12*(b*x^4)/c + (a*x^6)/6 + (b*x^6*ArcTan[c*x^2])/6 + (b*Log[1 + c^2*x^4]
)/(12*c^3)
```

Maple [A]
time = 0.20, size = 45, normalized size = 0.96

method	result	size
default	$\frac{x^6a}{6}+\frac{b\,x^6\arctan\left(c\,x^2\right)}{6}-\frac{b\,x^4}{12c}+\frac{b\ln\left(c^2x^4+1\right)}{12c^3}$	45
risch	$-\frac{ix^6b\ln\left(ic\,x^2+1\right)}{12}+\frac{ix^6b\ln\left(-ic\,x^2+1\right)}{12}+\frac{x^6a}{6}-\frac{b\,x^4}{12c}+\frac{b\ln\left(-c^2x^4-1\right)}{12c^3}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
[Out] 1/6*x^6*a+1/6*b*x^6*arctan(c*x^2)-1/12*b*x^4/c+1/12*b*ln(c^2*x^4+1)/c^3
```

Maxima [A]
time = 0.27, size = 48, normalized size = 1.02

$$\frac{1}{6}\,ax^6+\frac{1}{12}\left(2\,x^6\arctan\left(cx^2\right)-\left(\frac{x^4}{c^2}-\frac{\log\left(c^2x^4+1\right)}{c^4}\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/12*(2*x^6*arctan(c*x^2) - (x^4/c^2 - log(c^2*x^4 + 1)/c^4)*c)
*b
```

Fricas [A]
time = 2.86, size = 51, normalized size = 1.09

$$\frac{2\,bc^3x^6\arctan\left(cx^2\right)+2\,ac^3x^6-bc^2x^4+b\log\left(c^2x^4+1\right)}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*b*c^3*x^6*arctan(c*x^2) + 2*a*c^3*x^6 - b*c^2*x^4 + b*log(c^2*x^4 +
 1))/c^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(39) = 78$.
time = 21.11, size = 80, normalized size = 1.70

$$\begin{cases} \frac{ax^6}{6}+\frac{bx^6\operatorname{atan}\left(cx^2\right)}{6}-\frac{bx^4}{12c}+\frac{b\sqrt{-\frac{1}{c^2}}\operatorname{atan}\left(cx^2\right)}{6c^2}+\frac{b\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{6c^3} & \text{for}\ c\neq 0\\ \frac{ax^6}{6} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*x**6/6 + b*x**6*atan(c*x**2)/6 - b*x**4/(12*c) + b*sqrt(-1/c**
2)*atan(c*x**2)/(6*c**2) + b*log(x**2 + sqrt(-1/c**2))/(6*c**3), Ne(c, 0)),
 (a*x**6/6, True))
```

Giac [A]
time = 0.43, size = 47, normalized size = 1.00

$$\frac{2\,acx^6+\left(2\,cx^6\arctan\left(cx^2\right)-x^4+\frac{\log\left(c^2x^4+1\right)}{c^2}\right)b}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/12*(2*a*c*x^6 + (2*c*x^6*arctan(c*x^2) - x^4 + log(c^2*x^4 + 1)/c^2)*b)/c
```

Mupad [B]
time = 0.35, size = 44, normalized size = 0.94

$$\frac{a\,x^6}{6}+\frac{b\,\ln\left(c^2\,x^4+1\right)}{12\,c^3}-\frac{b\,x^4}{12\,c}+\frac{b\,x^6\,\mathrm{atan}\left(c\,x^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atan(c*x^2)),x)
```

```
[Out] (a*x^6)/6 + (b*log(c^2*x^4 + 1))/(12*c^3) - (b*x^4)/(12*c) + (b*x^6*atan(c*
x^2))/6
```

3.62 $\int x^3(a+b\mathbf{ArcTan}(cx^2))\,dx$

Optimal. Leaf size=43

$$-\frac{bx^2}{4c}+\frac{b\text{ArcTan}(cx^2)}{4c^2}+\frac{1}{4}x^4\left(a+b\text{ArcTan}\left(cx^2\right)\right)$$

```
[Out] -1/4*b*x^2/c+1/4*b*arctan(c*x^2)/c^2+1/4*x^4*(a+b*arctan(c*x^2))
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4946, 281, 327, 209}

$$\frac{1}{4}x^4\left(a+b\text{ArcTan}\left(cx^2\right)\right)+\frac{b\text{ArcTan}(cx^2)}{4c^2}-\frac{bx^2}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] -1/4*(b*x^2)/c + (b*ArcTan[c*x^2])/(4*c^2) + (x^4*(a + b*ArcTan[c*x^2]))/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
```

```
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)\,dx &= \frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{2}(bc)\int\frac{x^5}{1+c^2x^4}\,dx\\ &= \frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{4}(bc)\text{Subst}\left(\int\frac{x^2}{1+c^2x^2}\,dx,x,x^2\right)\\ &= -\frac{bx^2}{4c}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\text{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,x^2\right)}{4c}\\ &= -\frac{bx^2}{4c}+\frac{b\tan^{-1}\left(cx^2\right)}{4c^2}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^2\right)\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 1.12

$$-\frac{bx^2}{4c}+\frac{ax^4}{4}+\frac{b\text{ArcTan}(cx^2)}{4c^2}+\frac{1}{4}bx^4\text{ArcTan}\left(cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] -1/4*(b*x^2)/c + (a*x^4)/4 + (b*ArcTan[c*x^2])/(4*c^2) + (b*x^4*ArcTan[c*x^
2])/4
```

Maple [A]
time = 0.19, size = 41, normalized size = 0.95

method	result	size
default	$\frac{a\,x^4}{4}+\frac{b\,x^4\arctan(c\,x^2)}{4}-\frac{b\,x^2}{4c}+\frac{b\arctan(c\,x^2)}{4c^2}$	41
risch	$-\frac{ib\,x^4\ln(ic\,x^2+1)}{8}+\frac{ib\,x^4\ln(-ic\,x^2+1)}{8}+\frac{a\,x^4}{4}-\frac{b\,x^2}{4c}+\frac{b\arctan(c\,x^2)}{4c^2}+\frac{b^2}{16a\,c^2}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x^4+1/4*b*x^4*arctan(c*x^2)-1/4*b*x^2/c+1/4*b*arctan(c*x^2)/c^2
```

Maxima [A]
time = 0.46, size = 43, normalized size = 1.00

$$\frac{1}{4}\,ax^4+\frac{1}{4}\left(x^4\arctan\left(cx^2\right)-c\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2)),x, algorithm="maxima")
[Out] 1/4*a*x^4 + 1/4*(x^4*arctan(c*x^2) - c*(x^2/c^2 - arctan(c*x^2)/c^3))*b
```

Fricas [A]
time = 3.46, size = 38, normalized size = 0.88

$$\frac{ac^2x^4 - bcx^2 + (bc^2x^4 + b)\arctan(cx^2)}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2)),x, algorithm="fricas")
[Out] 1/4*(a*c^2*x^4 - b*c*x^2 + (b*c^2*x^4 + b)*arctan(c*x^2))/c^2
```

Sympy [A]
time = 10.19, size = 48, normalized size = 1.12

$$\begin{cases} \frac{ax^4}{4} + \frac{bx^4 \operatorname{atan}(cx^2)}{4} - \frac{bx^2}{4c} + \frac{b\operatorname{atan}(cx^2)}{4c^2} & \text{for } c \neq 0 \\ \frac{ax^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x**2)),x)
[Out] Piecewise((a*x**4/4 + b*x**4*atan(c*x**2)/4 - b*x**2/(4*c) + b*atan(c*x**2)
/(4*c**2), Ne(c, 0)), (a*x**4/4, True))
```

Giac [A]
time = 0.43, size = 43, normalized size = 1.00

$$\frac{acx^4 + \frac{(c^2x^4\arctan(cx^2) - cx^2 + \arctan(cx^2))b}{c}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2)),x, algorithm="giac")
[Out] 1/4*(a*c*x^4 + (c^2*x^4*arctan(c*x^2) - c*x^2 + arctan(c*x^2))*b/c)/c
```

Mupad [B]
time = 0.33, size = 40, normalized size = 0.93

$$\frac{a\,x^4}{4} - \frac{b\,x^2}{4\,c} + \frac{b\operatorname{atan}(c\,x^2)}{4\,c^2} + \frac{b\,x^4\operatorname{atan}(c\,x^2)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x^2)),x)
[Out] (a*x^4)/4 - (b*x^2)/(4*c) + (b*atan(c*x^2))/(4*c^2) + (b*x^4*atan(c*x^2))/4
```

3.63 $\int x(a+b\mathbf{ArcTan}(cx^2))\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}x^2(a+b\text{ArcTan}(cx^2))-\frac{b\log(1+c^2x^4)}{4c}$$

```
[Out] 1/2*x^2*(a+b*arctan(c*x^2))-1/4*b*ln(c^2*x^4+1)/c
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4946, 266}

$$\frac{1}{2}x^2(a+b\text{ArcTan}(cx^2))-\frac{b\log(c^2x^4+1)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (x^2*(a + b*ArcTan[c*x^2]))/2 - (b*Log[1 + c^2*x^4])/(4*c)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\tan^{-1}\left(cx^2\right)\right)\,dx&=\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^2\right)\right)-(bc)\int\frac{x^3}{1+c^2x^4}\,dx\\&=\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\log\left(1+c^2x^4\right)}{4c}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.14

$$\frac{ax^2}{2}+\frac{1}{2}bx^2\text{ArcTan}(cx^2)-\frac{b\log(1+c^2x^4)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (a*x^2)/2 + (b*x^2*ArcTan[c*x^2])/2 - (b*Log[1 + c^2*x^4])/(4*c)
```

Maple [A]
time = 0.15, size = 38, normalized size = 1.06

method	result	size
derivativedivides	$\frac{a c\,x^{2}+b c\,x^{2} \arctan\left(c\,x^{2}\right)-\frac{b \ln\left(c^{2} x^{4}+1\right)}{2}}{2 c}$	38
default	$\frac{a c\,x^{2}+b c\,x^{2} \arctan\left(c\,x^{2}\right)-\frac{b \ln\left(c^{2} x^{4}+1\right)}{2}}{2 c}$	38
risch	$-\frac{i x^{2} b \ln\left(i c\,x^{2}+1\right)}{4}+\frac{i b\,x^{2} \ln\left(-i c\,x^{2}+1\right)}{4}+\frac{a\,x^{2}}{2}-\frac{b \ln\left(-c^{2} x^{4}-1\right)}{4 c}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c*(a*c*x^2+b*c*x^2*arctan(c*x^2)-1/2*b*ln(c^2*x^4+1))
```

Maxima [A]
time = 0.26, size = 38, normalized size = 1.06

$$\frac{1}{2}\,ax^{2}+\frac{\left(2\,cx^{2}\arctan\left(cx^{2}\right)-\log\left(c^{2}x^{4}+1\right)\right)b}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/4*(2*c*x^2*arctan(c*x^2) - log(c^2*x^4 + 1))*b/c
```

Fricas [A]
time = 3.69, size = 39, normalized size = 1.08

$$\frac{2\,bcx^{2}\arctan\left(cx^{2}\right)+2\,acx^{2}-b\log\left(c^{2}x^{4}+1\right)}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*c*x^2*arctan(c*x^2) + 2*a*c*x^2 - b*log(c^2*x^4 + 1))/c
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(29) = 58$.

time = 5.67, size = 66, normalized size = 1.83

$$\begin{cases} \frac{ax^2}{2} + \frac{bx^2 \operatorname{atan}(cx^2)}{2} - \frac{b\sqrt{-\frac{1}{c^2}} \operatorname{atan}(cx^2)}{2} - \frac{b \log\left(x^2 + \sqrt{-\frac{1}{c^2}}\right)}{2c} & \text{for } c \neq 0 \\ \frac{ax^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*x**2/2 + b*x**2*atan(c*x**2)/2 - b*sqrt(-1/c**2)*atan(c*x**2)/
2 - b*log(x**2 + sqrt(-1/c**2))/(2*c), Ne(c, 0)), (a*x**2/2, True))
```

Giac [A]
time = 0.44, size = 40, normalized size = 1.11

$$\frac{2\,acx^2 + (2\,cx^2 \arctan(cx^2) - \log(c^2x^4 + 1))b}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/4*(2*a*c*x^2 + (2*c*x^2*arctan(c*x^2) - log(c^2*x^4 + 1))*b)/c
```

Mupad [B]
time = 0.31, size = 35, normalized size = 0.97

$$\frac{a\,x^2}{2} - \frac{b\,\ln\left(c^2\,x^4 + 1\right)}{4\,c} + \frac{b\,x^2\,\operatorname{atan}(c\,x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x^2)),x)
```

```
[Out] (a*x^2)/2 - (b*log(c^2*x^4 + 1))/(4*c) + (b*x^2*atan(c*x^2))/2
```

3.64 $\int \frac{a+b\mathbf{ArcTan}(cx^2)}{x}\,dx$

Optimal. Leaf size=39

$$a\log(x)+\frac{1}{4}ib\text{PolyLog}\left(2,-icx^2\right)-\frac{1}{4}ib\text{PolyLog}\left(2,icx^2\right)$$

```
[Out] a*ln(x)+1/4*I*b*polylog(2,-I*c*x^2)-1/4*I*b*polylog(2,I*c*x^2)
```

Rubi [A]
time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4944, 4940, 2438}

$$a\log(x)+\frac{1}{4}ib\text{Li}_2\left(-icx^2\right)-\frac{1}{4}ib\text{Li}_2\left(icx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x,x]
```

```
[Out] a*Log[x] + (I/4)*b*PolyLog[2, (-I)*c*x^2] - (I/4)*b*PolyLog[2, I*c*x^2]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}\left(cx^2\right)}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{a+b\tan^{-1}(cx)}{x}\,dx,x,x^2\right)\\ &= a\log(x)+\frac{1}{4}(ib)\text{Subst}\left(\int \frac{\log(1-icx)}{x}\,dx,x,x^2\right)-\frac{1}{4}(ib)\text{Subst}\left(\int \frac{\log(1+icx)}{x}\right.\\ &= a\log(x)+\frac{1}{4}ib\text{Li}_2\left(-icx^2\right)-\frac{1}{4}ib\text{Li}_2\left(icx^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.00

$$a\log(x)+\frac{1}{4}ib\text{PolyLog}\left(2,-icx^2\right)-\frac{1}{4}ib\text{PolyLog}\left(2,icx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x,x]
```

```
[Out] a*Log[x] + (I/4)*b*PolyLog[2, (-I)*c*x^2] - (I/4)*b*PolyLog[2, I*c*x^2]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.24, size = 63, normalized size = 1.62

method	result
default	$a\ln(x)+b\ln(x)\arctan(c\,x^2)-\frac{b\left(\sum_{\_R1=\text{RootOf}\left(c^2\_Z^4+1\right)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^2}\right)}{2c}$
risch	$-\frac{i\ln(x)\ln\left(1+ix\sqrt{-ic}\right)b}{2}-\frac{i\ln(x)\ln\left(1-ix\sqrt{-ic}\right)b}{2}-\frac{i\,\text{dilog}\left(1-ix\sqrt{-ic}\right)b}{2}-\frac{i\,\text{dilog}\left(1+ix\sqrt{-ic}\right)b}{2}+\frac{i\ln(x)\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] a*ln(x)+b*ln(x)*arctan(c*x^2)-1/2*b/c*sum(1/_R1^2*(ln(x)*ln((_R1-x)/_R1)+di
log((_R1-x)/_R1)),_R1=RootOf(_Z^4*c^2+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x,x, algorithm="maxima")
```

```
[Out] b*integrate(arctan(c*x^2)/x, x) + a*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x^2) + a)/x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atan}\left(cx^2\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x,x)
```

```
[Out] Integral((a + b*atan(c*x**2))/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)/x, x)
```

Mupad [**B**]
time = 0.33, size = 32, normalized size = 0.82

$$a\,\ln\left(x\right)-\frac{b\,\left(\mathrm{Li}_2(1-c\,x^2\,1{}\mathrm{i})-\mathrm{Li}_2(1{}\mathrm{i}\,c\,x^2+1)\right)\,1{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))/x,x)
```

```
[Out] a*log(x) - (b*(dilog(1 - c*x^2*1i) - dilog(c*x^2*1i + 1))*1i)/4
```

3.65 $\int \frac{a+b\mathbf{ArcTan}(cx^2)}{x^3}\,dx$

Optimal. Leaf size=39

$$-\frac{a+b\text{ArcTan}(cx^2)}{2x^2}+bc\log(x)-\frac{1}{4}bc\log\left(1+c^2x^4\right)$$

```
[Out] 1/2*(-a-b*arctan(c*x^2))/x^2+b*c*ln(x)-1/4*b*c*ln(c^2*x^4+1)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4946, 272, 36, 29, 31}

$$-\frac{a+b\text{ArcTan}(cx^2)}{2x^2}-\frac{1}{4}bc\log\left(c^2x^4+1\right)+bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x^3,x]
```

```
[Out] -1/2*(a + b*ArcTan[c*x^2])/x^2 + b*c*Log[x] - (b*c*Log[1 + c^2*x^4])/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}(cx^2)}{x^3}\,dx &= -\frac{a+b\tan^{-1}(cx^2)}{2x^2}+(bc)\int \frac{1}{x(1+c^2x^4)}\,dx\\ &= -\frac{a+b\tan^{-1}(cx^2)}{2x^2}+\frac{1}{4}(bc)\text{Subst}\left(\int \frac{1}{x(1+c^2x)}\,dx,x,x^4\right)\\ &= -\frac{a+b\tan^{-1}(cx^2)}{2x^2}+\frac{1}{4}(bc)\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^4\right)-\frac{1}{4}(bc^3)\,\text{Subst}\left(\int \frac{1}{1+c^2x}\,dx\right.\\ &= -\frac{a+b\tan^{-1}(cx^2)}{2x^2}+bc\log(x)-\frac{1}{4}bc\log\left(1+c^2x^4\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 1.13

$$-\frac{a}{2x^2}-\frac{b\text{ArcTan}(cx^2)}{2x^2}+bc\log(x)-\frac{1}{4}bc\log\left(1+c^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^3,x]
[Out] -1/2*a/x^2 - (b*ArcTan[c*x^2])/(2*x^2) + b*c*Log[x] - (b*c*Log[1 + c^2*x^4]
)/4
```

Maple [A]
time = 0.09, size = 39, normalized size = 1.00

method	result	size
default	$-\frac{a}{2x^2}-\frac{b\arctan(cx^2)}{2x^2}-\frac{bc\ln(c^2x^4+1)}{4}+bc\ln(x)$	39
risch	$\frac{ib\ln(icx^2+1)}{4x^2}-\frac{-4bc\ln(x)x^2+bc\ln(-c^2x^4-1)x^2+ib\ln(-icx^2+1)+2a}{4x^2}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*a/x^2-1/2*b/x^2*arctan(c*x^2)-1/4*b*c*ln(c^2*x^4+1)+b*c*ln(x)
```

Maxima [A]
time = 0.26, size = 41, normalized size = 1.05

$$-\frac{1}{4}\left(c\left(\log\left(c^2x^4+1\right)-\log\left(x^4\right)\right)+\frac{2\arctan\left(cx^2\right)}{x^2}\right)b-\frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^3,x, algorithm="maxima")
[Out] -1/4*(c*(log(c^2*x^4 + 1) - log(x^4)) + 2*arctan(c*x^2)/x^2)*b - 1/2*a/x^2
```

Fricas [A]
time = 2.43, size = 43, normalized size = 1.10

$$-\frac{bcx^2\log\left(c^2x^4+1\right)-4\,bcx^2\log\left(x\right)+2\,b\arctan\left(cx^2\right)+2\,a}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^3,x, algorithm="fricas")
[Out] -1/4*(b*c*x^2*log(c^2*x^4 + 1) - 4*b*c*x^2*log(x) + 2*b*arctan(c*x^2) + 2*a
)/x^2
```

Sympy [A]
time = 12.45, size = 75, normalized size = 1.92

$$\begin{cases}-\frac{a}{2x^2}+bc\log\left(x\right)-\frac{bc\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{2}+\frac{b\,\mathrm{atan}\left(cx^2\right)}{2\sqrt{-\frac{1}{c^2}}}-\frac{b\,\mathrm{atan}\left(cx^2\right)}{2x^2} & \text{for } c\neq 0\\ -\frac{a}{2x^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x**3,x)
[Out] Piecewise((-a/(2*x**2) + b*c*log(x) - b*c*log(x**2 + sqrt(-1/c**2))/2 + b*a
tan(c*x**2)/(2*sqrt(-1/c**2)) - b*atan(c*x**2)/(2*x**2), Ne(c, 0)), (-a/(2*
x**2), True))
```

Giac [A]
time = 0.44, size = 60, normalized size = 1.54

$$-\frac{bc^3x^2\log\left(c^2x^4+1\right)-2\,bc^3x^2\log\left(cx^2\right)+2\,bc^2\arctan\left(cx^2\right)+2\,ac^2}{4\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^3,x, algorithm="giac")
[Out] -1/4*(b*c^3*x^2*log(c^2*x^4 + 1) - 2*b*c^3*x^2*log(c*x^2) + 2*b*c^2*arctan(
c*x^2) + 2*a*c^2)/(c^2*x^2)
```

Mupad [B]
time = 0.34, size = 38, normalized size = 0.97

$$b\,c\,\ln\left(x\right)-\frac{a}{2\,x^2}-\frac{b\,\mathrm{atan}(c\,x^2)}{2\,x^2}-\frac{b\,c\,\ln\left(c^2\,x^4+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))/x^3,x)
```

```
[Out] b*c*log(x) - a/(2*x^2) - (b*atan(c*x^2))/(2*x^2) - (b*c*log(c^2*x^4 + 1))/4
```

3.66 $\int \frac{a+b\text{ArcTan}(cx^2)}{x^5}\,dx$

Optimal. Leaf size=41

$$-\frac{bc}{4x^2}-\frac{1}{4}bc^2\text{ArcTan}(cx^2)-\frac{a+b\text{ArcTan}(cx^2)}{4x^4}$$

```
[Out] -1/4*b*c/x^2-1/4*b*c^2*arctan(c*x^2)+1/4*(-a-b*arctan(c*x^2))/x^4
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4946, 281, 331, 209}

$$-\frac{a+b\text{ArcTan}(cx^2)}{4x^4}-\frac{1}{4}bc^2\text{ArcTan}(cx^2)-\frac{bc}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x^5,x]
```

```
[Out] -1/4*(b*c)/x^2 - (b*c^2*ArcTan[c*x^2])/4 - (a + b*ArcTan[c*x^2])/(4*x^4)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
```

```
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\tan^{-1}\left(cx^2\right)}{x^5}\,dx &= -\frac{a+b\tan^{-1}\left(cx^2\right)}{4x^4}+\frac{1}{2}(bc)\int \frac{1}{x^3\left(1+c^2x^4\right)}\,dx \\
&= -\frac{a+b\tan^{-1}\left(cx^2\right)}{4x^4}+\frac{1}{4}(bc)\text{Subst}\left(\int \frac{1}{x^2\left(1+c^2x^2\right)}\,dx,x,x^2\right) \\
&= -\frac{bc}{4x^2}-\frac{a+b\tan^{-1}\left(cx^2\right)}{4x^4}-\frac{1}{4}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1+c^2x^2}\,dx,x,x^2\right) \\
&= -\frac{bc}{4x^2}-\frac{1}{4}bc^2\tan^{-1}\left(cx^2\right)-\frac{a+b\tan^{-1}\left(cx^2\right)}{4x^4}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 48, normalized size = 1.17

$$-\frac{a}{4x^4}-\frac{b\text{ArcTan}\left(cx^2\right)}{4x^4}-\frac{bc\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-c^2x^4\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^5,x]
[Out] -1/4*a/x^4 - (b*ArcTan[c*x^2])/(4*x^4) - (b*c*Hypergeometric2F1[-1/2, 1, 1/
2, -(c^2*x^4)])/(4*x^2)
```

Maple [A]
time = 0.12, size = 39, normalized size = 0.95

method	result	size
default	$-\frac{a}{4x^4}-\frac{b\arctan\left(c\,x^2\right)}{4x^4}-\frac{b\,c^2\arctan\left(c\,x^2\right)}{4}-\frac{bc}{4x^2}$	39
risch	$\frac{ib\ln\left(ic\,x^2+1\right)}{8x^4}-\frac{ic^2b\ln\left(c\,x^2+i\right)x^4-ic^2b\ln\left(c\,x^2-i\right)x^4+2bc\,x^2+ib\ln\left(-ic\,x^2+1\right)+2a}{8x^4}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*a/x^4-1/4*b/x^4*arctan(c*x^2)-1/4*b*c^2*arctan(c*x^2)-1/4*b*c/x^2
```

Maxima [A]
time = 0.47, size = 35, normalized size = 0.85

$$-\frac{1}{4}\left(\left(c\arctan\left(cx^2\right)+\frac{1}{x^2}\right)c+\frac{\arctan\left(cx^2\right)}{x^4}\right)b-\frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*((c*arctan(c*x^2) + 1/x^2)*c + arctan(c*x^2)/x^4)*b - 1/4*a/x^4
```

Fricas [A]
time = 2.27, size = 30, normalized size = 0.73

$$-\frac{bcx^2 + (bc^2x^4 + b)\arctan(cx^2) + a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(b*c*x^2 + (b*c^2*x^4 + b)*arctan(c*x^2) + a)/x^4
```

Sympy [A]
time = 10.30, size = 42, normalized size = 1.02

$$-\frac{a}{4x^4} - \frac{bc^2\operatorname{atan}(cx^2)}{4} - \frac{bc}{4x^2} - \frac{b\operatorname{atan}(cx^2)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x**5,x)
```

```
[Out] -a/(4*x**4) - b*c**2*atan(c*x**2)/4 - b*c/(4*x**2) - b*atan(c*x**2)/(4*x**4
)
```

Giac [C] Result contains complex when optimal does not.
time = 0.46, size = 72, normalized size = 1.76

$$\frac{i\,bc^5x^4\log(i\,cx^2 + 1) - i\,bc^5x^4\log(-i\,cx^2 + 1) - 2\,bc^4x^2 - 2\,bc^3\arctan(cx^2) - 2\,ac^3}{8\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^5,x, algorithm="giac")
```

```
[Out] 1/8*(I*b*c^5*x^4*log(I*c*x^2 + 1) - I*b*c^5*x^4*log(-I*c*x^2 + 1) - 2*b*c^4
*x^2 - 2*b*c^3*arctan(c*x^2) - 2*a*c^3)/(c^3*x^4)
```

Mupad [B]
time = 0.36, size = 41, normalized size = 1.00

$$-\frac{\frac{b\,c\,x^2}{2} + \frac{a}{2}}{2\,x^4} - \frac{b\,c^2\,\mathrm{atan}(c\,x^2)}{4} - \frac{b\,\mathrm{atan}(c\,x^2)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))/x^5,x)
```

```
[Out] - (a/2 + (b*c*x^2)/2)/(2*x^4) - (b*c^2*atan(c*x^2))/4 - (b*atan(c*x^2))/(4*
x^4)
```

3.67 $\int \frac{a+b\text{ArcTan}(cx^2)}{x^7}\,dx$

Optimal. Leaf size=55

$$-\frac{bc}{12x^4}-\frac{a+b\text{ArcTan}(cx^2)}{6x^6}-\frac{1}{3}bc^3\log(x)+\frac{1}{12}bc^3\log\left(1+c^2x^4\right)$$

```
[Out] -1/12*b*c/x^4+1/6*(-a-b*arctan(c*x^2))/x^6-1/3*b*c^3*ln(x)+1/12*b*c^3*ln(c^
2*x^4+1)
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4946, 272, 46}

$$-\frac{a+b\text{ArcTan}(cx^2)}{6x^6}-\frac{1}{3}bc^3\log(x)+\frac{1}{12}bc^3\log\left(c^2x^4+1\right)-\frac{bc}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x^7,x]
```

```
[Out] -1/12*(b*c)/x^4 - (a + b*ArcTan[c*x^2])/(6*x^6) - (b*c^3*Log[x])/3 + (b*c^3
*Log[1 + c^2*x^4])/12
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx^2)}{x^7}\,dx = -\frac{a+b\tan^{-1}(cx^2)}{6x^6}+\frac{1}{3}(bc)\int \frac{1}{x^5(1+c^2x^4)}\,dx$$
$$= -\frac{a+b\tan^{-1}(cx^2)}{6x^6}+\frac{1}{12}(bc)\text{Subst}\left(\int \frac{1}{x^2(1+c^2x)}\,dx,x,x^4\right)$$
$$= -\frac{a+b\tan^{-1}(cx^2)}{6x^6}+\frac{1}{12}(bc)\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{c^2}{x}+\frac{c^4}{1+c^2x}\right)dx,x,x^4\right)$$
$$= -\frac{bc}{12x^4}-\frac{a+b\tan^{-1}(cx^2)}{6x^6}-\frac{1}{3}bc^3\log(x)+\frac{1}{12}bc^3\log\left(1+c^2x^4\right)$$

Mathematica [A]
time = 0.01, size = 60, normalized size = 1.09

$$-\frac{a}{6x^6}-\frac{bc}{12x^4}-\frac{b\text{ArcTan}(cx^2)}{6x^6}-\frac{1}{3}bc^3\log(x)+\frac{1}{12}bc^3\log\left(1+c^2x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^7,x]
```

```
[Out] -1/6*a/x^6 - (b*c)/(12*x^4) - (b*ArcTan[c*x^2])/(6*x^6) - (b*c^3*Log[x])/3
+ (b*c^3*Log[1 + c^2*x^4])/12
```

Maple [A]
time = 0.10, size = 51, normalized size = 0.93

method	result	size
default	$-\frac{a}{6x^6}-\frac{b\arctan(c\,x^2)}{6x^6}+\frac{b\,c^3\ln(c^2x^4+1)}{12}-\frac{bc}{12x^4}-\frac{b\,c^3\ln(x)}{3}$	51
risch	$\frac{ib\ln(ic\,x^2+1)}{12x^6}-\frac{4b\,c^3\ln(x)x^6-b\,c^3\ln(c^2x^4+1)x^6+bc\,x^2+ib\ln(-ic\,x^2+1)+2a}{12x^6}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a/x^6-1/6*b/x^6*arctan(c*x^2)+1/12*b*c^3*ln(c^2*x^4+1)-1/12*b*c/x^4-1/
3*b*c^3*ln(x)
```

Maxima [A]
time = 0.26, size = 53, normalized size = 0.96

$$\frac{1}{12}\left(\left(c^2\log\left(c^2x^4+1\right)-c^2\log\left(x^4\right)-\frac{1}{x^4}\right)c-\frac{2\arctan\left(cx^2\right)}{x^6}\right)b-\frac{a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^7,x, algorithm="maxima")
```

```
[Out] 1/12*((c^2*log(c^2*x^4 + 1) - c^2*log(x^4) - 1/x^4)*c - 2*arctan(c*x^2)/x^6
)*b - 1/6*a/x^6
```

Fricas [A]
time = 2.20, size = 54, normalized size = 0.98

$$\frac{bc^3x^6\log\left(c^2x^4+1\right)-4\,bc^3x^6\log\left(x\right)-bcx^2-2\,b\arctan\left(cx^2\right)-2\,a}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(b*c^3*x^6*log(c^2*x^4 + 1) - 4*b*c^3*x^6*log(x) - b*c*x^2 - 2*b*arcta
n(c*x^2) - 2*a)/x^6
```

Sympy [A]
time = 35.83, size = 92, normalized size = 1.67

$$\begin{cases} -\frac{a}{6x^6}-\frac{bc^3\log\left(x\right)}{3}+\frac{bc^3\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{6}-\frac{bc^2\,\mathrm{atan}\left(cx^2\right)}{6\sqrt{-\frac{1}{c^2}}}-\frac{bc}{12x^4}-\frac{b\,\mathrm{atan}\left(cx^2\right)}{6x^6} & \text{for}\ c\neq 0 \\ -\frac{a}{6x^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x**7,x)
```

```
[Out] Piecewise((-a/(6*x**6) - b*c**3*log(x)/3 + b*c**3*log(x**2 + sqrt(-1/c**2))
/6 - b*c**2*atan(c*x**2)/(6*sqrt(-1/c**2)) - b*c/(12*x**4) - b*atan(c*x**2)
/(6*x**6), Ne(c, 0)), (-a/(6*x**6), True))
```

Giac [A]
time = 0.46, size = 69, normalized size = 1.25

$$\frac{bc^7x^6\log\left(c^2x^4+1\right)-2\,bc^7x^6\log\left(cx^2\right)-bc^5x^2-2\,bc^4\arctan\left(cx^2\right)-2\,ac^4}{12\,c^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^7,x, algorithm="giac")
```

```
[Out] 1/12*(b*c^7*x^6*log(c^2*x^4 + 1) - 2*b*c^7*x^6*log(c*x^2) - b*c^5*x^2 - 2*b
*c^4*arctan(c*x^2) - 2*a*c^4)/(c^4*x^6)
```

Mupad [B]
time = 0.37, size = 50, normalized size = 0.91

$$\frac{b\,c^3\,\ln\left(c^2\,x^4+1\right)}{12}-\frac{a}{6\,x^6}-\frac{b\,c^3\,\ln\left(x\right)}{3}-\frac{b\,\mathrm{atan}\left(c\,x^2\right)}{6\,x^6}-\frac{b\,c}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))/x^7,x)
```

```
[Out] (b*c^3*log(c^2*x^4 + 1))/12 - a/(6*x^6) - (b*c^3*log(x))/3 - (b*atan(c*x^2)
)/(6*x^6) - (b*c)/(12*x^4)
```

3.68 $\int x^4(a+b\text{ArcTan}(cx^2))\,dx$

Optimal. Leaf size=161

$$-\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\text{ArcTan}\left(cx^2\right)\right)-\frac{b\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{c}\,x\right)}{5\sqrt{2}\,c^{5/2}}+\frac{b\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{c}\,x\right)}{5\sqrt{2}\,c^{5/2}}+\frac{b\log\left(1-\sqrt{2}\,\sqrt{\cdots}\right.}{10\sqrt{2}\,c^{\cdots}}$$

```
[Out] -2/15*b*x^3/c+1/5*x^5*(a+b*arctan(c*x^2))+1/10*b*arctan(-1+x*2^(1/2)*c^(1/2
))/c^(5/2)*2^(1/2)+1/10*b*arctan(1+x*2^(1/2)*c^(1/2))/c^(5/2)*2^(1/2)+1/20*
b*ln(1+c*x^2-x*2^(1/2)*c^(1/2))/c^(5/2)*2^(1/2)-1/20*b*ln(1+c*x^2+x*2^(1/2)
*c^(1/2))/c^(5/2)*2^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4946, 327, 303, 1176, 631, 210, 1179, 642}

$$\frac{1}{5}x^5\left(a+b\text{ArcTan}\left(cx^2\right)\right)-\frac{b\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{c}\,x\right)}{5\sqrt{2}\,c^{5/2}}+\frac{b\text{ArcTan}\left(\sqrt{2}\,\sqrt{c}\,x+1\right)}{5\sqrt{2}\,c^{5/2}}+\frac{b\log\left(cx^2-\sqrt{2}\,\sqrt{c}\,x+1\right)}{10\sqrt{2}\,c^{5/2}}-\frac{b\log\left(cx^2+\sqrt{2}\,\sqrt{c}\,x+1\right)}{10\sqrt{2}\,c^{5/2}}-\frac{2bx^3}{15c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (-2*b*x^3)/(15*c) + (x^5*(a + b*ArcTan[c*x^2]))/5 - (b*ArcTan[1 - Sqrt[2]*S
qrt[c]*x])/(5*Sqrt[2]*c^(5/2)) + (b*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(5*Sqrt[
2]*c^(5/2)) + (b*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt[2]*c^(5/2)) -
 (b*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt[2]*c^(5/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^4\left(a+b\tan^{-1}\left(cx^2\right)\right)\,dx &= \frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{5}(2bc)\int\frac{x^6}{1+c^2x^4}\,dx\\
&= -\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{(2b)\int\frac{x^2}{1+c^2x^4}\,dx}{5c}\\
&= -\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\int\frac{1-cx^2}{1+c^2x^4}\,dx}{5c^2}+\frac{b\int\frac{1+cx^2}{1+c^2x^4}\,dx}{5c^2}\\
&= -\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\int\frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{10c^3}+\frac{b\int\frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{10c^3}\\
&= -\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\log\left(1-\sqrt{2}\sqrt{c}\,x+cx^2\right)}{10\sqrt{2}\,c^{5/2}}-\frac{b\log\left(1+\right.}{}\\
&= -\frac{2bx^3}{15c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{5\sqrt{2}\,c^{5/2}}+\frac{b\tan^{-1}\left(1+\right.}{5\sqrt{2}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 179, normalized size = 1.11

$$
-\frac{2bx^3}{15c}+\frac{ax^5}{5}+\frac{1}{5}bx^5\text{ArcTan}\left(cx^2\right)+\frac{b\text{ArcTan}\left(\frac{-\sqrt{2}+2\sqrt{c}\,x}{\sqrt{2}}\right)}{5\sqrt{2}\,c^{5/2}}+\frac{b\text{ArcTan}\left(\frac{\sqrt{2}+2\sqrt{c}\,x}{\sqrt{2}}\right)}{5\sqrt{2}\,c^{5/2}}+\frac{b\log\left(1-\sqrt{2}\sqrt{c}\,x+cx^2\right)}{10\sqrt{2}\,c^{5/2}}-\frac{b\log\left(1+\sqrt{2}\sqrt{c}\,x+cx^2\right)}{10\sqrt{2}\,c^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (-2*b*x^3)/(15*c) + (a*x^5)/5 + (b*x^5*ArcTan[c*x^2])/5 + (b*ArcTan[(-Sqrt[
2] + 2*Sqrt[c]*x)/Sqrt[2]])/(5*Sqrt[2]*c^(5/2)) + (b*ArcTan[(Sqrt[2] + 2*Sq
rt[c]*x)/Sqrt[2]])/(5*Sqrt[2]*c^(5/2)) + (b*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x
^2])/(10*Sqrt[2]*c^(5/2)) - (b*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt
[2]*c^(5/2))
```

Maple [A]
time = 0.17, size = 140, normalized size = 0.87

method	result

default	$\frac{a\,x^{5}}{5}+\frac{x^{5} b \arctan\left(c\,x^{2}\right)}{5}-\frac{2 b\,x^{3}}{15 c}+\frac{b \sqrt{2}\, \ln\left(\frac{x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{1}{c^{2}}}}{x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{1}{c^{2}}}}\right)}{20 c^{3} \left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}+\frac{b \sqrt{2}\, \arctan\left(\frac{\sqrt{2}\, x}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}+1\right)}{10 c^{3} \left(\frac{1}{c^{2}}\right)^{\frac{1}{4}}}+\frac{b \sqrt{2}\, \arctan}{10 c^{3} (}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*x^5+1/5*x^5*b*arctan(c*x^2)-2/15*b*x^3/c+1/20*b/c^3/(1/c^2)^(1/4)*2^(
1/2)*ln((x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/4)*x*2^
(1/2)+(1/c^2)^(1/2)))+1/10*b/c^3/(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^
2)^(1/4)*x+1)+1/10*b/c^3/(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)
*x-1)
```

Maxima [A]
time = 0.48, size = 147, normalized size = 0.91

$$\frac{1}{5}\,ax^{5}+\frac{1}{60}\left(12\,x^{5}\arctan\left(cx^{2}\right)-c\left(\frac{8\,x^{3}}{c^{2}}-\frac{3\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(2\,cx+\sqrt{2}\,\sqrt{c}\right)}{2\,\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(2\,cx-\sqrt{2}\,\sqrt{c}\right)}{2\,\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{\sqrt{2}\,\log\left(cx^{2}+\sqrt{2}\,\sqrt{c}\,x+1\right)}{c^{\frac{3}{2}}}+\frac{\sqrt{2}\,\log\left(cx^{2}-\sqrt{2}\,\sqrt{c}\,x+1\right)}{c^{\frac{3}{2}}}\right)}{c^{2}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/60*(12*x^5*arctan(c*x^2) - c*(8*x^3/c^2 - 3*(2*sqrt(2)*arctan
(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/c^(3/2) + 2*sqrt(2)*arctan(
1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/c^(3/2) - sqrt(2)*log(c*x^2
+ sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log(c*x^2 - sqrt(2)*sqrt(c)*x +
1)/c^(3/2))/c^2))*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(116) = 232.
time = 1.37, size = 372, normalized size = 2.31

$$\frac{12\,bcx^{5}\arctan\left(cx^{2}\right)+12\,acx^{5}-8\,bx^{3}-12\,\sqrt{2}\,c\left(\frac{b^{4}}{c^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\,b^{3}c^{3}x\left(\frac{b^{4}}{c^{10}}\right)^{\frac{1}{4}}-\sqrt{2}\,\sqrt{\sqrt{2}\,b^{3}c^{7}x\left(\frac{b^{4}}{c^{10}}\right)^{\frac{3}{4}}+b^{4}c^{4}\sqrt{\frac{b^{4}}{c^{10}}}+b^{6}x^{2}}\,c^{3}\left(\frac{b^{4}}{c^{10}}\right)^{\frac{1}{4}}+b^{4}}{b^{4}}\right)-12\,\sqrt{2}\,c\left(\frac{b^{4}}{c^{10}}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\,b^{3}c^{3}x\left(\frac{b^{4}}{c^{10}}\right)^{\frac{1}{4}}-\sqrt{2}\,\sqrt{-\sqrt{2}\,b^{3}c^{7}x\left(\frac{b^{4}}{c^{10}}\right)^{\frac{3}{4}}+b^{4}c^{4}\sqrt{\frac{b^{4}}{c^{10}}}+b^{6}x^{2}}\,c^{3}\left(\frac{b^{4}}{c^{10}}\right)^{\frac{1}{4}}-b^{4}}{b^{4}}\right)-3\,\sqrt{2}\,c\left(\frac{b^{4}}{c^{10}}\right)^{\frac{1}{4}}\log\left(\sqrt{2}\,b^{3}c^{7}x\left(\frac{b^{4}}{c^{10}}\right)^{\frac{3}{4}}+b^{4}c^{4}\sqrt{\frac{b^{4}}{c^{10}}}+b^{6}x^{2}\right)+3\,\sqrt{2}\,c\left(\frac{b^{4}}{c^{10}}\right)^{\frac{1}{4}}\log\left(-\sqrt{2}\,b^{3}c^{7}x\left(\frac{b^{4}}{c^{10}}\right)^{\frac{3}{4}}+b^{4}c^{4}\sqrt{\frac{b^{4}}{c^{10}}}+b^{6}x^{2}\right)}{60\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b*c*x^5*arctan(c*x^2) + 12*a*c*x^5 - 8*b*x^3 - 12*sqrt(2)*c*(b^4/c
^10)^(1/4)*arctan(-(sqrt(2)*b^3*c^3*x*(b^4/c^10)^(1/4) - sqrt(2)*sqrt(sqrt(
```

```
2)*b^3*c^7*x*(b^4/c^10)^(3/4) + b^4*c^4*sqrt(b^4/c^10) + b^6*x^2)*c^3*(b^4/
c^10)^(1/4) + b^4)/b^4) - 12*sqrt(2)*c*(b^4/c^10)^(1/4)*arctan(-(sqrt(2)*b^
3*c^3*x*(b^4/c^10)^(1/4) - sqrt(2)*sqrt(-sqrt(2)*b^3*c^7*x*(b^4/c^10)^(3/4)
 + b^4*c^4*sqrt(b^4/c^10) + b^6*x^2)*c^3*(b^4/c^10)^(1/4) - b^4)/b^4) - 3*s
qrt(2)*c*(b^4/c^10)^(1/4)*log(sqrt(2)*b^3*c^7*x*(b^4/c^10)^(3/4) + b^4*c^4*
sqrt(b^4/c^10) + b^6*x^2) + 3*sqrt(2)*c*(b^4/c^10)^(1/4)*log(-sqrt(2)*b^3*c
^7*x*(b^4/c^10)^(3/4) + b^4*c^4*sqrt(b^4/c^10) + b^6*x^2))/c
```

Sympy [A]
time = 14.35, size = 153, normalized size = 0.95

$$\begin{cases} \frac{ax^5}{5} + \frac{bx^5 \operatorname{atan}(cx^2)}{5} - \frac{2bx^3}{15c} + \frac{b \log\left(x - \sqrt[4]{-\frac{1}{c^2}}\right)}{5c^3 \sqrt[4]{-\frac{1}{c^2}}} - \frac{b \log\left(x^2 + \sqrt{-\frac{1}{c^2}}\right)}{10c^3 \sqrt[4]{-\frac{1}{c^2}}} + \frac{b \operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{c^2}}}\right)}{5c^3 \sqrt[4]{-\frac{1}{c^2}}} - \frac{b \operatorname{atan}(cx^2)}{5c^6 \left(-\frac{1}{c^2}\right)^{\frac{7}{4}}} & \text{for}\: c \neq 0 \\ \frac{ax^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*x**5/5 + b*x**5*atan(c*x**2)/5 - 2*b*x**3/(15*c) + b*log(x - (
-1/c**2)**(1/4))/(5*c**3*(-1/c**2)**(1/4)) - b*log(x**2 + sqrt(-1/c**2))/(1
0*c**3*(-1/c**2)**(1/4)) + b*atan(x/(-1/c**2)**(1/4))/(5*c**3*(-1/c**2)**(1
/4)) - b*atan(c*x**2)/(5*c**6*(-1/c**2)**(7/4)), Ne(c, 0)), (a*x**5/5, True
))
```

Giac [A]
time = 0.49, size = 169, normalized size = 1.05

$$\frac{1}{20}\,bc^9 \left(\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x + \frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^{12}} + \frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x - \frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^{12}} - \frac{\sqrt{2}\log\left(x^2 + \frac{\sqrt{2}\,x}{\sqrt{|c|}} + \frac{1}{|c|}\right)}{c^{10}|c|^{\frac{3}{2}}} + \frac{\sqrt{2}\sqrt{|c|}\log\left(x^2 - \frac{\sqrt{2}\,x}{\sqrt{|c|}} + \frac{1}{|c|}\right)}{c^{12}}\right) + \frac{3\,bcx^5 \arctan(cx^2) + 3\,acx^5 - 2\,bx^3}{15\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/20*b*c^9*(2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(a
bs(c)))*sqrt(abs(c)))/c^12 + 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/c^12 - sqrt(2)*log(x^2 + sqrt(2)*x/s
qrt(abs(c)) + 1/abs(c))/(c^10*abs(c)^(3/2)) + sqrt(2)*sqrt(abs(c))*log(x^2
- sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^12) + 1/15*(3*b*c*x^5*arctan(c*x^2)
+ 3*a*c*x^5 - 2*b*x^3)/c
```

Mupad [B]
time = 0.36, size = 64, normalized size = 0.40

$$\frac{a\,x^5}{5} - \frac{2\,b\,x^3}{15\,c} + \frac{b\,x^5\,\mathrm{atan}(c\,x^2)}{5} + \frac{{(-1)}^{1/4}\,b\,\mathrm{atan}\left({(-1)}^{1/4}\,\sqrt{c}\,x\right)}{5\,c^{5/2}} + \frac{{(-1)}^{1/4}\,b\,\mathrm{atan}\left({(-1)}^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{5\,c^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atan(c*x^2)),x)
```

```
[Out] (a*x^5)/5 - (2*b*x^3)/(15*c) + (b*x^5*atan(c*x^2))/5 + ((-1)^(1/4)*b*atan((
-1)^(1/4)*c^(1/2)*x))/(5*c^(5/2)) + ((-1)^(1/4)*b*atan((-1)^(1/4)*c^(1/2)*x
*1i)*1i)/(5*c^(5/2))
```

3.69 $\int x^2(a + b\mathbf{ArcTan}(cx^2))\, dx$

Optimal. Leaf size=159

$$-\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\text{ArcTan}\left(cx^2\right)\right)-\frac{b\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{c}\,x\right)}{3\sqrt{2}\,c^{3/2}}+\frac{b\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{c}\,x\right)}{3\sqrt{2}\,c^{3/2}}-\frac{b\log\left(1-\sqrt{2}\,\sqrt{c}\right.}{6\sqrt{2}\,c^{3/}}$$

```
[Out] -2/3*b*x/c+1/3*x^3*(a+b*arctan(c*x^2))+1/6*b*arctan(-1+x*2^(1/2)*c^(1/2))/c
^(3/2)*2^(1/2)+1/6*b*arctan(1+x*2^(1/2)*c^(1/2))/c^(3/2)*2^(1/2)-1/12*b*ln(
1+c*x^2-x*2^(1/2)*c^(1/2))/c^(3/2)*2^(1/2)+1/12*b*ln(1+c*x^2+x*2^(1/2)*c^(1
/2))/c^(3/2)*2^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$,
Rules used = {4946, 327, 217, 1179, 642, 1176, 631, 210}

$$\frac{1}{3}x^3\left(a+b\text{ArcTan}\left(cx^2\right)\right)-\frac{b\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{c}\,x\right)}{3\sqrt{2}\,c^{3/2}}+\frac{b\text{ArcTan}\left(\sqrt{2}\,\sqrt{c}\,x+1\right)}{3\sqrt{2}\,c^{3/2}}-\frac{b\log\left(cx^2-\sqrt{2}\,\sqrt{c}\,x+1\right)}{6\sqrt{2}\,c^{3/2}}+\frac{b\log\left(cx^2+\sqrt{2}\,\sqrt{c}\,x+1\right)}{6\sqrt{2}\,c^{3/2}}-\frac{2bx}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (-2*b*x)/(3*c) + (x^3*(a + b*ArcTan[c*x^2]))/3 - (b*ArcTan[1 - Sqrt[2]*Sqrt
[c]*x])/(3*Sqrt[2]*c^(3/2)) + (b*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(3*Sqrt[2]*
c^(3/2)) - (b*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2]*c^(3/2)) + (b*
Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2]*c^(3/2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int x^2\left(a+b\tan^{-1}\left(cx^2\right)\right)\,dx=\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{1}{3}(2bc)\int\frac{x^4}{1+c^2x^4}\,dx$$
$$=-\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{(2b)\int\frac{1}{1+c^2x^4}\,dx}{3c}$$
$$=-\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\int\frac{1-cx^2}{1+c^2x^4}\,dx}{3c}+\frac{b\int\frac{1+cx^2}{1+c^2x^4}\,dx}{3c}$$
$$=-\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)+\frac{b\int\frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{6c^2}+\frac{b\int\frac{1}{\frac{1}{c}+\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx}{6c^2}-$$
$$=-\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\log\left(1-\sqrt{2}\sqrt{c}\,x+cx^2\right)}{6\sqrt{2}\,c^{3/2}}+\frac{b\log\left(1+\right.}{6}$$
$$=-\frac{2bx}{3c}+\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)-\frac{b\tan^{-1}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{3\sqrt{2}\,c^{3/2}}+\frac{b\tan^{-1}\left(1+\right.}{3\sqrt{2}\,c}$$

Mathematica [A]
time = 0.03, size = 177, normalized size = 1.11

$$-\frac{2bx}{3c}+\frac{ax^3}{3}+\frac{1}{3}bx^3\text{ArcTan}(cx^2)+\frac{b\text{ArcTan}\left(\frac{-\sqrt{2}+2\sqrt{c}\,x}{\sqrt{2}}\right)}{3\sqrt{2}\,c^{3/2}}+\frac{b\text{ArcTan}\left(\frac{\sqrt{2}+2\sqrt{c}\,x}{\sqrt{2}}\right)}{3\sqrt{2}\,c^{3/2}}-\frac{b\log\left(1-\sqrt{2}\sqrt{c}\,x+cx^2\right)}{6\sqrt{2}\,c^{3/2}}+\frac{b\log\left(1+\sqrt{2}\sqrt{c}\,x+cx^2\right)}{6\sqrt{2}\,c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] (-2*b*x)/(3*c) + (a*x^3)/3 + (b*x^3*ArcTan[c*x^2])/3 + (b*ArcTan[(-Sqrt[2]
+ 2*Sqrt[c]*x)/Sqrt[2]])/(3*Sqrt[2]*c^(3/2)) + (b*ArcTan[(Sqrt[2] + 2*Sqrt[
c]*x)/Sqrt[2]])/(3*Sqrt[2]*c^(3/2)) - (b*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2]
)/(6*Sqrt[2]*c^(3/2)) + (b*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2]*c
^(3/2))
```

Maple [A]
time = 0.08, size = 138, normalized size = 0.87

method	result
default	$\frac{a\,x^3}{3}+\frac{b\,x^3\arctan(c\,x^2)}{3}-\frac{2bx}{3c}+\frac{b\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)}{12c}+\frac{b\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)}{6c}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x^2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*x^3+1/3*b*x^3*arctan(c*x^2)-2/3*b*x/c+1/12*b/c*(1/c^2)^(1/4)*2^(1/2)*
ln((x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2-(1/c^2)^(1/4)*x*2^(1/2)
+(1/c^2)^(1/2)))+1/6*b/c*(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)
*x+1)+1/6*b/c*(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1)
```

Maxima [**A**]
time = 0.49, size = 145, normalized size = 0.91

$$\frac{1}{3}\,ax^3+\frac{1}{12}\left(4\,x^3\arctan\left(cx^2\right)-c\left(\frac{8\,x}{c^2}-\frac{\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(2\,cx+\sqrt{2}\,\sqrt{c}\right)}{2\,\sqrt{c}}\right)}{\sqrt{c}}+\frac{2\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(2\,cx-\sqrt{2}\,\sqrt{c}\right)}{2\,\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{2}\,\log\left(cx^2+\sqrt{2}\,\sqrt{c}\,x+1\right)}{\sqrt{c}}-\frac{\sqrt{2}\,\log\left(cx^2-\sqrt{2}\,\sqrt{c}\,x+1\right)}{\sqrt{c}}}{c^2}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/12*(4*x^3*arctan(c*x^2) - c*(8*x/c^2 - (2*sqrt(2)*arctan(1/2*
sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + 2*sqrt(2)*arctan(1/2*s
qrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + sqrt(2)*log(c*x^2 + sqr
t(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1)/sq
rt(c))/c^2))*b
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(114) = 228$.
time = 2.40, size = 337, normalized size = 2.12

$$\frac{4\,bcx^3\arctan\left(cx^2\right)+4\,acx^3-4\,\sqrt{2}\,c\left(\frac{b^4}{c^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\,bc^5x\left(\frac{b^4}{c^6}\right)^{\frac{3}{4}}-\sqrt{2}\,\sqrt{b^2x^2+\sqrt{2}\,bcx\left(\frac{b^4}{c^6}\right)^{\frac{1}{4}}+c^2\sqrt{\frac{b^4}{c^6}}}\,c^5\left(\frac{b^4}{c^6}\right)^{\frac{3}{4}}+b^4}{b^4}\right)-4\,\sqrt{2}\,c\left(\frac{b^4}{c^6}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\,bc^5x\left(\frac{b^4}{c^6}\right)^{\frac{3}{4}}-\sqrt{2}\,\sqrt{b^2x^2-\sqrt{2}\,bcx\left(\frac{b^4}{c^6}\right)^{\frac{1}{4}}+c^2\sqrt{\frac{b^4}{c^6}}}\,c^5\left(\frac{b^4}{c^6}\right)^{\frac{3}{4}}-b^4}{b^4}\right)+\sqrt{2}\,c\left(\frac{b^4}{c^6}\right)^{\frac{1}{4}}\log\left(b^2x^2+\sqrt{2}\,bcx\left(\frac{b^4}{c^6}\right)^{\frac{1}{4}}+c^2\sqrt{\frac{b^4}{c^6}}\right)-\sqrt{2}\,c\left(\frac{b^4}{c^6}\right)^{\frac{1}{4}}\log\left(b^2x^2-\sqrt{2}\,bcx\left(\frac{b^4}{c^6}\right)^{\frac{1}{4}}+c^2\sqrt{\frac{b^4}{c^6}}\right)-8\,bx}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*b*c*x^3*arctan(c*x^2) + 4*a*c*x^3 - 4*sqrt(2)*c*(b^4/c^6)^(1/4)*arc
tan(-(sqrt(2)*b*c^5*x*(b^4/c^6)^(3/4) - sqrt(2)*sqrt(b^2*x^2 + sqrt(2)*b*c*
x*(b^4/c^6)^(1/4) + c^2*sqrt(b^4/c^6))*c^5*(b^4/c^6)^(3/4) + b^4)/b^4) - 4*
sqrt(2)*c*(b^4/c^6)^(1/4)*arctan(-(sqrt(2)*b*c^5*x*(b^4/c^6)^(3/4) - sqrt(2
)*sqrt(b^2*x^2 - sqrt(2)*b*c*x*(b^4/c^6)^(1/4) + c^2*sqrt(b^4/c^6))*c^5*(b^
4/c^6)^(3/4) - b^4)/b^4) + sqrt(2)*c*(b^4/c^6)^(1/4)*log(b^2*x^2 + sqrt(2)*
b*c*x*(b^4/c^6)^(1/4) + c^2*sqrt(b^4/c^6)) - sqrt(2)*c*(b^4/c^6)^(1/4)*log(
b^2*x^2 - sqrt(2)*b*c*x*(b^4/c^6)^(1/4) + c^2*sqrt(b^4/c^6)) - 8*b*x)/c
```

Sympy [A]
time = 7.25, size = 143, normalized size = 0.90

$$\begin{cases} \frac{ax^3}{3} + \frac{bx^3 \operatorname{atan}(cx^2)}{3} + \frac{b\left(-\frac{1}{c^2}\right)^{\frac{3}{4}} \operatorname{atan}(cx^2)}{3} - \frac{2bx}{3c} - \frac{b\sqrt[4]{-\frac{1}{c^2}} \log\left(x - \sqrt[4]{-\frac{1}{c^2}}\right)}{3c} + \frac{b\sqrt[4]{-\frac{1}{c^2}} \log\left(x^2 + \sqrt{-\frac{1}{c^2}}\right)}{6c} + \frac{b\sqrt[4]{-\frac{1}{c^2}} \operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{c^2}}}\right)}{3c} & \text{for } c \neq 0 \\ \frac{ax^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x**2)),x)
```

```
[Out] Piecewise((a*x**3/3 + b*x**3*atan(c*x**2)/3 + b*(-1/c**2)**(3/4)*atan(c*x**
2)/3 - 2*b*x/(3*c) - b*(-1/c**2)**(1/4)*log(x - (-1/c**2)**(1/4))/(3*c) + b
*(-1/c**2)**(1/4)*log(x**2 + sqrt(-1/c**2))/(6*c) + b*(-1/c**2)**(1/4)*atan
(x/(-1/c**2)**(1/4))/(3*c), Ne(c, 0)), (a*x**3/3, True))
```

Giac [A]
time = 0.46, size = 165, normalized size = 1.04

$$\frac{1}{12}bc^5\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x + \frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^6\sqrt{|c|}} + \frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x - \frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^6\sqrt{|c|}} + \frac{\sqrt{2}\log\left(x^2 + \frac{\sqrt{2}x}{\sqrt{|c|}} + \frac{1}{|c|}\right)}{c^6\sqrt{|c|}} - \frac{\sqrt{2}\log\left(x^2 - \frac{\sqrt{2}x}{\sqrt{|c|}} + \frac{1}{|c|}\right)}{c^6\sqrt{|c|}}\right) + \frac{bcx^3\arctan(cx^2) + acx^3 - 2bx}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] 1/12*b*c^5*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt(
abs(c)))/(c^6*sqrt(abs(c))) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/s
qrt(abs(c)))*sqrt(abs(c)))/(c^6*sqrt(abs(c))) + sqrt(2)*log(x^2 + sqrt(2)*x
/sqrt(abs(c)) + 1/abs(c))/(c^6*sqrt(abs(c))) - sqrt(2)*log(x^2 - sqrt(2)*x/
sqrt(abs(c)) + 1/abs(c))/(c^6*sqrt(abs(c)))) + 1/3*(b*c*x^3*arctan(c*x^2) +
 a*c*x^3 - 2*b*x)/c
```

Mupad [B]
time = 0.39, size = 62, normalized size = 0.39

$$\frac{a\,x^3}{3} + \frac{b\,x^3\,\mathrm{atan}(c\,x^2)}{3} - \frac{2\,b\,x}{3\,c} - \frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\right)\,1\mathrm{i}}{3\,c^{3/2}} - \frac{(-1)^{1/4}\,b\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)}{3\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x^2)),x)
```

```
[Out] (a*x^3)/3 + (b*x^3*atan(c*x^2))/3 - (2*b*x)/(3*c) - ((-1)^(1/4)*b*atan((-1)
^(1/4)*c^(1/2)*x)*1i)/(3*c^(3/2)) - ((-1)^(1/4)*b*atan((-1)^(1/4)*c^(1/2)*x
*1i))/(3*c^(3/2))
```

3.70 $\int (a + b\mathbf{ArcTan}(cx^2))\, dx$

Optimal. Leaf size=140

$$ax+bx\text{ArcTan}(cx^2)+\frac{b\text{ArcTan}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{\sqrt{2}\sqrt{c}}-\frac{b\text{ArcTan}\left(1+\sqrt{2}\sqrt{c}\,x\right)}{\sqrt{2}\sqrt{c}}-\frac{b\log\left(1-\sqrt{2}\sqrt{c}\,x+cx^2\right)}{2\sqrt{2}\sqrt{c}}+ b$$

```
[Out] a*x+b*x*arctan(c*x^2)-1/2*b*arctan(-1+x*2^(1/2)*c^(1/2))*2^(1/2)/c^(1/2)-1/
2*b*arctan(1+x*2^(1/2)*c^(1/2))*2^(1/2)/c^(1/2)-1/4*b*ln(1+c*x^2-x*2^(1/2)*
c^(1/2))*2^(1/2)/c^(1/2)+1/4*b*ln(1+c*x^2+x*2^(1/2)*c^(1/2))*2^(1/2)/c^(1/2
)
```

Rubi [A]
time = 0.08, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {4930, 303, 1176, 631, 210, 1179, 642}

$$ax+bx\text{ArcTan}(cx^2)+\frac{b\text{ArcTan}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{\sqrt{2}\sqrt{c}}-\frac{b\text{ArcTan}\left(\sqrt{2}\sqrt{c}\,x+1\right)}{\sqrt{2}\sqrt{c}}-\frac{b\log\left(cx^2-\sqrt{2}\sqrt{c}\,x+1\right)}{2\sqrt{2}\sqrt{c}}+\frac{b\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{2\sqrt{2}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTan[c*x^2],x]
```

```
[Out] a*x + b*x*ArcTan[c*x^2] + (b*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/(Sqrt[2]*Sqrt[c
]) - (b*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(Sqrt[2]*Sqrt[c]) - (b*Log[1 - Sqrt[
2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]*Sqrt[c]) + (b*Log[1 + Sqrt[2]*Sqrt[c]*x +
 c*x^2])/(2*Sqrt[2]*Sqrt[c])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$
\begin{aligned}
\int \left(a+b\tan^{-1}\left(cx^2\right)\right)\,dx &= ax+b\int \tan^{-1}\left(cx^2\right)\,dx\\
&= ax+bx\tan^{-1}\left(cx^2\right)-(2bc)\int \frac{x^2}{1+c^2x^4}\,dx\\
&= ax+bx\tan^{-1}\left(cx^2\right)+b\int \frac{1-cx^2}{1+c^2x^4}\,dx-b\int \frac{1+cx^2}{1+c^2x^4}\,dx\\
&= ax+bx\tan^{-1}\left(cx^2\right)-\frac{b\int \frac{1}{\frac{1}{c}-\frac{\sqrt{2}\,x}{\sqrt{c}}+x^2}\,dx}{2c}-\frac{b\int \frac{1}{\frac{1}{c}+\frac{\sqrt{2}\,x}{\sqrt{c}}+x^2}\,dx}{2c}-\frac{b\int \frac{\frac{\sqrt{2}}{\sqrt{c}}+2x}{-\frac{1}{c}-\frac{\sqrt{2}\,x}{\sqrt{c}}-\cdots}}{2\sqrt{2}\,\sqrt{c}}\\
&= ax+bx\tan^{-1}\left(cx^2\right)-\frac{b\log\left(1-\sqrt{2}\,\sqrt{c}\,x+cx^2\right)}{2\sqrt{2}\,\sqrt{c}}+\frac{b\log\left(1+\sqrt{2}\,\sqrt{c}\,x+cx^2\right)}{2\sqrt{2}\,\sqrt{c}}\\
&= ax+bx\tan^{-1}\left(cx^2\right)+\frac{b\tan^{-1}\left(1-\sqrt{2}\,\sqrt{c}\,x\right)}{\sqrt{2}\,\sqrt{c}}-\frac{b\tan^{-1}\left(1+\sqrt{2}\,\sqrt{c}\,x\right)}{\sqrt{2}\,\sqrt{c}}-\cdots
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 107, normalized size = 0.76

$$ax+bx\text{ArcTan}\left(cx^2\right)-\frac{b\left(-2\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{c}\,x\right)+2\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{c}\,x\right)+\log\left(1-\sqrt{2}\,\sqrt{c}\,x+cx^2\right)-\log\left(1+\sqrt{2}\,\sqrt{c}\,x+cx^2\right)\right)}{2\sqrt{2}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTan[c*x^2],x]
```

```
[Out] a*x + b*x*ArcTan[c*x^2] - (b*(-2*ArcTan[1 - Sqrt[2]*Sqrt[c]*x] + 2*ArcTan[1
 + Sqrt[2]*Sqrt[c]*x] + Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2] - Log[1 + Sqrt[2
]*Sqrt[c]*x + c*x^2]))/(2*Sqrt[2]*Sqrt[c])
```

Maple [A]
time = 0.08, size = 125, normalized size = 0.89

method	result
default	$ax+bx\arctan\left(c\,x^2\right)-\frac{b\sqrt{2}\,\ln\left(\frac{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)}{4c\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-\frac{b\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)}{2c\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-\frac{b\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\cdots}{\left(\frac{1}{c^2}\cdots\right)}\right)}{2c\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctan(c*x^2),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*x*arctan(c*x^2)-1/4*b/c/(1/c^2)^(1/4)*2^(1/2)*ln((x^2-(1/c^2)^(1/4)*x
*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2)))-1/2*b/
c/(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)-1/2*b/c/(1/c^2)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1)
```

Maxima [**A**]
time = 0.47, size = 127, normalized size = 0.91

$$-\frac{1}{4}\left(c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}x+1\right)}{c^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(cx^2-\sqrt{2}\sqrt{c}x+1\right)}{c^{\frac{3}{2}}}\right)-4x\arctan\left(cx^2\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x^2),x, algorithm="maxima")
```

```
[Out] -1/4*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/c^
(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/c^(
3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log(c*x
^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) - 4*x*arctan(c*x^2))*b + a*x
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(104) = 208$.
time = 1.39, size = 319, normalized size = 2.28

$$bx\arctan\left(cx^2\right)+ax+\sqrt{2}\left(\frac{b^4}{c^2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\frac{b^4}{c^2}\right)^{\frac{1}{4}}b^3cx+b^4-\sqrt{2}\sqrt{b^6x^2+\sqrt{2}\left(\frac{b^4}{c^2}\right)^{\frac{3}{4}}b^3cx+\sqrt{\frac{b^4}{c^2}}b^4}\left(\frac{b^4}{c^2}\right)^{\frac{1}{4}}c}{b^4}\right)+\sqrt{2}\left(\frac{b^4}{c^2}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\frac{b^4}{c^2}\right)^{\frac{1}{4}}b^3cx-b^4-\sqrt{2}\sqrt{b^6x^2-\sqrt{2}\left(\frac{b^4}{c^2}\right)^{\frac{3}{4}}b^3cx+\sqrt{\frac{b^4}{c^2}}b^4}\left(\frac{b^4}{c^2}\right)^{\frac{1}{4}}c}{b^4}\right)+\frac{1}{4}\sqrt{2}\left(\frac{b^4}{c^2}\right)^{\frac{1}{4}}\log\left(b^6x^2+\sqrt{2}\left(\frac{b^4}{c^2}\right)^{\frac{3}{4}}b^3cx+\sqrt{\frac{b^4}{c^2}}b^4\right)-\frac{1}{4}\sqrt{2}\left(\frac{b^4}{c^2}\right)^{\frac{1}{4}}\log\left(b^6x^2-\sqrt{2}\left(\frac{b^4}{c^2}\right)^{\frac{3}{4}}b^3cx+\sqrt{\frac{b^4}{c^2}}b^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x^2),x, algorithm="fricas")
```

```
[Out] b*x*arctan(c*x^2) + a*x + sqrt(2)*(b^4/c^2)^(1/4)*arctan(-(sqrt(2)*(b^4/c^2
)^(1/4)*b^3*c*x + b^4 - sqrt(2)*sqrt(b^6*x^2 + sqrt(2)*(b^4/c^2)^(3/4)*b^3*
c*x + sqrt(b^4/c^2)*b^4)*(b^4/c^2)^(1/4)*c)/b^4) + sqrt(2)*(b^4/c^2)^(1/4)*
arctan(-(sqrt(2)*(b^4/c^2)^(1/4)*b^3*c*x - b^4 - sqrt(2)*sqrt(b^6*x^2 - sqr
t(2)*(b^4/c^2)^(3/4)*b^3*c*x + sqrt(b^4/c^2)*b^4)*(b^4/c^2)^(1/4)*c)/b^4) +
 1/4*sqrt(2)*(b^4/c^2)^(1/4)*log(b^6*x^2 + sqrt(2)*(b^4/c^2)^(3/4)*b^3*c*x
+ sqrt(b^4/c^2)*b^4) - 1/4*sqrt(2)*(b^4/c^2)^(1/4)*log(b^6*x^2 - sqrt(2)*(b
^4/c^2)^(3/4)*b^3*c*x + sqrt(b^4/c^2)*b^4)
```

Sympy [**A**]
time = 4.14, size = 617, normalized size = 4.41

$$ax+b\left(\begin{cases}-\infty ix & \text{for}\ c=-\frac{i}{x^2}\\ \infty ix & \text{for}\ c=\frac{i}{x^2}\\ 0 & \text{for}\ c=0\\ \frac{2c^5x^5\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}\operatorname{atan}\left(cx^2\right)}{2c^5x^4\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}+2c^3\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}}-\frac{2c^4x^4\left(-\frac{1}{c^2}\right)^{\frac{3}{2}}\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)}{2c^5x^4\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}+2c^3\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}}+\frac{c^4x^4\left(-\frac{1}{c^2}\right)^{\frac{3}{2}}\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{2c^5x^4\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}+2c^3\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}}-\frac{2c^4x^4\left(-\frac{1}{c^2}\right)^{\frac{3}{2}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{c^2}}}\right)}{2c^5x^4\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}+2c^3\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}}+\frac{2c^3x\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}\operatorname{atan}\left(cx^2\right)}{2c^5x^4\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}+2c^3\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}}-\frac{2c^2\left(-\frac{1}{c^2}\right)^{\frac{3}{2}}\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)}{2c^5x^4\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}+2c^3\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}}+\frac{c^2\left(-\frac{1}{c^2}\right)^{\frac{3}{2}}\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{2c^5x^4\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}+2c^3\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}}-\frac{2c^2\left(-\frac{1}{c^2}\right)^{\frac{3}{2}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{c^2}}}\right)}{2c^5x^4\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}+2c^3\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}}+\frac{2cx^4\operatorname{atan}\left(cx^2\right)}{2c^5x^4\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}+2c^3\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}}+\frac{2\operatorname{atan}\left(cx^2\right)}{2c^6x^4\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}+2c^4\left(-\frac{1}{c^2}\right)^{\frac{7}{4}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atan(c*x**2),x)
```

```
[Out] a*x + b*Piecewise((-oo*I*x, Eq(c, -I/x**2)), (oo*I*x, Eq(c, I/x**2)), (0, E
q(c, 0)), (2*c**5*x**5*(-1/c**2)**(7/4)*atan(c*x**2)/(2*c**5*x**4*(-1/c**2)
**(7/4) + 2*c**3*(-1/c**2)**(7/4)) - 2*c**4*x**4*(-1/c**2)**(3/2)*log(x - (
-1/c**2)**(1/4))/(2*c**5*x**4*(-1/c**2)**(7/4) + 2*c**3*(-1/c**2)**(7/4)) +
 c**4*x**4*(-1/c**2)**(3/2)*log(x**2 + sqrt(-1/c**2))/(2*c**5*x**4*(-1/c**2
)**(7/4) + 2*c**3*(-1/c**2)**(7/4)) - 2*c**4*x**4*(-1/c**2)**(3/2)*atan(x/(
-1/c**2)**(1/4))/(2*c**5*x**4*(-1/c**2)**(7/4) + 2*c**3*(-1/c**2)**(7/4)) +
 2*c**3*x*(-1/c**2)**(7/4)*atan(c*x**2)/(2*c**5*x**4*(-1/c**2)**(7/4) + 2*c
**3*(-1/c**2)**(7/4)) - 2*c**2*(-1/c**2)**(3/2)*log(x - (-1/c**2)**(1/4))/(
2*c**5*x**4*(-1/c**2)**(7/4) + 2*c**3*(-1/c**2)**(7/4)) + c**2*(-1/c**2)**(
3/2)*log(x**2 + sqrt(-1/c**2))/(2*c**5*x**4*(-1/c**2)**(7/4) + 2*c**3*(-1/c
**2)**(7/4)) - 2*c**2*(-1/c**2)**(3/2)*atan(x/(-1/c**2)**(1/4))/(2*c**5*x**
4*(-1/c**2)**(7/4) + 2*c**3*(-1/c**2)**(7/4)) + 2*c*x**4*atan(c*x**2)/(2*c*
*5*x**4*(-1/c**2)**(7/4) + 2*c**3*(-1/c**2)**(7/4)) + 2*atan(c*x**2)/(2*c**
6*x**4*(-1/c**2)**(7/4) + 2*c**4*(-1/c**2)**(7/4)), True))
```

Giac [A]
time = 0.41, size = 149, normalized size = 1.06

$$-\frac{1}{4}\left(c\left(\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2}+\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2}-\frac{\sqrt{2}\sqrt{|c|}\log\left(x^2+\frac{\sqrt{2}x}{\sqrt{|c|}}+\frac{1}{|c|}\right)}{c^2}+\frac{\sqrt{2}\sqrt{|c|}\log\left(x^2-\frac{\sqrt{2}x}{\sqrt{|c|}}+\frac{1}{|c|}\right)}{c^2}\right)-4x\arctan\left(cx^2\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x^2),x, algorithm="giac")
```

```
[Out] -1/4*(c*(2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(
c)))*sqrt(abs(c)))/c^2 + 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x - s
qrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/c^2 - sqrt(2)*sqrt(abs(c))*log(x^2 + sqr
t(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2 + sqrt(2)*sqrt(abs(c))*log(x^2 - sqrt(2
)*x/sqrt(abs(c)) + 1/abs(c))/c^2) - 4*x*arctan(c*x^2))*b + a*x
```

Mupad [B]
time = 0.39, size = 49, normalized size = 0.35

$$a\,x+b\,x\,\mathrm{atan}\left(c\,x^2\right)-\frac{{\left(-1\right)}^{1/4}\,b\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{c}\,x\right)}{\sqrt{c}}-\frac{{\left(-1\right)}^{1/4}\,b\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{c}\,x\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atan(c*x^2),x)
```

```
[Out] a*x + b*x*atan(c*x^2) - ((-1)^(1/4)*b*atan((-1)^(1/4)*c^(1/2)*x))/c^(1/2) -
 ((-1)^(1/4)*b*atan((-1)^(1/4)*c^(1/2)*x*1i)*1i)/c^(1/2)
```

3.71 $\int \frac{a+b\mathbf{ArcTan}(cx^2)}{x^2}\,dx$

Optimal. Leaf size=143

$$-\frac{a+b\text{ArcTan}(cx^2)}{x}-\frac{b\sqrt{c}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{c}\,x\right)}{\sqrt{2}}+\frac{b\sqrt{c}\,\text{ArcTan}\left(1+\sqrt{2}\,\sqrt{c}\,x\right)}{\sqrt{2}}-\frac{b\sqrt{c}\,\log\left(1-\sqrt{2}\,\sqrt{c}\right.}{2\sqrt{2}}$$

```
[Out] (-a-b*arctan(c*x^2))/x+1/2*b*arctan(-1+x*2^(1/2)*c^(1/2))*c^(1/2)*2^(1/2)+1
/2*b*arctan(1+x*2^(1/2)*c^(1/2))*c^(1/2)*2^(1/2)-1/4*b*ln(1+c*x^2-x*2^(1/2)
*c^(1/2))*c^(1/2)*2^(1/2)+1/4*b*ln(1+c*x^2+x*2^(1/2)*c^(1/2))*c^(1/2)*2^(1/
2)
```

Rubi [A]
time = 0.06, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4946, 217, 1179, 642, 1176, 631, 210}

$$-\frac{a+b\text{ArcTan}(cx^2)}{x}-\frac{b\sqrt{c}\,\text{ArcTan}\left(1-\sqrt{2}\,\sqrt{c}\,x\right)}{\sqrt{2}}+\frac{b\sqrt{c}\,\text{ArcTan}\left(\sqrt{2}\,\sqrt{c}\,x+1\right)}{\sqrt{2}}-\frac{b\sqrt{c}\,\log\left(cx^2-\sqrt{2}\,\sqrt{c}\,x+1\right)}{2\sqrt{2}}+\frac{b\sqrt{c}\,\log\left(cx^2+\sqrt{2}\,\sqrt{c}\,x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x^2,x]
```

```
[Out] -((a + b*ArcTan[c*x^2])/x) - (b*Sqrt[c]*ArcTan[1 - Sqrt[2]*Sqrt[c]*x])/Sqrt
[2] + (b*Sqrt[c]*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/Sqrt[2] - (b*Sqrt[c]*Log[1
- Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]) + (b*Sqrt[c]*Log[1 + Sqrt[2]*Sqrt
[c]*x + c*x^2])/(2*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx^2)}{x^2}\,dx = -\frac{a+b\tan^{-1}(cx^2)}{x} + (2bc)\int \frac{1}{1+c^2x^4}\,dx$$

$$= -\frac{a+b\tan^{-1}(cx^2)}{x} + (bc)\int \frac{1-cx^2}{1+c^2x^4}\,dx + (bc)\int \frac{1+cx^2}{1+c^2x^4}\,dx$$

$$(b\sqrt{}$$

$$= -\frac{a+b\tan^{-1}(cx^2)}{x} + \frac{1}{2}b\int \frac{1}{\frac{1}{c}-\frac{\sqrt{2}\,x}{\sqrt{c}}+x^2}\,dx + \frac{1}{2}b\int \frac{1}{\frac{1}{c}+\frac{\sqrt{2}\,x}{\sqrt{c}}+x^2}\,dx - $$

$$= -\frac{a+b\tan^{-1}(cx^2)}{x} - \frac{b\sqrt{c}\,\log\left(1-\sqrt{2}\sqrt{c}\,x+cx^2\right)}{2\sqrt{2}} + \frac{b\sqrt{c}\,\log\left(1+\sqrt{2}\sqrt{c}\,x\right.}{2\sqrt{2}}$$

$$= -\frac{a+b\tan^{-1}(cx^2)}{x} - \frac{b\sqrt{c}\,\tan^{-1}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{\sqrt{2}} + \frac{b\sqrt{c}\,\tan^{-1}\left(1+\sqrt{2}\sqrt{c}\,x\right)}{\sqrt{2}}$$

Mathematica [A]
time = 0.03, size = 158, normalized size = 1.10

$$-\frac{a}{x} - \frac{b\text{ArcTan}(cx^2)}{x} + \frac{b\sqrt{c}\,\text{ArcTan}\left(\frac{-\sqrt{2}+2\sqrt{c}\,x}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{b\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}+2\sqrt{c}\,x}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{b\sqrt{c}\,\log\left(1-\sqrt{2}\sqrt{c}\,x+cx^2\right)}{2\sqrt{2}} + \frac{b\sqrt{c}\,\log\left(1+\sqrt{2}\sqrt{c}\,x+cx^2\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^2,x]
```

```
[Out] -(a/x) - (b*ArcTan[c*x^2])/x + (b*Sqrt[c]*ArcTan[(-Sqrt[2] + 2*Sqrt[c]*x)/S
qrt[2]])/Sqrt[2] + (b*Sqrt[c]*ArcTan[(Sqrt[2] + 2*Sqrt[c]*x)/Sqrt[2]])/Sqrt
[2] - (b*Sqrt[c]*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2]) + (b*Sqrt[
c]*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(2*Sqrt[2])
```

Maple [A]
time = 0.10, size = 125, normalized size = 0.87

method	result
default	$-\frac{a}{x} - \frac{b\arctan(c\,x^2)}{x} + \frac{bc\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)}{2} + \frac{bc\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)}{4} + \frac{bc\left(\frac{1}{c^2}\right)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/x-b/x*arctan(c*x^2)+1/2*b*c*(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)
^(1/4)*x-1)+1/4*b*c*(1/c^2)^(1/4)*2^(1/2)*ln((x^2+(1/c^2)^(1/4)*x*2^(1/2)+(
1/c^2)^(1/2))/(x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2)))+1/2*b*c*(1/c^2)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)
```

Maxima [A]
time = 0.49, size = 132, normalized size = 0.92

$$\frac{1}{4}\left(c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{\sqrt{c}}+\frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{\sqrt{c}}-\frac{\sqrt{2}\log\left(cx^2-\sqrt{2}\sqrt{c}\,x+1\right)}{\sqrt{c}}\right)-\frac{4\arctan\left(cx^2\right)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sqr
t(c) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sqrt
(c) + sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c*x^
2 - sqrt(2)*sqrt(c)*x + 1)/sqrt(c)) - 4*arctan(c*x^2)/x)*b - a/x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(107) = 214$.
time = 2.09, size = 322, normalized size = 2.25

$$-\frac{4\sqrt{2}\left(b^4c^2\right)^{\frac{1}{4}}x\arctan\left(-\frac{b^4c^2+\sqrt{2}\left(b^4c^2\right)^{\frac{3}{4}}bcx-\sqrt{2}\left(b^4c^2\right)^{\frac{3}{4}}\sqrt{b^2c^2x^2+\sqrt{2}\left(b^4c^2\right)^{\frac{1}{4}}bcx+\sqrt{b^4c^2}}}{b^4c^2}\right)+4\sqrt{2}\left(b^4c^2\right)^{\frac{1}{4}}x\arctan\left(\frac{b^4c^2-\sqrt{2}\left(b^4c^2\right)^{\frac{3}{4}}bcx+\sqrt{2}\left(b^4c^2\right)^{\frac{3}{4}}\sqrt{b^2c^2x^2-\sqrt{2}\left(b^4c^2\right)^{\frac{1}{4}}bcx+\sqrt{b^4c^2}}}{b^4c^2}\right)-\sqrt{2}\left(b^4c^2\right)^{\frac{1}{4}}x\log\left(b^2c^2x^2+\sqrt{2}\left(b^4c^2\right)^{\frac{1}{4}}bcx+\sqrt{b^4c^2}\right)+\sqrt{2}\left(b^4c^2\right)^{\frac{1}{4}}x\log\left(b^2c^2x^2-\sqrt{2}\left(b^4c^2\right)^{\frac{1}{4}}bcx+\sqrt{b^4c^2}\right)+4\,b\arctan\left(cx^2\right)+4\,a}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*sqrt(2)*(b^4*c^2)^(1/4)*x*arctan(-(b^4*c^2 + sqrt(2)*(b^4*c^2)^(3/4
)*b*c*x - sqrt(2)*(b^4*c^2)^(3/4)*sqrt(b^2*c^2*x^2 + sqrt(2)*(b^4*c^2)^(1/4
)*b*c*x + sqrt(b^4*c^2)))/(b^4*c^2)) + 4*sqrt(2)*(b^4*c^2)^(1/4)*x*arctan((
b^4*c^2 - sqrt(2)*(b^4*c^2)^(3/4)*b*c*x + sqrt(2)*(b^4*c^2)^(3/4)*sqrt(b^2*
c^2*x^2 - sqrt(2)*(b^4*c^2)^(1/4)*b*c*x + sqrt(b^4*c^2)))/(b^4*c^2)) - sqrt
(2)*(b^4*c^2)^(1/4)*x*log(b^2*c^2*x^2 + sqrt(2)*(b^4*c^2)^(1/4)*b*c*x + sqr
t(b^4*c^2)) + sqrt(2)*(b^4*c^2)^(1/4)*x*log(b^2*c^2*x^2 - sqrt(2)*(b^4*c^2)
^(1/4)*b*c*x + sqrt(b^4*c^2)) + 4*b*arctan(c*x^2) + 4*a)/x
```

Sympy [A]
time = 8.22, size = 121, normalized size = 0.85

$$\begin{cases} -\frac{a}{x}-bc\sqrt[4]{-\frac{1}{c^2}}\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)+\frac{bc\sqrt[4]{-\frac{1}{c^2}}\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{2}+bc\sqrt[4]{-\frac{1}{c^2}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{c^2}}}\right)-\frac{b\operatorname{atan}\left(cx^2\right)}{\sqrt[4]{-\frac{1}{c^2}}}-\frac{b\operatorname{atan}\left(cx^2\right)}{x} & \text{for}\ c\neq 0\\ -\frac{a}{x} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x**2,x)
```

```
[Out] Piecewise((-a/x - b*c*(-1/c**2)**(1/4)*log(x - (-1/c**2)**(1/4)) + b*c*(-1/
c**2)**(1/4)*log(x**2 + sqrt(-1/c**2))/2 + b*c*(-1/c**2)**(1/4)*atan(x/(-1/
c**2)**(1/4)) - b*atan(c*x**2)/(-1/c**2)**(1/4) - b*atan(c*x**2)/x, Ne(c, 0
)), (-a/x, True))
```

Giac [A]
time = 0.45, size = 138, normalized size = 0.97

$$\frac{1}{4}\,bc\left(\frac{2\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(2\,x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{2\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\left(2\,x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{\sqrt{2}\,\log\left(x^2+\frac{\sqrt{2}\,x}{\sqrt{|c|}}+\frac{1}{|c|}\right)}{\sqrt{|c|}}-\frac{\sqrt{2}\,\log\left(x^2-\frac{\sqrt{2}\,x}{\sqrt{|c|}}+\frac{1}{|c|}\right)}{\sqrt{|c|}}\right)-\frac{b\arctan\left(cx^2\right)+a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt(abs
(c)))/sqrt(abs(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(abs(c
)))*sqrt(abs(c)))/sqrt(abs(c)) + sqrt(2)*log(x^2 + sqrt(2)*x/sqrt(abs(c)) +
 1/abs(c))/sqrt(abs(c)) - sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(c)) + 1/abs(
c))/sqrt(abs(c))) - (b*arctan(c*x^2) + a)/x
```

Mupad [B]
time = 0.20, size = 55, normalized size = 0.38

$$-\frac{a}{x}-\frac{b\,\mathrm{atan}\left(c\,x^2\right)}{x}-{\left(-1\right)}^{1/4}\,b\,\sqrt{c}\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{c}\,x\right)\,1{}\mathrm{i}-{\left(-1\right)}^{1/4}\,b\,\sqrt{c}\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{c}\,x\,1{}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))/x^2,x)
```

```
[Out] - a/x - (b*atan(c*x^2))/x - (-1)^(1/4)*b*c^(1/2)*atan((-1)^(1/4)*c^(1/2)*x)
*1i - (-1)^(1/4)*b*c^(1/2)*atan((-1)^(1/4)*c^(1/2)*x*1i)
```

3.72 $\int \frac{a+b\text{ArcTan}(cx^2)}{x^4} dx$

Optimal. Leaf size=159

$$-\frac{2bc}{3x}-\frac{a+b\text{ArcTan}(cx^2)}{3x^3}+\frac{bc^{3/2}\text{ArcTan}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{3\sqrt{2}}-\frac{bc^{3/2}\text{ArcTan}\left(1+\sqrt{2}\sqrt{c}\,x\right)}{3\sqrt{2}}-\frac{bc^{3/2}\log\left(1-\sqrt{2}\right.}{6\sqrt{2}}$$

```
[Out] -2/3*b*c/x+1/3*(-a-b*arctan(c*x^2))/x^3-1/6*b*c^(3/2)*arctan(-1+x*2^(1/2)*c
^(1/2))*2^(1/2)-1/6*b*c^(3/2)*arctan(1+x*2^(1/2)*c^(1/2))*2^(1/2)-1/12*b*c^
(3/2)*ln(1+c*x^2-x*2^(1/2)*c^(1/2))*2^(1/2)+1/12*b*c^(3/2)*ln(1+c*x^2+x*2^(
1/2)*c^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4946, 331, 303, 1176, 631, 210, 1179, 642}

$$-\frac{a+b\text{ArcTan}(cx^2)}{3x^3}+\frac{bc^{3/2}\text{ArcTan}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{3\sqrt{2}}-\frac{bc^{3/2}\text{ArcTan}\left(\sqrt{2}\sqrt{c}\,x+1\right)}{3\sqrt{2}}-\frac{bc^{3/2}\log\left(cx^2-\sqrt{2}\sqrt{c}\,x+1\right)}{6\sqrt{2}}+\frac{bc^{3/2}\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{6\sqrt{2}}-\frac{2bc}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x^4,x]
```

```
[Out] (-2*b*c)/(3*x) - (a + b*ArcTan[c*x^2])/(3*x^3) + (b*c^(3/2)*ArcTan[1 - Sqrt
[2]*Sqrt[c]*x])/(3*Sqrt[2]) - (b*c^(3/2)*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/(3*
Sqrt[2]) - (b*c^(3/2)*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2]) + (b*
c^(3/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 303

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx^2)}{x^4}\,dx = -\frac{a+b\tan^{-1}(cx^2)}{3x^3} + \frac{1}{3}(2bc)\int \frac{1}{x^2(1+c^2x^4)}\,dx$$
$$= -\frac{2bc}{3x} - \frac{a+b\tan^{-1}(cx^2)}{3x^3} - \frac{1}{3}(2bc^3)\int \frac{x^2}{1+c^2x^4}\,dx$$
$$= -\frac{2bc}{3x} - \frac{a+b\tan^{-1}(cx^2)}{3x^3} + \frac{1}{3}(bc^2)\int \frac{1-cx^2}{1+c^2x^4}\,dx - \frac{1}{3}(bc^2)\int \frac{1+cx^2}{1+c^2x^4}\,dx$$

$$= -\frac{2bc}{3x} - \frac{a+b\tan^{-1}(cx^2)}{3x^3} - \frac{1}{6}(bc)\int \frac{1}{\frac{1}{c} - \frac{\sqrt{2}x}{\sqrt{c}} + x^2}\,dx - \frac{1}{6}(bc)\int \frac{1}{\frac{1}{c} + \frac{\sqrt{2}x}{\sqrt{c}} + x^2}$$
$$= -\frac{2bc}{3x} - \frac{a+b\tan^{-1}(cx^2)}{3x^3} - \frac{bc^{3/2}\log\left(1-\sqrt{2}\sqrt{c}\,x+cx^2\right)}{6\sqrt{2}} + \frac{bc^{3/2}\log\left(1+\sqrt{2}\right.}{6\sqrt{2}}$$
$$= -\frac{2bc}{3x} - \frac{a+b\tan^{-1}(cx^2)}{3x^3} + \frac{bc^{3/2}\tan^{-1}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{3\sqrt{2}} - \frac{bc^{3/2}\tan^{-1}\left(1+\sqrt{2}\right.}{3\sqrt{2}}$$

Mathematica [A]
time = 0.04, size = 177, normalized size = 1.11

$$-\frac{a}{3x^3} - \frac{2bc}{3x} - \frac{b\text{ArcTan}(cx^2)}{3x^3} - \frac{bc^{3/2}\text{ArcTan}\left(\frac{-\sqrt{2}+2\sqrt{c}\,x}{\sqrt{2}}\right)}{3\sqrt{2}} - \frac{bc^{3/2}\text{ArcTan}\left(\frac{\sqrt{2}+2\sqrt{c}\,x}{\sqrt{2}}\right)}{3\sqrt{2}} - \frac{bc^{3/2}\log\left(1-\sqrt{2}\sqrt{c}\,x+cx^2\right)}{6\sqrt{2}} + \frac{bc^{3/2}\log\left(1+\sqrt{2}\sqrt{c}\,x+cx^2\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^4,x]
```

```
[Out] -1/3*a/x^3 - (2*b*c)/(3*x) - (b*ArcTan[c*x^2])/(3*x^3) - (b*c^(3/2)*ArcTan[
(-Sqrt[2] + 2*Sqrt[c]*x)/Sqrt[2]])/(3*Sqrt[2]) - (b*c^(3/2)*ArcTan[(Sqrt[2]
 + 2*Sqrt[c]*x)/Sqrt[2]])/(3*Sqrt[2]) - (b*c^(3/2)*Log[1 - Sqrt[2]*Sqrt[c]*
x + c*x^2])/(6*Sqrt[2]) + (b*c^(3/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(6
*Sqrt[2])
```

Maple [A]
time = 0.11, size = 132, normalized size = 0.83

method	result

default	$-\frac{a}{3x^3}-\frac{b\arctan\left(c\,x^2\right)}{3x^3}-\frac{bc\sqrt{2}\,\ln\left(\frac{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)}{12\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-\frac{bc\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}+1\right)}{6\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-\frac{bc\sqrt{2}\,\arctan\left(\frac{\sqrt{}}{\left(\frac{1}{c^{}}\right.}\right.}{6\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/x^3-1/3*b/x^3*arctan(c*x^2)-1/12*b*c/(1/c^2)^(1/4)*2^(1/2)*ln((x^2-(
1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^
(1/2)))-1/6*b*c/(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)-1/6
*b*c/(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1)-2/3*b*c/x
```

Maxima [A]
time = 0.48, size = 142, normalized size = 0.89

$$-\frac{1}{12}\left(\left(c^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx+\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}+\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(2cx-\sqrt{2}\sqrt{c}\right)}{2\sqrt{c}}\right)}{c^{\frac{3}{2}}}-\frac{\sqrt{2}\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{c^{\frac{3}{2}}}+\frac{\sqrt{2}\log\left(cx^2-\sqrt{2}\sqrt{c}\,x+1\right)}{c^{\frac{3}{2}}}\right)+\frac{8}{x}\right)c+\frac{4\arctan\left(cx^2\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^4,x, algorithm="maxima")
```

```
[Out] -1/12*((c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c)
)/c^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))
/c^(3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log
(c*x^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) + 8/x)*c + 4*arctan(c*x^2)/x^3)*b
- 1/3*a/x^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 389 vs. 2(114) = 228.
time = 1.28, size = 389, normalized size = 2.45

$$\frac{4\sqrt{2}\left(b^4c^6\right)^{\frac{1}{4}}x^3\arctan\left(-\frac{b^4c^6+\sqrt{2}\left(b^4c^6\right)^{\frac{1}{4}}b^3c^5x-\sqrt{2}\sqrt{b^6c^{10}x^2+\sqrt{b^4c^6}\,b^4c^6+\sqrt{2}\left(b^4c^6\right)^{\frac{3}{4}}b^3c^5x}\left(b^4c^6\right)^{\frac{1}{4}}}{b^4c^6}\right)+4\sqrt{2}\left(b^4c^6\right)^{\frac{1}{4}}x^3\arctan\left(\frac{b^4c^6-\sqrt{2}\left(b^4c^6\right)^{\frac{1}{4}}b^3c^5x+\sqrt{2}\sqrt{b^6c^{10}x^2+\sqrt{b^4c^6}\,b^4c^6-\sqrt{2}\left(b^4c^6\right)^{\frac{3}{4}}b^3c^5x}\left(b^4c^6\right)^{\frac{1}{4}}}{b^4c^6}\right)+\sqrt{2}\left(b^4c^6\right)^{\frac{1}{4}}x^3\log\left(b^6c^{10}x^2+\sqrt{b^4c^6}\,b^4c^6+\sqrt{2}\left(b^4c^6\right)^{\frac{3}{4}}b^3c^5x\right)-\sqrt{2}\left(b^4c^6\right)^{\frac{1}{4}}x^3\log\left(b^6c^{10}x^2+\sqrt{b^4c^6}\,b^4c^6-\sqrt{2}\left(b^4c^6\right)^{\frac{3}{4}}b^3c^5x\right)-8\,bcx^2-4\,b\arctan\left(cx^2\right)-4\,a}{12\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(2)*(b^4*c^6)^(1/4)*x^3*arctan(-(b^4*c^6 + sqrt(2)*(b^4*c^6)^(1
/4)*b^3*c^5*x - sqrt(2)*sqrt(b^6*c^10*x^2 + sqrt(b^4*c^6)*b^4*c^6 + sqrt(2)
*(b^4*c^6)^(3/4)*b^3*c^5*x)*(b^4*c^6)^(1/4))/(b^4*c^6)) + 4*sqrt(2)*(b^4*c^
6)^(1/4)*x^3*arctan((b^4*c^6 - sqrt(2)*(b^4*c^6)^(1/4)*b^3*c^5*x + sqrt(2)*
sqrt(b^6*c^10*x^2 + sqrt(b^4*c^6)*b^4*c^6 - sqrt(2)*(b^4*c^6)^(3/4)*b^3*c^5
*x)*(b^4*c^6)^(1/4))/(b^4*c^6)) + sqrt(2)*(b^4*c^6)^(1/4)*x^3*log(b^6*c^10*
```

```
x^2 + sqrt(b^4*c^6)*b^4*c^6 + sqrt(2)*(b^4*c^6)^(3/4)*b^3*c^5*x) - sqrt(2)*
(b^4*c^6)^(1/4)*x^3*log(b^6*c^10*x^2 + sqrt(b^4*c^6)*b^4*c^6 - sqrt(2)*(b^4
*c^6)^(3/4)*b^3*c^5*x) - 8*b*c*x^2 - 4*b*arctan(c*x^2) - 4*a)/x^3
```

Sympy [C] Result contains complex when optimal does not.
time = 16.59, size = 529, normalized size = 3.33

$$\begin{cases} -\frac{a-\infty ib}{3x^3} & \text{for } c = -\frac{i}{x^2} \\ -\frac{a+\infty ib}{3x^3} & \text{for } c = \frac{i}{x^2} \\ -\frac{a}{3x^3} & \text{for } c = 0 \\ -\frac{2ax^4}{6x^7+\frac{6x^3}{c^2}} - \frac{2a}{6c^2x^7+6x^3} + \frac{2bc^3x^7\left(-\frac{1}{c^2}\right)^{\frac{3}{4}}\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)}{6x^7+\frac{6x^3}{c^2}} - \frac{bc^3x^7\left(-\frac{1}{c^2}\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{6x^7+\frac{6x^3}{c^2}} + \frac{2bc^3x^7\left(-\frac{1}{c^2}\right)^{\frac{3}{4}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{c^2}}}\right)}{6x^7+\frac{6x^3}{c^2}} + \frac{2bc^2x^7\sqrt[4]{-\frac{1}{c^2}}\operatorname{atan}\left(cx^2\right)}{6x^7+\frac{6x^3}{c^2}} - \frac{4bcx^6}{6x^7+\frac{6x^3}{c^2}} + \frac{2bcx^3\left(-\frac{1}{c^2}\right)^{\frac{3}{4}}\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)}{6x^7+\frac{6x^3}{c^2}} - \frac{bcx^3\left(-\frac{1}{c^2}\right)^{\frac{3}{4}}\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{6x^7+\frac{6x^3}{c^2}} + \frac{2bcx^3\left(-\frac{1}{c^2}\right)^{\frac{3}{4}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{c^2}}}\right)}{6x^7+\frac{6x^3}{c^2}} - \frac{2bx^4\operatorname{atan}\left(cx^2\right)}{6x^7+\frac{6x^3}{c^2}} + \frac{2bx^3\sqrt[4]{-\frac{1}{c^2}}\operatorname{atan}\left(cx^2\right)}{6x^7+\frac{6x^3}{c^2}} - \frac{4bx^2}{6cx^7+\frac{6x^3}{c}} - \frac{2b\operatorname{atan}\left(cx^2\right)}{6c^2x^7+6x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x**4,x)
```

```
[Out] Piecewise((-(a - oo*I*b)/(3*x**3), Eq(c, -I/x**2)), (-(a + oo*I*b)/(3*x**3)
, Eq(c, I/x**2)), (-a/(3*x**3), Eq(c, 0)), (-2*a*x**4/(6*x**7 + 6*x**3/c**2
) - 2*a/(6*c**2*x**7 + 6*x**3) + 2*b*c**3*x**7*(-1/c**2)**(3/4)*log(x - (-1
/c**2)**(1/4))/(6*x**7 + 6*x**3/c**2) - b*c**3*x**7*(-1/c**2)**(3/4)*log(x*
*2 + sqrt(-1/c**2))/(6*x**7 + 6*x**3/c**2) + 2*b*c**3*x**7*(-1/c**2)**(3/4)
*atan(x/(-1/c**2)**(1/4))/(6*x**7 + 6*x**3/c**2) + 2*b*c**2*x**7*(-1/c**2)*
*(1/4)*atan(c*x**2)/(6*x**7 + 6*x**3/c**2) - 4*b*c*x**6/(6*x**7 + 6*x**3/c*
*2) + 2*b*c*x**3*(-1/c**2)**(3/4)*log(x - (-1/c**2)**(1/4))/(6*x**7 + 6*x**
3/c**2) - b*c*x**3*(-1/c**2)**(3/4)*log(x**2 + sqrt(-1/c**2))/(6*x**7 + 6*x
**3/c**2) + 2*b*c*x**3*(-1/c**2)**(3/4)*atan(x/(-1/c**2)**(1/4))/(6*x**7 +
6*x**3/c**2) - 2*b*x**4*atan(c*x**2)/(6*x**7 + 6*x**3/c**2) + 2*b*x**3*(-1/
c**2)**(1/4)*atan(c*x**2)/(6*x**7 + 6*x**3/c**2) - 4*b*x**2/(6*c*x**7 + 6*x
**3/c) - 2*b*atan(c*x**2)/(6*c**2*x**7 + 6*x**3), True))
```

Giac [A]
time = 0.48, size = 159, normalized size = 1.00

$$-\frac{1}{12}bc^3\left(\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2}+\frac{2\sqrt{2}\sqrt{|c|}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{c^2}-\frac{\sqrt{2}\sqrt{|c|}\log\left(x^2+\frac{\sqrt{2}x}{\sqrt{|c|}}+\frac{1}{|c|}\right)}{c^2}+\frac{\sqrt{2}\sqrt{|c|}\log\left(x^2-\frac{\sqrt{2}x}{\sqrt{|c|}}+\frac{1}{|c|}\right)}{c^2}\right)-\frac{2bcx^2+b\arctan\left(cx^2\right)+a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^4,x, algorithm="giac")
```

```
[Out] -1/12*b*c^3*(2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(
abs(c)))*sqrt(abs(c)))/c^2 + 2*sqrt(2)*sqrt(abs(c))*arctan(1/2*sqrt(2)*(2*x
 - sqrt(2)/sqrt(abs(c)))*sqrt(abs(c)))/c^2 - sqrt(2)*sqrt(abs(c))*log(x^2 +
 sqrt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2 + sqrt(2)*sqrt(abs(c))*log(x^2 - sq
rt(2)*x/sqrt(abs(c)) + 1/abs(c))/c^2) - 1/3*(2*b*c*x^2 + b*arctan(c*x^2) +
a)/x^3
```

Mupad [B]
time = 0.43, size = 63, normalized size = 0.40

$$-\frac{2\,b\,c\,x^2+a}{3\,x^3}-\frac{b\,\mathrm{atan}\left(c\,x^2\right)}{3\,x^3}-\frac{{\left(-1\right)}^{1/4}\,b\,c^{3/2}\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{c}\,x\right)}{3}-\frac{{\left(-1\right)}^{1/4}\,b\,c^{3/2}\,\mathrm{atan}\left({\left(-1\right)}^{1/4}\,\sqrt{c}\,x\,1{}\mathrm{i}\right)\,1{}\mathrm{i}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))/x^4,x)
```

```
[Out] - (a + 2*b*c*x^2)/(3*x^3) - (b*atan(c*x^2))/(3*x^3) - ((-1)^(1/4)*b*c^(3/2)
*atan((-1)^(1/4)*c^(1/2)*x))/3 - ((-1)^(1/4)*b*c^(3/2)*atan((-1)^(1/4)*c^(1
/2)*x*1i)*1i)/3
```

3.73 $\int \frac{a+b\mathbf{ArcTan}(cx^2)}{x^6}\,dx$

Optimal. Leaf size=159

$$-\frac{2bc}{15x^3}-\frac{a+b\text{ArcTan}(cx^2)}{5x^5}+\frac{bc^{5/2}\text{ArcTan}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{5\sqrt{2}}-\frac{bc^{5/2}\text{ArcTan}\left(1+\sqrt{2}\sqrt{c}\,x\right)}{5\sqrt{2}}+\frac{bc^{5/2}\log\left(1-\sqrt{2}\right.}{10\sqrt{}}$$

```
[Out] -2/15*b*c/x^3+1/5*(-a-b*arctan(c*x^2))/x^5-1/10*b*c^(5/2)*arctan(-1+x*2^(1/
2)*c^(1/2))*2^(1/2)-1/10*b*c^(5/2)*arctan(1+x*2^(1/2)*c^(1/2))*2^(1/2)+1/20
*b*c^(5/2)*ln(1+c*x^2-x*2^(1/2)*c^(1/2))*2^(1/2)-1/20*b*c^(5/2)*ln(1+c*x^2+
x*2^(1/2)*c^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$,
Rules used = {4946, 331, 217, 1179, 642, 1176, 631, 210}

$$-\frac{a+b\text{ArcTan}(cx^2)}{5x^5}+\frac{bc^{5/2}\text{ArcTan}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{5\sqrt{2}}-\frac{bc^{5/2}\text{ArcTan}\left(\sqrt{2}\sqrt{c}\,x+1\right)}{5\sqrt{2}}+\frac{bc^{5/2}\log\left(cx^2-\sqrt{2}\sqrt{c}\,x+1\right)}{10\sqrt{2}}-\frac{bc^{5/2}\log\left(cx^2+\sqrt{2}\sqrt{c}\,x+1\right)}{10\sqrt{2}}-\frac{2bc}{15x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])/x^6,x]
```

```
[Out] (-2*b*c)/(15*x^3) - (a + b*ArcTan[c*x^2])/(5*x^5) + (b*c^(5/2)*ArcTan[1 - S
qrt[2]*Sqrt[c]*x])/(5*Sqrt[2]) - (b*c^(5/2)*ArcTan[1 + Sqrt[2]*Sqrt[c]*x])/
(5*Sqrt[2]) + (b*c^(5/2)*Log[1 - Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt[2]) -
 (b*c^(5/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2])/(10*Sqrt[2])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
```

```
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx^2)}{x^6}\,dx = -\frac{a+b\tan^{-1}(cx^2)}{5x^5}+\frac{1}{5}(2bc)\int \frac{1}{x^4(1+c^2x^4)}\,dx$$

$$= -\frac{2bc}{15x^3}-\frac{a+b\tan^{-1}(cx^2)}{5x^5}-\frac{1}{5}(2bc^3)\int \frac{1}{1+c^2x^4}\,dx$$

$$= -\frac{2bc}{15x^3}-\frac{a+b\tan^{-1}(cx^2)}{5x^5}-\frac{1}{5}(bc^3)\int \frac{1-cx^2}{1+c^2x^4}\,dx-\frac{1}{5}(bc^3)\int \frac{1+cx^2}{1+c^2x^4}\,dx$$

$$= -\frac{2bc}{15x^3}-\frac{a+b\tan^{-1}(cx^2)}{5x^5}-\frac{1}{10}(bc^2)\int \frac{1}{\frac{1}{c}-\frac{\sqrt{2}x}{\sqrt{c}}+x^2}\,dx-\frac{1}{10}(bc^2)\int \frac{1}{\frac{1}{c}+\frac{\sqrt{2}}{\sqrt{c}}$$

$$= -\frac{2bc}{15x^3}-\frac{a+b\tan^{-1}(cx^2)}{5x^5}+\frac{bc^{5/2}\log\left(1-\sqrt{2}\sqrt{c}\,x+cx^2\right)}{10\sqrt{2}}-\frac{bc^{5/2}\log\left(1+\sqrt{2}\right.}{10\sqrt{2}}$$

$$= -\frac{2bc}{15x^3}-\frac{a+b\tan^{-1}(cx^2)}{5x^5}+\frac{bc^{5/2}\tan^{-1}\left(1-\sqrt{2}\sqrt{c}\,x\right)}{5\sqrt{2}}-\frac{bc^{5/2}\tan^{-1}\left(1+\sqrt{2}\right.}{5\sqrt{2}}$$

Mathematica [A]
time = 0.04, size = 177, normalized size = 1.11

$$-\frac{a}{5x^5}-\frac{2bc}{15x^3}-\frac{b\text{ArcTan}(cx^2)}{5x^5}-\frac{bc^{5/2}\text{ArcTan}\left(\frac{-\sqrt{2}+2\sqrt{c}\,x}{\sqrt{2}}\right)}{5\sqrt{2}}-\frac{bc^{5/2}\text{ArcTan}\left(\frac{\sqrt{2}+2\sqrt{c}\,x}{\sqrt{2}}\right)}{5\sqrt{2}}+\frac{bc^{5/2}\log\left(1-\sqrt{2}\sqrt{c}\,x+cx^2\right)}{10\sqrt{2}}-\frac{bc^{5/2}\log\left(1+\sqrt{2}\sqrt{c}\,x+cx^2\right)}{10\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])/x^6,x]
```

```
[Out] -1/5*a/x^5 - (2*b*c)/(15*x^3) - (b*ArcTan[c*x^2])/(5*x^5) - (b*c^(5/2)*ArcT
an[(-Sqrt[2] + 2*Sqrt[c]*x)/Sqrt[2]])/(5*Sqrt[2]) - (b*c^(5/2)*ArcTan[(Sqrt
[2] + 2*Sqrt[c]*x)/Sqrt[2]])/(5*Sqrt[2]) + (b*c^(5/2)*Log[1 - Sqrt[2]*Sqrt[
c]*x + c*x^2])/(10*Sqrt[2]) - (b*c^(5/2)*Log[1 + Sqrt[2]*Sqrt[c]*x + c*x^2]
)/(10*Sqrt[2])
```

Maple [A]
time = 0.12, size = 138, normalized size = 0.87

method	result
default	$-\frac{a}{5x^5}-\frac{b\arctan(c\,x^2)}{5x^5}-\frac{b\,c^3\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\,\ln\left(\frac{x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}{x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{1}{c^2}}}\right)}{20}-\frac{b\,c^3\left(\frac{1}{c^2}\right)^{\frac{1}{4}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{1}{c^2}\right)^{\frac{1}{4}}}-1\right)}{10}-\frac{b\,c^3\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a/x^5-1/5*b/x^5*arctan(c*x^2)-1/20*b*c^3*(1/c^2)^(1/4)*2^(1/2)*ln((x^2
+(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2)^(1/2))/(x^2-(1/c^2)^(1/4)*x*2^(1/2)+(1/c^2
)^(1/2)))-1/10*b*c^3*(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x-1
)-1/10*b*c^3*(1/c^2)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/c^2)^(1/4)*x+1)-2/15*b
*c/x^3
```

Maxima [A]
time = 0.48, size = 138, normalized size = 0.87

$$-\frac{1}{60}\left(\left(6\sqrt{2}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\,cx+\sqrt{2}\,\sqrt{c}\right)}{2\,\sqrt{c}}\right)+6\sqrt{2}\,c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(2\,cx-\sqrt{2}\,\sqrt{c}\right)}{2\,\sqrt{c}}\right)+3\sqrt{2}\,c^{\frac{3}{2}}\log\left(cx^{2}+\sqrt{2}\,\sqrt{c}\,x+1\right)-3\sqrt{2}\,c^{\frac{3}{2}}\log\left(cx^{2}-\sqrt{2}\,\sqrt{c}\,x+1\right)+\frac{8}{x^{3}}\right)c+\frac{12\arctan\left(cx^{2}\right)}{x^{5}}\right)b-\frac{a}{5\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^6,x, algorithm="maxima")
```

```
[Out] -1/60*((6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt
(c)) + 6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(
c)) + 3*sqrt(2)*c^(3/2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1) - 3*sqrt(2)*c^(3
/2)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1) + 8/x^3)*c + 12*arctan(c*x^2)/x^5)*b
 - 1/5*a/x^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(114) = 228.
time = 1.95, size = 350, normalized size = 2.20

$$\frac{12\sqrt{2}\left(b^{4}c^{10}\right)^{\frac{1}{4}}x^{5}\arctan\left(-\frac{b^{4}c^{10}+\sqrt{2}\left(b^{4}c^{10}\right)^{\frac{3}{4}}bc^{3}x-\sqrt{2}\left(b^{4}c^{10}\right)^{\frac{3}{4}}\sqrt{b^{2}c^{6}x^{2}+\sqrt{2}\left(b^{4}c^{10}\right)^{\frac{1}{4}}bc^{3}x+\sqrt{b^{4}c^{10}}}}{b^{4}c^{10}}\right)+12\sqrt{2}\left(b^{4}c^{10}\right)^{\frac{1}{4}}x^{5}\arctan\left(\frac{b^{4}c^{10}-\sqrt{2}\left(b^{4}c^{10}\right)^{\frac{3}{4}}bc^{3}x+\sqrt{2}\left(b^{4}c^{10}\right)^{\frac{3}{4}}\sqrt{b^{2}c^{6}x^{2}-\sqrt{2}\left(b^{4}c^{10}\right)^{\frac{1}{4}}bc^{3}x+\sqrt{b^{4}c^{10}}}}{b^{4}c^{10}}\right)-3\sqrt{2}\left(b^{4}c^{10}\right)^{\frac{1}{4}}x^{5}\log\left(b^{2}c^{6}x^{2}+\sqrt{2}\left(b^{4}c^{10}\right)^{\frac{1}{4}}bc^{3}x+\sqrt{b^{4}c^{10}}\right)+3\sqrt{2}\left(b^{4}c^{10}\right)^{\frac{1}{4}}x^{5}\log\left(b^{2}c^{6}x^{2}-\sqrt{2}\left(b^{4}c^{10}\right)^{\frac{1}{4}}bc^{3}x+\sqrt{b^{4}c^{10}}\right)-8\,bcx^{2}-12\,b\arctan\left(cx^{2}\right)-12\,a}{60\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(12*sqrt(2)*(b^4*c^10)^(1/4)*x^5*arctan(-(b^4*c^10 + sqrt(2)*(b^4*c^10
)^(3/4)*b*c^3*x - sqrt(2)*(b^4*c^10)^(3/4)*sqrt(b^2*c^6*x^2 + sqrt(2)*(b^4*
c^10)^(1/4)*b*c^3*x + sqrt(b^4*c^10)))/(b^4*c^10)) + 12*sqrt(2)*(b^4*c^10)^
(1/4)*x^5*arctan((b^4*c^10 - sqrt(2)*(b^4*c^10)^(3/4)*b*c^3*x + sqrt(2)*(b^
4*c^10)^(3/4)*sqrt(b^2*c^6*x^2 - sqrt(2)*(b^4*c^10)^(1/4)*b*c^3*x + sqrt(b^
4*c^10)))/(b^4*c^10)) - 3*sqrt(2)*(b^4*c^10)^(1/4)*x^5*log(b^2*c^6*x^2 + sq
rt(2)*(b^4*c^10)^(1/4)*b*c^3*x + sqrt(b^4*c^10)) + 3*sqrt(2)*(b^4*c^10)^(1/
4)*x^5*log(b^2*c^6*x^2 - sqrt(2)*(b^4*c^10)^(1/4)*b*c^3*x + sqrt(b^4*c^10))
 - 8*b*c*x^2 - 12*b*arctan(c*x^2) - 12*a)/x^5
```

Sympy [A]
time = 27.33, size = 155, normalized size = 0.97

$$\begin{cases} -\frac{a}{5x^5} + \frac{bc^3\sqrt[4]{-\frac{1}{c^2}}\log\left(x-\sqrt[4]{-\frac{1}{c^2}}\right)}{5} - \frac{bc^3\sqrt[4]{-\frac{1}{c^2}}\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{10} - \frac{bc^3\sqrt[4]{-\frac{1}{c^2}}\operatorname{atan}\left(\frac{x}{\sqrt[4]{-\frac{1}{c^2}}}\right)}{5} + \frac{bc^2\operatorname{atan}(cx^2)}{5\sqrt[4]{-\frac{1}{c^2}}} - \frac{2bc}{15x^3} - \frac{b\operatorname{atan}(cx^2)}{5x^5} & \text{for } c \neq 0 \\ -\frac{a}{5x^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))/x**6,x)
```

```
[Out] Piecewise((-a/(5*x**5) + b*c**3*(-1/c**2)**(1/4)*log(x - (-1/c**2)**(1/4))/
5 - b*c**3*(-1/c**2)**(1/4)*log(x**2 + sqrt(-1/c**2))/10 - b*c**3*(-1/c**2)
**(1/4)*atan(x/(-1/c**2)**(1/4))/5 + b*c**2*atan(c*x**2)/(5*(-1/c**2)**(1/4
)) - 2*b*c/(15*x**3) - b*atan(c*x**2)/(5*x**5), Ne(c, 0)), (-a/(5*x**5), Tr
ue))
```

Giac [A]
time = 0.50, size = 150, normalized size = 0.94

$$-\frac{1}{20}bc^3\left(\frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}} + \frac{2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\frac{\sqrt{2}}{\sqrt{|c|}}\right)\sqrt{|c|}\right)}{\sqrt{|c|}} + \frac{\sqrt{2}\log\left(x^2+\frac{\sqrt{2}x}{\sqrt{|c|}}+\frac{1}{|c|}\right)}{\sqrt{|c|}} - \frac{\sqrt{2}\log\left(x^2-\frac{\sqrt{2}x}{\sqrt{|c|}}+\frac{1}{|c|}\right)}{\sqrt{|c|}}\right) - \frac{2bcx^2+3b\arctan(cx^2)+3a}{15x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))/x^6,x, algorithm="giac")
```

```
[Out] -1/20*b*c^3*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)/sqrt(abs(c)))*sqrt
(abs(c)))/sqrt(abs(c)) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)/sqrt(a
bs(c)))*sqrt(abs(c)))/sqrt(abs(c)) + sqrt(2)*log(x^2 + sqrt(2)*x/sqrt(abs(c
)) + 1/abs(c))/sqrt(abs(c)) - sqrt(2)*log(x^2 - sqrt(2)*x/sqrt(abs(c)) + 1/
abs(c))/sqrt(abs(c))) - 1/15*(2*b*c*x^2 + 3*b*arctan(c*x^2) + 3*a)/x^5
```

Mupad [B]
time = 0.45, size = 63, normalized size = 0.40

$$-\frac{\frac{2\,b\,c\,x^2}{3}+a}{5\,x^5} - \frac{b\,\mathrm{atan}(c\,x^2)}{5\,x^5} + \frac{(-1)^{1/4}\,b\,c^{5/2}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\right)\,1\mathrm{i}}{5} + \frac{(-1)^{1/4}\,b\,c^{5/2}\,\mathrm{atan}\left((-1)^{1/4}\,\sqrt{c}\,x\,1\mathrm{i}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))/x^6,x)
```

```
[Out] ((-1)^(1/4)*b*c^(5/2)*atan((-1)^(1/4)*c^(1/2)*x)*1i)/5 - (b*atan(c*x^2))/(5
*x^5) - (a + (2*b*c*x^2)/3)/(5*x^5) + ((-1)^(1/4)*b*c^(5/2)*atan((-1)^(1/4)
*c^(1/2)*x*1i))/5
```

3.74 $\int x^7(a+b\mathbf{ArcTan}(cx^2))^2\,dx$

Optimal. Leaf size=124

$$\frac{abx^2}{4c^3}+\frac{b^2x^4}{24c^2}+\frac{b^2x^2\text{ArcTan}(cx^2)}{4c^3}-\frac{bx^6(a+b\text{ArcTan}(cx^2))}{12c}-\frac{(a+b\text{ArcTan}(cx^2))^2}{8c^4}+\frac{1}{8}x^8(a+b\text{ArcTan}(cx^2))^2$$

```
[Out] 1/4*a*b*x^2/c^3+1/24*b^2*x^4/c^2+1/4*b^2*x^2*arctan(c*x^2)/c^3-1/12*b*x^6*(
a+b*arctan(c*x^2))/c-1/8*(a+b*arctan(c*x^2))^2/c^4+1/8*x^8*(a+b*arctan(c*x^
2))^2-1/6*b^2*ln(c^2*x^4+1)/c^4
```

Rubi [A]
time = 0.18, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4948, 4946, 5036, 272, 45, 4930, 266, 5004}

$$-\frac{(a+b\text{ArcTan}(cx^2))^2}{8c^4}+\frac{1}{8}x^8(a+b\text{ArcTan}(cx^2))^2-\frac{bx^6(a+b\text{ArcTan}(cx^2))}{12c}+\frac{abx^2}{4c^3}+\frac{b^2x^2\text{ArcTan}(cx^2)}{4c^3}+\frac{b^2x^4}{24c^2}-\frac{b^2\log(c^2x^4+1)}{6c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (a*b*x^2)/(4*c^3) + (b^2*x^4)/(24*c^2) + (b^2*x^2*ArcTan[c*x^2])/(4*c^3) -
(b*x^6*(a + b*ArcTan[c*x^2]))/(12*c) - (a + b*ArcTan[c*x^2])^2/(8*c^4) + (x
^8*(a + b*ArcTan[c*x^2])^2)/8 - (b^2*Log[1 + c^2*x^4])/(6*c^4)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
```

```
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$
\begin{aligned}
\int x^7\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\,dx &= \int \left(\frac{1}{4}x^7\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{2}bx^7\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+\right.\right.\\
&= \frac{1}{4}\int x^7\left(2a+ib\log\left(1-icx^2\right)\right)^2\,dx+\frac{1}{2}b\int x^7\left(-2ia+b\log\left(1-icx^2\right)\right)\log\\
&= \frac{1}{8}\mathrm{Subst}\left(\int x^3(2a+ib\log(1-icx))^2\,dx,x,x^2\right)+\frac{1}{4}b\mathrm{Subst}\left(\int x^3(-2ia+b\right.\\
&= \frac{1}{32}x^8\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{1}{16}bx^8\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right.\\
&= \frac{1}{32}x^8\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{1}{16}bx^8\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right.\\
&= \frac{1}{32}x^8\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{192}ib\left(2a+ib\log\left(1-icx^2\right)\right)\left(\frac{48(1-icx^2}{c^4}\right.\\
&= \frac{abx^2}{8c^3}-\frac{bx^4(2ia-b\log(1-icx^2))}{32c^2}+\frac{ibx^6(2ia-b\log(1-icx^2))}{48c}+\frac{1}{64}bx^8(\\
&= \frac{abx^2}{8c^3}-\frac{bx^4(2ia-b\log(1-icx^2))}{32c^2}+\frac{ibx^6(2ia-b\log(1-icx^2))}{48c}+\frac{1}{64}bx^8(\\
&= \frac{abx^2}{8c^3}-\frac{55ib^2x^2}{192c^3}-\frac{5b^2x^4}{384c^2}+\frac{ib^2x^6}{576c}+\frac{b^2x^8}{256}-\frac{3b^2(1-icx^2)^2}{32c^4}+\frac{b^2(1-icx^2)^3}{36c^4}\\
&= \frac{abx^2}{8c^3}-\frac{55ib^2x^2}{192c^3}-\frac{5b^2x^4}{384c^2}+\frac{ib^2x^6}{576c}+\frac{b^2x^8}{256}-\frac{3b^2(1-icx^2)^2}{32c^4}+\frac{b^2(1-icx^2)^3}{36c^4}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 121, normalized size = 0.98

$$
\frac{cx^2(6ab+b^2cx^2-2abc^2x^4+3a^2c^3x^6)-2b(bcx^2(-3+c^2x^4)+a(3-3c^4x^8))\,\mathrm{ArcTan}(cx^2)+3b^2(-1+c^4x^8)\,\mathrm{ArcTan}(cx^2)^2-4b^2\log\left(1+c^2x^4\right)}{24c^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (c*x^2*(6*a*b + b^2*c*x^2 - 2*a*b*c^2*x^4 + 3*a^2*c^3*x^6) - 2*b*(b*c*x^2*(
-3 + c^2*x^4) + a*(3 - 3*c^4*x^8))*ArcTan[c*x^2] + 3*b^2*(-1 + c^4*x^8)*Arc
Tan[c*x^2]^2 - 4*b^2*Log[1 + c^2*x^4])/(24*c^4)
```

Maple [A]
time = 0.22, size = 151, normalized size = 1.22

method	result
default	$\frac{x^8 a^2}{8}+\frac{b^2 x^8 \arctan\left(c\,x^2\right)^2}{8}-\frac{b^2 \arctan\left(c\,x^2\right) x^6}{12c}+\frac{b^2 x^2 \arctan\left(c\,x^2\right)}{4c^3}-\frac{b^2 \arctan\left(c\,x^2\right)^2}{8c^4}+\frac{b^2 x^4}{24c^2}-\frac{b^2 \ln\left(c^2 x^4+1\right)}{6c^4}+\frac{ab\,x^8\,\mathrm{a}}{}$
risch	$-\frac{b^2\left(c^4 x^8-1\right) \ln\left(ic\,x^2+1\right)^2}{32c^4}-\frac{ib\left(6a\,c^4 x^8+3ib\,c^4 x^8 \ln\left(-ic\,x^2+1\right)-2b\,c^3 x^6+6bc\,x^2-3ib\ln\left(-ic\,x^2+1\right)\right)\ln\left(ic\,x^2+1\right)}{48c^4}-\frac{b^2 x^8 \ln\left(-ic\,x\right.}{32}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arctan(c*x^2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^8*a^2+1/8*b^2*x^8*arctan(c*x^2)^2-1/12*b^2*arctan(c*x^2)/c*x^6+1/4*b^
2*x^2*arctan(c*x^2)/c^3-1/8*b^2/c^4*arctan(c*x^2)^2+1/24*b^2*x^4/c^2-1/6*b^
2*ln(c^2*x^4+1)/c^4+1/4*a*b*x^8*arctan(c*x^2)-1/12*a*b/c*x^6+1/4*a*b*x^2/c^
3-1/4*a*b/c^4*arctan(c*x^2)
```

Maxima [A]

time = 0.52, size = 169, normalized size = 1.36

$$\frac{1}{8} b^2 x^8 \arctan\left(cx^2\right)^2+\frac{1}{8} a^2 x^8+\frac{1}{12}\left(3 x^8 \arctan\left(cx^2\right)-c\left(\frac{c^2 x^6-3 x^2}{c^4}+\frac{3 \arctan\left(cx^2\right)}{c^5}\right)\right) ab-\frac{1}{24}\left(2 c\left(\frac{c^2 x^6-3 x^2}{c^4}+\frac{3 \arctan\left(cx^2\right)}{c^5}\right) \arctan\left(cx^2\right)-\frac{c^2 x^4+3 \arctan\left(cx^2\right)^2-3 \log\left(12 c^7 x^4+12 c^5\right)-\log\left(c^2 x^4+1\right)}{c^4}\right) b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^2*x^8*arctan(c*x^2)^2 + 1/8*a^2*x^8 + 1/12*(3*x^8*arctan(c*x^2) - c*(
(c^2*x^6 - 3*x^2)/c^4 + 3*arctan(c*x^2)/c^5))*a*b - 1/24*(2*c*((c^2*x^6 - 3
*x^2)/c^4 + 3*arctan(c*x^2)/c^5)*arctan(c*x^2) - (c^2*x^4 + 3*arctan(c*x^2)
^2 - 3*log(12*c^7*x^4 + 12*c^5) - log(c^2*x^4 + 1))/c^4)*b^2
```

Fricas [A]

time = 1.18, size = 137, normalized size = 1.10

$$\frac{3 a^2 c^4 x^8-2 abc^3 x^6+b^2 c^2 x^4+6 abcx^2+3\left(b^2 c^4 x^8-b^2\right) \arctan\left(cx^2\right)^2+6 ab \arctan\left(\frac{1}{cx^2}\right)-4 b^2 \log\left(c^2 x^4+1\right)+2\left(3 abc^4 x^8-b^2 c^3 x^6+3 b^2 cx^2\right) \arctan\left(cx^2\right)}{24 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*a^2*c^4*x^8 - 2*a*b*c^3*x^6 + b^2*c^2*x^4 + 6*a*b*c*x^2 + 3*(b^2*c^
4*x^8 - b^2)*arctan(c*x^2)^2 + 6*a*b*arctan(1/(c*x^2)) - 4*b^2*log(c^2*x^4
+ 1) + 2*(3*a*b*c^4*x^8 - b^2*c^3*x^6 + 3*b^2*c*x^2)*arctan(c*x^2))/c^4
```

Sympy [A]

time = 41.04, size = 199, normalized size = 1.60

$$\begin{cases} \frac{a^2 x^8}{8}+\frac{abx^8 \operatorname{atan}\left(cx^2\right)}{4}-\frac{abx^6}{12c}+\frac{abx^2}{4c^3}-\frac{ab \operatorname{atan}\left(cx^2\right)}{4c^4}+\frac{b^2 x^8 \operatorname{atan}^2\left(cx^2\right)}{8}-\frac{b^2 x^6 \operatorname{atan}\left(cx^2\right)}{12c}+\frac{b^2 x^4}{24c^2}+\frac{b^2 x^2 \operatorname{atan}\left(cx^2\right)}{4c^3}-\frac{b^2 \sqrt{-\frac{1}{c^2}} \operatorname{atan}\left(cx^2\right)}{3c^3}-\frac{b^2 \log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{3c^4}-\frac{b^2 \operatorname{atan}^2\left(cx^2\right)}{8c^4} & \text{for}\ c \neq 0 \\ \frac{a^2 x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atan(c*x**2))**2,x)
```

```
[Out] Piecewise((a**2*x**8/8 + a*b*x**8*atan(c*x**2)/4 - a*b*x**6/(12*c) + a*b*x*
*2/(4*c**3) - a*b*atan(c*x**2)/(4*c**4) + b**2*x**8*atan(c*x**2)**2/8 - b**
2*x**6*atan(c*x**2)/(12*c) + b**2*x**4/(24*c**2) + b**2*x**2*atan(c*x**2)/(
4*c**3) - b**2*sqrt(-1/c**2)*atan(c*x**2)/(3*c**3) - b**2*log(x**2 + sqrt(-
1/c**2))/(3*c**4) - b**2*atan(c*x**2)**2/(8*c**4), Ne(c, 0)), (a**2*x**8/8,
 True))
```

Giac [A]
time = 0.45, size = 145, normalized size = 1.17

$$\frac{3\,a^2cx^8+2\left(3\,cx^8\arctan\left(cx^2\right)-\frac{3\,\arctan\left(cx^2\right)}{c^3}-\frac{c^9x^6-3\,c^7x^2}{c^9}\right)ab+\left(3\,cx^8\arctan\left(cx^2\right)^2-\frac{2\,c^3x^6\arctan\left(cx^2\right)-c^2x^4-6\,cx^2\arctan\left(cx^2\right)+3\,\arctan\left(cx^2\right)^2+4\,\log\left(c^2x^4+1\right)}{c^3}\right)b^2}{24\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*a^2*c*x^8 + 2*(3*c*x^8*arctan(c*x^2) - 3*arctan(c*x^2)/c^3 - (c^9*x
^6 - 3*c^7*x^2)/c^9)*a*b + (3*c*x^8*arctan(c*x^2)^2 - (2*c^3*x^6*arctan(c*x
^2) - c^2*x^4 - 6*c*x^2*arctan(c*x^2) + 3*arctan(c*x^2)^2 + 4*log(c^2*x^4 +
 1))/c^3)*b^2)/c
```

Mupad [B]
time = 1.02, size = 150, normalized size = 1.21

$$\frac{a^2\,x^8}{8}-\frac{b^2\,\mathrm{atan}\left(c\,x^2\right)^2}{8\,c^4}+\frac{b^2\,x^8\,\mathrm{atan}\left(c\,x^2\right)^2}{8}-\frac{b^2\,\ln\left(c^2\,x^4+1\right)}{6\,c^4}+\frac{b^2\,x^4}{24\,c^2}+\frac{b^2\,x^2\,\mathrm{atan}\left(c\,x^2\right)}{4\,c^3}-\frac{b^2\,x^6\,\mathrm{atan}\left(c\,x^2\right)}{12\,c}+\frac{a\,b\,x^2}{4\,c^3}-\frac{a\,b\,x^6}{12\,c}-\frac{a\,b\,\mathrm{atan}\left(c\,x^2\right)}{4\,c^4}+\frac{a\,b\,x^8\,\mathrm{atan}\left(c\,x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*atan(c*x^2))^2,x)
```

```
[Out] (a^2*x^8)/8 - (b^2*atan(c*x^2)^2)/(8*c^4) + (b^2*x^8*atan(c*x^2)^2)/8 - (b^
2*log(c^2*x^4 + 1))/(6*c^4) + (b^2*x^4)/(24*c^2) + (b^2*x^2*atan(c*x^2))/(4
*c^3) - (b^2*x^6*atan(c*x^2))/(12*c) + (a*b*x^2)/(4*c^3) - (a*b*x^6)/(12*c)
 - (a*b*atan(c*x^2))/(4*c^4) + (a*b*x^8*atan(c*x^2))/4
```

3.75 $\int x^5(a+b\text{ArcTan}(cx^2))^2\,dx$

Optimal. Leaf size=154

$$\frac{b^2x^2}{6c^2}-\frac{b^2\text{ArcTan}(cx^2)}{6c^3}-\frac{bx^4(a+b\text{ArcTan}(cx^2))}{6c}-\frac{i(a+b\text{ArcTan}(cx^2))^2}{6c^3}+\frac{1}{6}x^6\left(a+b\text{ArcTan}\left(cx^2\right)\right)^2-\frac{b(a+bA}{}$$

```
[Out] 1/6*b^2*x^2/c^2-1/6*b^2*arctan(c*x^2)/c^3-1/6*b*x^4*(a+b*arctan(c*x^2))/c-1
/6*I*(a+b*arctan(c*x^2))^2/c^3+1/6*x^6*(a+b*arctan(c*x^2))^2-1/3*b*(a+b*arc
tan(c*x^2))*ln(2/(1+I*c*x^2))/c^3-1/6*I*b^2*polylog(2,1-2/(1+I*c*x^2))/c^3
```

Rubi [A]
time = 0.17, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {4948, 4946, 5036, 327, 209, 5040, 4964, 2449, 2352}

$$-\frac{i(a+b\text{ArcTan}(cx^2))^2}{6c^3}-\frac{b\log\left(\frac{2}{1+icx^2}\right)(a+b\text{ArcTan}(cx^2))}{3c^3}+\frac{1}{6}x^6\left(a+b\text{ArcTan}\left(cx^2\right)\right)^2-\frac{bx^4(a+b\text{ArcTan}(cx^2))}{6c}-\frac{b^2\text{ArcTan}(cx^2)}{6c^3}-\frac{ib^2\text{Li}_2\left(1-\frac{2}{icx^2+1}\right)}{6c^3}+\frac{b^2x^2}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (b^2*x^2)/(6*c^2) - (b^2*ArcTan[c*x^2])/(6*c^3) - (b*x^4*(a + b*ArcTan[c*x^
2]))/(6*c) - ((I/6)*(a + b*ArcTan[c*x^2])^2)/c^3 + (x^6*(a + b*ArcTan[c*x^2
])^2)/6 - (b*(a + b*ArcTan[c*x^2])*Log[2/(1 + I*c*x^2)])/(3*c^3) - ((I/6)*b
^2*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x^5 \left(a+b\tan^{-1}\left(cx^2\right)\right)^2\,dx = \int \left(\frac{1}{4}x^5\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{2}bx^5\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+ic\right.\right.$$

$$=\frac{1}{4}\int x^5\left(2a+ib\log\left(1-icx^2\right)\right)^2\,dx+\frac{1}{2}b\int x^5\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(\right.$$

$$=\frac{1}{8}\text{Subst}\left(\int x^2(2a+ib\log(1-icx))^2\,dx,x,x^2\right)+\frac{1}{4}b\text{Subst}\left(\int x^2(-2ia+b\,\text{l}\right.$$

$$=\frac{1}{24}x^6\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{1}{12}bx^6\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)$$

$$=\frac{1}{24}x^6\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{1}{12}bx^6\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)$$

$$=\frac{1}{24}x^6\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{72}ib\left(2a+ib\log\left(1-icx^2\right)\right)\left(\frac{18i(1-icx^2)}{c^3}\right.$$

$$=-\frac{iabx^2}{6c^2}+\frac{ibx^4(2ia-b\log(1-icx^2))}{24c}+\frac{1}{36}bx^6\left(2ia-b\log\left(1-icx^2\right)\right)+\frac{1}{24}$$

$$=-\frac{iabx^2}{6c^2}+\frac{ibx^4(2ia-b\log(1-icx^2))}{24c}+\frac{1}{36}bx^6\left(2ia-b\log\left(1-icx^2\right)\right)+\frac{1}{24}$$

$$=-\frac{iabx^2}{6c^2}+\frac{13b^2x^2}{72c^2}+\frac{ib^2x^4}{144c}+\frac{b^2x^6}{108}-\frac{ib^2(1-icx^2)^2}{16c^3}+\frac{ib^2(1-icx^2)^3}{108c^3}+\frac{ib^2(1}{}$$

$$=-\frac{iabx^2}{6c^2}+\frac{13b^2x^2}{72c^2}+\frac{ib^2x^4}{144c}+\frac{b^2x^6}{108}-\frac{ib^2(1-icx^2)^2}{16c^3}+\frac{ib^2(1-icx^2)^3}{108c^3}+\frac{ib^2(1}{}$$

Mathematica [A]
time = 0.23, size = 141, normalized size = 0.92

$$\frac{b^2cx^2-abc^2x^4+a^2c^3x^6+b^2(i+c^3x^6)\,\text{ArcTan}(cx^2)^2-b\text{ArcTan}(cx^2)\left(b+bc^2x^4-2ac^3x^6+2b\log\left(1+e^{2i\text{ArcTan}(cx^2)}\right)\right)+ab\log\left(1+c^2x^4\right)+ib^2\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx^2)}\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (b^2*c*x^2 - a*b*c^2*x^4 + a^2*c^3*x^6 + b^2*(I + c^3*x^6)*ArcTan[c*x^2]^2
 - b*ArcTan[c*x^2]*(b + b*c^2*x^4 - 2*a*c^3*x^6 + 2*b*Log[1 + E^((2*I)*ArcTa
n[c*x^2])]) + a*b*Log[1 + c^2*x^4] + I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x^
2])])/(6*c^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(136) = 272.
time = 0.43, size = 367, normalized size = 2.38

method	result
risch	$\frac{x^6a^2}{6}+\frac{ba\ln(c^2x^4+1)}{6c^3}+\frac{ib^2\ln(-icx^2+1)^2}{24c^3}+\frac{b^2\ln(icx^2+1)\ln(-icx^2+1)x^6}{12}-\frac{abx^4}{6c}+\frac{ib^2\ln\left(\frac{1}{2}+\frac{icx^2}{2}\right)\ln(-icx^2+1)}{6c^3}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x^2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^6*a^2+1/6*b*a/c^3*ln(c^2*x^4+1)-1/6*I*b^2/c^3*dilog(1/2-1/2*I*c*x^2)+
1/12*b^2*ln(1+I*c*x^2)*ln(1-I*c*x^2)*x^6-1/6*a*b/c*x^4+1/24*I/c^3*b^2*ln(1-
I*c*x^2)^2+1/6*I*b^2/c^3*ln(1/2+1/2*I*c*x^2)*ln(1-I*c*x^2)-1/6*I*b^2/c^3*ln
(1/2+1/2*I*c*x^2)*ln(1/2-1/2*I*c*x^2)-1/6*I*b*a*x^6*ln(1+I*c*x^2)-1/12*I/c*
b^2*x^4*ln(1-I*c*x^2)-1/6*b^2*arctan(c*x^2)/c^3+1/6*b^2*x^2/c^2-1/12*I*b^2/
c^3*ln(1+I*c*x^2)*ln(1-I*c*x^2)-1/24*I/c^3*b^2*ln(1+I*c*x^2)^2+1/12*I/c*b^2
*x^4*ln(1+I*c*x^2)-1/24*b^2*x^6*ln(1+I*c*x^2)^2-17/108*I*b^2/c^3-1/24*b^2*x
^6*ln(1-I*c*x^2)^2+1/6*I*a*b*x^6*ln(1-I*c*x^2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/6*a^2*x^6 + 1/6*(2*x^6*arctan(c*x^2) - (x^4/c^2 - log(c^2*x^4 + 1)/c^4)*c
)*a*b + 1/96*(4*x^6*arctan(c*x^2)^2 - x^6*log(c^2*x^4 + 1)^2 + 96*integrate
(1/48*(4*c^2*x^9*log(c^2*x^4 + 1) - 8*c*x^7*arctan(c*x^2) + 36*(c^2*x^9 + x
^5)*arctan(c*x^2)^2 + 3*(c^2*x^9 + x^5)*log(c^2*x^4 + 1)^2)/(c^2*x^4 + 1),
x))*b^2
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^5*arctan(c*x^2)^2 + 2*a*b*x^5*arctan(c*x^2) + a^2*x^5, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\left(a+b\operatorname{atan}\left(cx^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x**2))**2,x)
```

```
[Out] Integral(x**5*(a + b*atan(c*x**2))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2*x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,{\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atan(c*x^2))^2,x)
```

```
[Out] int(x^5*(a + b*atan(c*x^2))^2, x)
```

3.76 $\int x^3(a+b\textbf{ArcTan}(cx^2))^2\,dx$

Optimal. Leaf size=90

$$-\frac{abx^2}{2c}-\frac{b^2x^2\text{ArcTan}(cx^2)}{2c}+\frac{(a+b\text{ArcTan}(cx^2))^2}{4c^2}+\frac{1}{4}x^4\left(a+b\text{ArcTan}\left(cx^2\right)\right)^2+\frac{b^2\log\left(1+c^2x^4\right)}{4c^2}$$

```
[Out] -1/2*a*b*x^2/c-1/2*b^2*x^2*arctan(c*x^2)/c+1/4*(a+b*arctan(c*x^2))^2/c^2+1/
4*x^4*(a+b*arctan(c*x^2))^2+1/4*b^2*ln(c^2*x^4+1)/c^2
```

Rubi [A]
time = 0.10, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4948, 4946, 5036, 4930, 266, 5004}

$$\frac{(a+b\text{ArcTan}(cx^2))^2}{4c^2}+\frac{1}{4}x^4\left(a+b\text{ArcTan}\left(cx^2\right)\right)^2-\frac{abx^2}{2c}-\frac{b^2x^2\text{ArcTan}(cx^2)}{2c}+\frac{b^2\log\left(c^2x^4+1\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] -1/2*(a*b*x^2)/c - (b^2*x^2*ArcTan[c*x^2])/(2*c) + (a + b*ArcTan[c*x^2])^2/
(4*c^2) + (x^4*(a + b*ArcTan[c*x^2])^2)/4 + (b^2*Log[1 + c^2*x^4])/(4*c^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
```

```
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\int x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\,dx=\int\left(\frac{1}{4}x^3\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{2}bx^3\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+i\right.\right.$$

$$=\frac{1}{4}\int x^3\left(2a+ib\log\left(1-icx^2\right)\right)^2\,dx+\frac{1}{2}b\int x^3\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(\right.$$

$$=\frac{1}{8}\text{Subst}\left(\int x(2a+ib\log(1-icx))^2\,dx,x,x^2\right)+\frac{1}{4}b\text{Subst}\left(\int x(-2ia+b\log\right.$$

$$=-\frac{1}{8}bx^4\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)+\frac{1}{8}\text{Subst}\left(\int\left(-\frac{i(2a+ib\log(}{c}\right.\right.$$

$$=-\frac{1}{8}bx^4\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\frac{i\text{Subst}\left(\int(2a+ib\log(1-icx\right.}{8c}$$

$$=-\frac{1}{8}bx^4\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)-\frac{1}{8}b\text{Subst}\left(\int x(-2ia+b\log(\right.$$

$$=-\frac{abx^2}{4c}+\frac{1}{16}bx^4\left(2ia-b\log\left(1-icx^2\right)\right)+\frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8c^2}$$

$$=-\frac{3abx^2}{4c}-\frac{3ib^2x^2}{8c}+\frac{b^2\left(1-icx^2\right)^2}{32c^2}+\frac{b^2\left(1+icx^2\right)^2}{32c^2}+\frac{1}{16}bx^4\left(2ia-b\log\left(1-\right.\right.$$

$$=-\frac{3abx^2}{4c}+\frac{b^2x^4}{16}+\frac{b^2\left(1-icx^2\right)^2}{32c^2}+\frac{b^2\left(1+icx^2\right)^2}{32c^2}-\frac{b^2\log\left(i-cx^2\right)}{16c^2}+\frac{3b^2(1-}{}$$

Mathematica [A]

time = 0.04, size = 85, normalized size = 0.94

$$\frac{acx^2(-2b+acx^2)+2b(a-bcx^2+ac^2x^4)\,\mathrm{ArcTan}(cx^2)+b^2(1+c^2x^4)\,\mathrm{ArcTan}(cx^2)^2+b^2\log\left(1+c^2x^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (a*c*x^2*(-2*b + a*c*x^2) + 2*b*(a - b*c*x^2 + a*c^2*x^4)*ArcTan[c*x^2] + b
^2*(1 + c^2*x^4)*ArcTan[c*x^2]^2 + b^2*Log[1 + c^2*x^4])/(4*c^2)
```

Maple [A]
time = 0.13, size = 113, normalized size = 1.26

method	result
default	$\frac{x^4a^2}{4}+\frac{b^2x^4\arctan(cx^2)^2}{4}-\frac{b^2x^2\arctan(cx^2)}{2c}+\frac{b^2\arctan(cx^2)^2}{4c^2}+\frac{b^2\ln(c^2x^4+1)}{4c^2}+\frac{ab\,x^4\arctan(cx^2)}{2}-\frac{ab\,x^2}{2c}+\frac{ab\,\mathrm{a}}{}$
risch	$-\frac{b^2(c^2x^4+1)\ln(icx^2+1)^2}{16c^2}-\frac{ib(4a^2c^2x^4+2iab\,c^2x^4\ln(-icx^2+1)-4abc\,x^2+b^2+2iab\ln(-icx^2+1))\ln(icx^2+1)}{16a\,c^2}+\frac{iab\,x^4\ln(-}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x^2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*a^2+1/4*b^2*x^4*arctan(c*x^2)^2-1/2*b^2*x^2*arctan(c*x^2)/c+1/4*b^2
/c^2*arctan(c*x^2)^2+1/4*b^2*ln(c^2*x^4+1)/c^2+1/2*a*b*x^4*arctan(c*x^2)-1/
2*a*b*x^2/c+1/2*a*b/c^2*arctan(c*x^2)
```

Maxima [A]
time = 0.52, size = 126, normalized size = 1.40

$$\frac{1}{4}\,b^2x^4\arctan\left(cx^2\right)^2+\frac{1}{4}\,a^2x^4+\frac{1}{2}\left(x^4\arctan\left(cx^2\right)-c\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)\right)ab-\frac{1}{4}\left(2\,c\left(\frac{x^2}{c^2}-\frac{\arctan\left(cx^2\right)}{c^3}\right)\arctan\left(cx^2\right)+\frac{\arctan\left(cx^2\right)^2-\log\left(4\,c^5x^4+4\,c^3\right)}{c^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctan(c*x^2)^2 + 1/4*a^2*x^4 + 1/2*(x^4*arctan(c*x^2) - c*(x^2
/c^2 - arctan(c*x^2)/c^3))*a*b - 1/4*(2*c*(x^2/c^2 - arctan(c*x^2)/c^3)*arc
tan(c*x^2) + (arctan(c*x^2)^2 - log(4*c^5*x^4 + 4*c^3))/c^2)*b^2
```

Fricas [A]
time = 1.80, size = 100, normalized size = 1.11

$$\frac{a^2c^2x^4-2\,abcx^2+\left(b^2c^2x^4+b^2\right)\arctan\left(cx^2\right)^2-2\,ab\arctan\left(\frac{1}{cx^2}\right)+b^2\log\left(c^2x^4+1\right)+2\left(abc^2x^4-b^2cx^2\right)\arctan\left(cx^2\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(a^2*c^2*x^4 - 2*a*b*c*x^2 + (b^2*c^2*x^4 + b^2)*arctan(c*x^2)^2 - 2*a*
b*arctan(1/(c*x^2)) + b^2*log(c^2*x^4 + 1) + 2*(a*b*c^2*x^4 - b^2*c*x^2)*ar
ctan(c*x^2))/c^2
```

Sympy [A]
time = 14.78, size = 155, normalized size = 1.72

$$\begin{cases} \frac{a^2x^4}{4} + \frac{abx^4 \operatorname{atan}(cx^2)}{2} - \frac{abx^2}{2c} + \frac{ab \operatorname{atan}(cx^2)}{2c^2} + \frac{b^2x^4 \operatorname{atan}^2(cx^2)}{4} - \frac{b^2x^2 \operatorname{atan}(cx^2)}{2c} + \frac{b^2\sqrt{-\frac{1}{c^2}} \operatorname{atan}(cx^2)}{2c} + \frac{b^2 \log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{2c^2} + \frac{b^2 \operatorname{atan}^2(cx^2)}{4c^2} & \text{for } c \neq 0 \\ \frac{a^2x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x**2))**2,x)
```

```
[Out] Piecewise((a**2*x**4/4 + a*b*x**4*atan(c*x**2)/2 - a*b*x**2/(2*c) + a*b*ata
n(c*x**2)/(2*c**2) + b**2*x**4*atan(c*x**2)**2/4 - b**2*x**2*atan(c*x**2)/(
2*c) + b**2*sqrt(-1/c**2)*atan(c*x**2)/(2*c) + b**2*log(x**2 + sqrt(-1/c**2
))/(2*c**2) + b**2*atan(c*x**2)**2/(4*c**2), Ne(c, 0)), (a**2*x**4/4, True)
)
```

Giac [A]
time = 0.43, size = 100, normalized size = 1.11

$$\frac{a^2cx^4 + \frac{2\left(c^2x^4 \arctan(cx^2) - cx^2 + \arctan(cx^2)\right)ab}{c} + \frac{\left(c^2x^4 \arctan(cx^2)^2 - 2\,cx^2 \arctan(cx^2) + \arctan(cx^2)^2 + \log(c^2x^4+1)\right)b^2}{c}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] 1/4*(a^2*c*x^4 + 2*(c^2*x^4*arctan(c*x^2) - c*x^2 + arctan(c*x^2))*a*b/c +
(c^2*x^4*arctan(c*x^2)^2 - 2*c*x^2*arctan(c*x^2) + arctan(c*x^2)^2 + log(c^
2*x^4 + 1))*b^2/c)/c
```

Mupad [B]
time = 0.68, size = 112, normalized size = 1.24

$$\frac{a^2\,x^4}{4} + \frac{b^2\,\mathrm{atan}(c\,x^2)^2}{4\,c^2} + \frac{b^2\,x^4\,\mathrm{atan}(c\,x^2)^2}{4} + \frac{b^2\,\ln\left(c^2\,x^4+1\right)}{4\,c^2} - \frac{b^2\,x^2\,\mathrm{atan}(c\,x^2)}{2\,c} - \frac{a\,b\,x^2}{2\,c} + \frac{a\,b\,\mathrm{atan}(c\,x^2)}{2\,c^2} + \frac{a\,b\,x^4\,\mathrm{atan}(c\,x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x^2))^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*atan(c*x^2)^2)/(4*c^2) + (b^2*x^4*atan(c*x^2)^2)/4 + (b^
2*log(c^2*x^4 + 1))/(4*c^2) - (b^2*x^2*atan(c*x^2))/(2*c) - (a*b*x^2)/(2*c)
 + (a*b*atan(c*x^2))/(2*c^2) + (a*b*x^4*atan(c*x^2))/2
```

3.77 $\int x(a+b\textbf{ArcTan}(cx^2))^2\,dx$

Optimal. Leaf size=101

$$\frac{i(a+b\text{ArcTan}(cx^2))^2}{2c}+\frac{1}{2}x^2\big(a+b\text{ArcTan}\big(cx^2\big)\big)^2+\frac{b(a+b\text{ArcTan}(cx^2))\log\left(\frac{2}{1+icx^2}\right)}{c}+\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+ \right.}{2c}$$

```
[Out] 1/2*I*(a+b*arctan(c*x^2))^2/c+1/2*x^2*(a+b*arctan(c*x^2))^2+b*(a+b*arctan(c
*x^2))*ln(2/(1+I*c*x^2))/c+1/2*I*b^2*polylog(2,1-2/(1+I*c*x^2))/c
```

Rubi [A]
time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4948, 4930, 5040, 4964, 2449, 2352}

$$\frac{1}{2}x^2\big(a+b\text{ArcTan}\big(cx^2\big)\big)^2+\frac{i(a+b\text{ArcTan}(cx^2))^2}{2c}+\frac{b\log\left(\frac{2}{1+icx^2}\right)(a+b\text{ArcTan}(cx^2))}{c}+\frac{ib^2\text{Li}_2\left(1-\frac{2}{icx^2+1}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] ((I/2)*(a + b*ArcTan[c*x^2])^2)/c + (x^2*(a + b*ArcTan[c*x^2])^2)/2 + (b*(a
 + b*ArcTan[c*x^2])*Log[2/(1 + I*c*x^2)])/c + ((I/2)*b^2*PolyLog[2, 1 - 2/(
1 + I*c*x^2)])/c
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
```

```
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\,dx &= \int\left(\frac{1}{4}x\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{2}bx\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right.\right.\\
&= \frac{1}{4}\int x\left(2a+ib\log\left(1-icx^2\right)\right)^2\,dx+\frac{1}{2}b\int x\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+\right.\\
&= \frac{1}{8}\text{Subst}\left(\int(2a+ib\log(1-icx))^2\,dx,x,x^2\right)+\frac{1}{4}b\text{Subst}\left(\int(-2ia+b\log(1-\right.\right.\\
&= -\frac{1}{4}bx^2\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)+\frac{i\text{Subst}\left(\int(2a+ib\log(x))^2\,dx,\right.}{8c}\\
&= \frac{i(1-icx^2)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8c}-\frac{1}{4}bx^2\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+ic\right.\\
&= -\frac{1}{2}iabx^2-\frac{b^2x^2}{4}+\frac{i(1-icx^2)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8c}-\frac{ib^2(1+icx^2)\log(1\;}{4c}\\
&= -\frac{1}{2}b^2x^2+\frac{ib^2(1-icx^2)\log\left(1-icx^2\right)}{4c}+\frac{i(1-icx^2)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8c}-\\
&= -\frac{1}{4}b^2x^2+\frac{ib^2(1-icx^2)\log\left(1-icx^2\right)}{4c}+\frac{i(1-icx^2)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8c}-\\
&= \frac{i(1-icx^2)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{8c}+\frac{ib(2ia-b\log\left(1-icx^2\right))\log\left(\frac{1}{2}(1+icx\right.}{4c}
\end{aligned}
$$

Mathematica [A]

time = 0.06, size = 107, normalized size = 1.06

$$\frac{b^2(-i+cx^2)\operatorname{ArcTan}(cx^2)^2+2b\operatorname{ArcTan}(cx^2)\left(acx^2+b\log\left(1+e^{2i\operatorname{ArcTan}(cx^2)}\right)\right)+a(acx^2-b\log(1+c^2x^4))-ib^2\operatorname{PolyLog}\left(2,-e^{2i\operatorname{ArcTan}(cx^2)}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (b^2*(-I + c*x^2)*ArcTan[c*x^2]^2 + 2*b*ArcTan[c*x^2]*(a*c*x^2 + b*Log[1 +
E^((2*I)*ArcTan[c*x^2])]) + a*(a*c*x^2 - b*Log[1 + c^2*x^4]) - I*b^2*PolyLo
g[2, -E^((2*I)*ArcTan[c*x^2])])/(2*c)
```

Maple [A]
time = 0.18, size = 142, normalized size = 1.41

method	result
derivativedivides	$\frac{c x^2 a^2-i\arctan(c x^2)^2 b^2+\arctan(c x^2)^2 b^2 c x^2+2\arctan(c x^2)\ln\left(1+\frac{(i c x^2+1)^2}{c^2 x^4+1}\right) b^2-i\,\mathrm{polylog}\left(2,-\frac{(i c x^2+1)^2}{c^2 x^4+1}\right) b^2+2}{2c}$
default	$\frac{c x^2 a^2-i\arctan(c x^2)^2 b^2+\arctan(c x^2)^2 b^2 c x^2+2\arctan(c x^2)\ln\left(1+\frac{(i c x^2+1)^2}{c^2 x^4+1}\right) b^2-i\,\mathrm{polylog}\left(2,-\frac{(i c x^2+1)^2}{c^2 x^4+1}\right) b^2+2}{2c}$
risch	$-\frac{i\ln(-i c x^2+1)^2 b^2}{8c}-\frac{ab\ln(c^2x^4+1)}{2c}+\frac{i\ln(-i c x^2+1)x^2 ab}{2}+\frac{b^2\ln(i c x^2+1)\ln(-i c x^2+1)x^2}{4}+\frac{ib^2\,\mathrm{dilog}\left(\frac{1}{2}-\right.}{2c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x^2))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c*(c*x^2*a^2-I*arctan(c*x^2)^2*b^2+arctan(c*x^2)^2*b^2*c*x^2+2*arctan(c
*x^2)*ln(1+(1+I*c*x^2)^2/(c^2*x^4+1))*b^2-I*polylog(2,-(1+I*c*x^2)^2/(c^2*x
^4+1))*b^2+2*a*b*c*x^2*arctan(c*x^2)-a*b*ln(c^2*x^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + 1/32*(4*x^2*arctan(c*x^2)^2 - x^2*log(c^2*x^4 + 1)^2 + 384*c^
2*integrate(1/16*x^5*arctan(c*x^2)^2/(c^2*x^4 + 1), x) + 32*c^2*integrate(1
/16*x^5*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 128*c^2*integrate(1/16*x^5*l
og(c^2*x^4 + 1)/(c^2*x^4 + 1), x) + 4*arctan(c*x^2)^3/c - 256*c*integrate(1
/16*x^3*arctan(c*x^2)/(c^2*x^4 + 1), x) + 32*integrate(1/16*x*log(c^2*x^4 +
 1)^2/(c^2*x^4 + 1), x))*b^2 + 1/2*(2*c*x^2*arctan(c*x^2) - log(c^2*x^4 + 1
))*a*b/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
[Out] integral(b^2*x*arctan(c*x^2)^2 + 2*a*b*x*arctan(c*x^2) + a^2*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\operatorname{atan}\left(cx^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x**2))**2,x)
[Out] Integral(x*(a + b*atan(c*x**2))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^2*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x^2))^2,x)
[Out] int(x*(a + b*atan(c*x^2))^2, x)
```

3.78 $\int \frac{\left(a+b\mathbf{ArcTan}\left(cx^2\right)\right)^2}{x}\,dx$

Optimal. Leaf size=151

$$(a+b\text{ArcTan}(cx^2))^2\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)-\frac{1}{2}ib(a+b\text{ArcTan}(cx^2))\,\text{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)+\frac{1}{2}ib(a$$

```
[Out] -(a+b*arctan(c*x^2))^2*arctanh(-1+2/(1+I*c*x^2))-1/2*I*b*(a+b*arctan(c*x^2)
)*polylog(2,1-2/(1+I*c*x^2))+1/2*I*b*(a+b*arctan(c*x^2))*polylog(2,-1+2/(1+
I*c*x^2))-1/4*b^2*polylog(3,1-2/(1+I*c*x^2))+1/4*b^2*polylog(3,-1+2/(1+I*c*
x^2))
```

Rubi [A]
time = 0.21, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4944, 4942, 5108, 5004, 5114, 6745}

$$-\frac{1}{2}ib\text{Li}_2\left(1-\frac{2}{icx^2+1}\right)(a+b\text{ArcTan}(cx^2))+\frac{1}{2}ib\text{Li}_2\left(\frac{2}{icx^2+1}-1\right)(a+b\text{ArcTan}(cx^2))+\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)(a+b\text{ArcTan}(cx^2))^2-\frac{1}{4}b^2\text{Li}_3\left(1-\frac{2}{icx^2+1}\right)+\frac{1}{4}b^2\text{Li}_3\left(\frac{2}{icx^2+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x,x]
```

```
[Out] (a + b*ArcTan[c*x^2])^2*ArcTanh[1 - 2/(1 + I*c*x^2)] - (I/2)*b*(a + b*ArcTa
n[c*x^2])*PolyLog[2, 1 - 2/(1 + I*c*x^2)] + (I/2)*b*(a + b*ArcTan[c*x^2])*P
olyLog[2, -1 + 2/(1 + I*c*x^2)] - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x^2)])/4 +
 (b^2*PolyLog[3, -1 + 2/(1 + I*c*x^2)])/4
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx, x, x^2\right)$$

$$= \left(a+b\tan^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)-(2bc)\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)}{1+}\right.$$

$$= \left(a+b\tan^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)+(bc)\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)}{1+c^2x^2}\right.$$

$$= \left(a+b\tan^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)-\frac{1}{2}ib\left(a+b\tan^{-1}\left(cx^2\right)\right)\text{Li}_2\left(1-\right.$$

$$= \left(a+b\tan^{-1}\left(cx^2\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)-\frac{1}{2}ib\left(a+b\tan^{-1}\left(cx^2\right)\right)\text{Li}_2\left(1-\right.$$

Mathematica [A]
time = 0.07, size = 165, normalized size = 1.09

$$\left(a+b\text{ArcTan}\left(cx^2\right)\right)^2\tanh^{-1}\left(1+\frac{2i}{-i+cx^2}\right)+\frac{1}{4}b\left(2i\left(a+b\text{ArcTan}\left(cx^2\right)\right)\text{PolyLog}\left(2,\frac{i+cx^2}{i-cx^2}\right)-2i\left(a+b\text{ArcTan}\left(cx^2\right)\right)\text{PolyLog}\left(2,\frac{i+cx^2}{-i+cx^2}\right)+b\left(\text{PolyLog}\left(3,\frac{i+cx^2}{i-cx^2}\right)-\text{PolyLog}\left(3,\frac{i+cx^2}{-i+cx^2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x,x]
```

```
[Out] (a + b*ArcTan[c*x^2])^2*ArcTanh[1 + (2*I)/(-I + c*x^2)] + (b*((2*I)*(a + b*
ArcTan[c*x^2])*PolyLog[2, (I + c*x^2)/(I - c*x^2)] - (2*I)*(a + b*ArcTan[c*
x^2])*PolyLog[2, (I + c*x^2)/(-I + c*x^2)] + b*(PolyLog[3, (I + c*x^2)/(I -
 c*x^2)] - PolyLog[3, (I + c*x^2)/(-I + c*x^2)])))/4
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b \arctan \left(c\,x^{2}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2/x,x)
```

```
[Out] int((a+b*arctan(c*x^2))^2/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + 1/16*integrate((12*b^2*arctan(c*x^2)^2 + b^2*log(c^2*x^4 + 1)^
2 + 32*a*b*arctan(c*x^2))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b \operatorname{atan}\left(cx^{2}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2/x,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))^2/x,x)
[Out] int((a + b*atan(c*x^2))^2/x, x)
```

3.79 $\int \frac{\left(a+b\mathbf{ArcTan}\left(cx^2\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=97

$$-\frac{1}{2}ic\left(a+b\text{ArcTan}\left(cx^2\right)\right)^2-\frac{\left(a+b\text{ArcTan}\left(cx^2\right)\right)^2}{2x^2}+bc\left(a+b\text{ArcTan}\left(cx^2\right)\right)\log\left(2-\frac{2}{1-icx^2}\right)-\frac{1}{2}ib^2c\text{PolyI}$$

```
[Out] -1/2*I*c*(a+b*arctan(c*x^2))^2-1/2*(a+b*arctan(c*x^2))^2/x^2+b*c*(a+b*arcta
n(c*x^2))*ln(2-2/(1-I*c*x^2))-1/2*I*b^2*c*polylog(2,-1+2/(1-I*c*x^2))
```

Rubi [A]
time = 0.13, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {4948, 4946, 5044, 4988, 2497}

$$-\frac{1}{2}ic\left(a+b\text{ArcTan}\left(cx^2\right)\right)^2-\frac{\left(a+b\text{ArcTan}\left(cx^2\right)\right)^2}{2x^2}+bc\log\left(2-\frac{2}{1-icx^2}\right)\left(a+b\text{ArcTan}\left(cx^2\right)\right)-\frac{1}{2}ib^2c\text{Li}_2\left(\frac{2}{1-icx^2}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x^3,x]
```

```
[Out] (-1/2*I)*c*(a + b*ArcTan[c*x^2])^2 - (a + b*ArcTan[c*x^2])^2/(2*x^2) + b*c*
(a + b*ArcTan[c*x^2])*Log[2 - 2/(1 - I*c*x^2)] - (I/2)*b^2*c*PolyLog[2, -1
+ 2/(1 - I*c*x^2)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
```

```
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx^2))^2}{x^3}\,dx = \int \left(\frac{(2a+ib\log(1-icx^2))^2}{4x^3} + \frac{b(-2ia+b\log(1-icx^2))\log(1+icx^2)}{2x^3} - \frac{b^2\log\right.$$

$$= \frac{1}{4}\int \frac{(2a+ib\log(1-icx^2))^2}{x^3}\,dx + \frac{1}{2}b\int \frac{(-2ia+b\log(1-icx^2))\log(1+icx^2)}{x^3}$$

$$= \frac{1}{8}\text{Subst}\left(\int \frac{(2a+ib\log(1-icx))^2}{x^2}\,dx, x, x^2\right) + \frac{1}{4}b\text{Subst}\left(\int \frac{(-2ia+b\log(1-\right.$$

$$= -\frac{(1-icx^2)(2a+ib\log(1-icx^2))^2}{8x^2} + \frac{b(2ia-b\log(1-icx^2))\log(1+icx^2)}{4x^2} +$$

$$= abc\log(x) - \frac{(1-icx^2)(2a+ib\log(1-icx^2))^2}{8x^2} + \frac{b(2ia-b\log(1-icx^2))\log(}{4x^2}$$

$$= abc\log(x) - \frac{(1-icx^2)(2a+ib\log(1-icx^2))^2}{8x^2} + \frac{b(2ia-b\log(1-icx^2))\log(}{4x^2}$$

$$= 2abc\log(x) - \frac{(1-icx^2)(2a+ib\log(1-icx^2))^2}{8x^2} + \frac{1}{4}ibc(2ia-b\log(1-icx^2))$$

$$= 2abc\log(x) - \frac{(1-icx^2)(2a+ib\log(1-icx^2))^2}{8x^2} + \frac{1}{4}ibc(2ia-b\log(1-icx^2))$$

$$= 2abc\log(x) - \frac{(1-icx^2)(2a+ib\log(1-icx^2))^2}{8x^2} + \frac{1}{4}ibc(2ia-b\log(1-icx^2))$$

Mathematica [A]

time = 0.10, size = 127, normalized size = 1.31

$$-\frac{a^2}{2x^2}+abc\left(-\frac{\text{ArcTan}(cx^2)}{cx^2}+\log\left(cx^2\right)-\frac{1}{2}\log\left(1+c^2x^4\right)\right)+\frac{1}{2}b^2c\left(-\frac{\text{ArcTan}(cx^2)^2}{cx^2}+2\text{ArcTan}(cx^2)\log\left(1-e^{2i\text{ArcTan}(cx^2)}\right)-i\left(\text{ArcTan}(cx^2)^2+\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx^2)}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x^3,x]
```

```
[Out] -1/2*a^2/x^2 + a*b*c*(-(ArcTan[c*x^2]/(c*x^2)) + Log[c*x^2] - Log[1 + c^2*x
^4]/2) + (b^2*c*(-(ArcTan[c*x^2]^2/(c*x^2)) + 2*ArcTan[c*x^2]*Log[1 - E^((2
*I)*ArcTan[c*x^2])] - I*(ArcTan[c*x^2]^2 + PolyLog[2, E^((2*I)*ArcTan[c*x^2
])])))/2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\arctan\left(c\,x^{2}\right)\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2/x^3,x)
```

```
[Out] int((a+b*arctan(c*x^2))^2/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^4 + 1) - log(x^4)) + 2*arctan(c*x^2)/x^2)*a*b + 1/32*(32
*x^2*integrate(-1/16*(4*c^2*x^4*log(c^2*x^4 + 1) - 8*c*x^2*arctan(c*x^2) -
12*(c^2*x^4 + 1)*arctan(c*x^2)^2 - (c^2*x^4 + 1)*log(c^2*x^4 + 1)^2)/(c^2*x
^7 + x^3), x) - 4*arctan(c*x^2)^2 + log(c^2*x^4 + 1)^2)*b^2/x^2 - 1/2*a^2/x
^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}\left(cx^{2}\right)\right)^{2}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**2/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)}^2}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))^2/x^3,x)
```

```
[Out] int((a + b*atan(c*x^2))^2/x^3, x)
```

3.80 $\int \frac{\left(a+b\text{ArcTan}(cx^2)\right)^2}{x^5} dx$

Optimal. Leaf size=87

$$-\frac{bc(a+b\text{ArcTan}(cx^2))}{2x^2}-\frac{1}{4}c^2\left(a+b\text{ArcTan}(cx^2)\right)^2-\frac{(a+b\text{ArcTan}(cx^2))^2}{4x^4}+b^2c^2\log(x)-\frac{1}{4}b^2c^2\log\left(1+c^2x^4\right)$$

```
[Out] -1/2*b*c*(a+b*arctan(c*x^2))/x^2-1/4*c^2*(a+b*arctan(c*x^2))^2-1/4*(a+b*arc
tan(c*x^2))^2/x^4+b^2*c^2*ln(x)-1/4*b^2*c^2*ln(c^2*x^4+1)
```

Rubi [A]
time = 0.12, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4948, 4946, 5038, 272, 36, 29, 31, 5004}

$$-\frac{1}{4}c^2\left(a+b\text{ArcTan}(cx^2)\right)^2-\frac{bc(a+b\text{ArcTan}(cx^2))}{2x^2}-\frac{(a+b\text{ArcTan}(cx^2))^2}{4x^4}-\frac{1}{4}b^2c^2\log\left(c^2x^4+1\right)+b^2c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x^5,x]
```

```
[Out] -1/2*(b*c*(a + b*ArcTan[c*x^2]))/x^2 - (c^2*(a + b*ArcTan[c*x^2])^2)/4 - (a
 + b*ArcTan[c*x^2])^2/(4*x^4) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 + c^2*x^4])
/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^5}\,dx = \int\left(\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x^5}+\frac{b\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{2x^5}-\frac{b^2 l\cdots}{}\right.$$

$$=\frac{1}{4}\int\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{x^5}\,dx+\frac{1}{2}b\int\frac{\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+icx^{\cdots}\right.}{x^5}$$

$$=\frac{1}{8}\text{Subst}\left(\int\frac{(2a+ib\log(1-icx))^2}{x^3}\,dx,x,x^2\right)+\frac{1}{4}b\text{Subst}\left(\int\frac{(-2ia+b\log(1-\cdots}{}\right.$$

$$=-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16x^4}+\frac{b\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{8x^4}+\frac{b^2\log^2\left(\cdots\right.}{16\cdots}$$

$$=-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16x^4}+\frac{b\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{8x^4}+\frac{b^2\log^2\left(\cdots\right.}{16\cdots}$$

$$=-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16x^4}+\frac{b\left(2ia-b\log\left(1-icx^2\right)\right)\log\left(1+icx^2\right)}{8x^4}+\frac{b^2\log^2\left(\cdots\right.}{16\cdots}$$

$$=-\frac{1}{2}iabc^2\log(x)+\frac{ibc\left(2ia-b\log\left(1-icx^2\right)\right)}{8x^2}-\frac{bc\left(1-icx^2\right)\left(2a+ib\log\left(1-ic\cdots\right.\right.}{8x^2}$$

$$=\frac{1}{4}b^2c^2\log(x)+\frac{ibc\left(2ia-b\log\left(1-icx^2\right)\right)}{8x^2}-\frac{bc\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^{2}\cdots\right.\right.}{8x^2}$$

$$=\frac{1}{2}b^2c^2\log(x)+\frac{ibc\left(2ia-b\log\left(1-icx^2\right)\right)}{8x^2}-\frac{bc\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^{2}\cdots\right.\right.}{8x^2}$$

Mathematica [A]
time = 0.05, size = 98, normalized size = 1.13

$$-\frac{a^2+2abcx^2+2b\left(a+bcx^2+ac^2x^4\right)\text{ArcTan}\left(cx^2\right)+b^2\left(1+c^2x^4\right)\text{ArcTan}\left(cx^2\right)^2-4b^2c^2x^4\log(x)+b^2c^2x^4\log\left(1+c^2x^4\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x^5,x]
```

```
[Out] -1/4*(a^2 + 2*a*b*c*x^2 + 2*b*(a + b*c*x^2 + a*c^2*x^4)*ArcTan[c*x^2] + b^2
*(1 + c^2*x^4)*ArcTan[c*x^2]^2 - 4*b^2*c^2*x^4*Log[x] + b^2*c^2*x^4*Log[1 +
 c^2*x^4])/x^4
```

Maple [A]
time = 0.13, size = 118, normalized size = 1.36

method	result
default	$-\frac{a^2}{4x^4}-\frac{b^2\arctan\left(cx^2\right)^2}{4x^4}-\frac{b^2c\arctan\left(cx^2\right)}{2x^2}-\frac{b^2\arctan\left(cx^2\right)^2c^2}{4}+b^2c^2\ln\left(x\right)-\frac{b^2c^2\ln\left(c^2x^4+1\right)}{4}-\frac{ab\arctan\left(cx^2\right)}{2x^4}$

risch	$\frac{b^2(c^2x^4+1)\ln(icx^2+1)^2}{16x^4}+\frac{ib(ibc^2x^4\ln(-icx^2+1)+2bcx^2+2a+ib\ln(-icx^2+1))\ln(icx^2+1)}{8x^4}-\frac{4i\ln((-5ibc+ac)x^2+5b+ia)abc$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a^2/x^4-1/4*b^2/x^4*arctan(c*x^2)^2-1/2*b^2*c*arctan(c*x^2)/x^2-1/4*b^
2*arctan(c*x^2)^2*c^2+b^2*c^2*ln(x)-1/4*b^2*c^2*ln(c^2*x^4+1)-1/2*a*b/x^4*a
rctan(c*x^2)-1/2*a*b*c/x^2-1/2*a*b*arctan(c*x^2)*c^2
```

Maxima [A]
time = 0.54, size = 110, normalized size = 1.26

$$-\frac{1}{2}\left(\left(c\arctan\left(cx^2\right)+\frac{1}{x^2}\right)c+\frac{\arctan\left(cx^2\right)}{x^4}\right)ab+\frac{1}{4}\left(\left(\arctan\left(cx^2\right)^2-\log\left(c^2x^4+1\right)+4\log\left(x\right)\right)c^2-2\left(c\arctan\left(cx^2\right)+\frac{1}{x^2}\right)c\arctan\left(cx^2\right)\right)b^2-\frac{b^2\arctan\left(cx^2\right)^2}{4x^4}-\frac{a^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/2*((c*arctan(c*x^2) + 1/x^2)*c + arctan(c*x^2)/x^4)*a*b + 1/4*((arctan(c
*x^2)^2 - log(c^2*x^4 + 1) + 4*log(x))*c^2 - 2*(c*arctan(c*x^2) + 1/x^2)*c*
arctan(c*x^2))*b^2 - 1/4*b^2*arctan(c*x^2)^2/x^4 - 1/4*a^2/x^4
```

Fricas [A]
time = 2.37, size = 115, normalized size = 1.32

$$\frac{2abc^2x^4\arctan\left(\frac{1}{cx^2}\right)-b^2c^2x^4\log\left(c^2x^4+1\right)+4b^2c^2x^4\log\left(x\right)-2abcx^2-\left(b^2c^2x^4+b^2\right)\arctan\left(cx^2\right)^2-a^2-2\left(b^2cx^2+ab\right)\arctan\left(cx^2\right)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*c^2*x^4*arctan(1/(c*x^2)) - b^2*c^2*x^4*log(c^2*x^4 + 1) + 4*b^2
*c^2*x^4*log(x) - 2*a*b*c*x^2 - (b^2*c^2*x^4 + b^2)*arctan(c*x^2)^2 - a^2 -
 2*(b^2*c*x^2 + a*b)*arctan(c*x^2))/x^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(80) = 160.
time = 21.55, size = 167, normalized size = 1.92

$$\begin{cases}-\frac{a^2}{4x^4}-\frac{abc^2\operatorname{atan}\left(cx^2\right)}{2}-\frac{abc}{2x^2}-\frac{ab\operatorname{atan}\left(cx^2\right)}{2x^4}+b^2c^2\log\left(x\right)-\frac{b^2c^2\log\left(x^2+\sqrt{-\frac{1}{c^2}}\right)}{2}-\frac{b^2c^2\operatorname{atan}^2\left(cx^2\right)}{4}+\frac{b^2c\operatorname{atan}\left(cx^2\right)}{2\sqrt{-\frac{1}{c^2}}}-\frac{b^2c\operatorname{atan}\left(cx^2\right)}{2x^2}-\frac{b^2\operatorname{atan}^2\left(cx^2\right)}{4x^4} & \text{for}\ c\neq 0\\-\frac{a^2}{4x^4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2/x**5,x)
```

```
[Out] Piecewise((-a**2/(4*x**4) - a*b*c**2*atan(c*x**2)/2 - a*b*c/(2*x**2) - a*b*
atan(c*x**2)/(2*x**4) + b**2*c**2*log(x) - b**2*c**2*log(x**2 + sqrt(-1/c**
2))/2 - b**2*c**2*atan(c*x**2)**2/4 + b**2*c*atan(c*x**2)/(2*sqrt(-1/c**2))
 - b**2*c*atan(c*x**2)/(2*x**2) - b**2*atan(c*x**2)**2/(4*x**4), Ne(c, 0)),
 (-a**2/(4*x**4), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2/x^5, x)
```

Mupad [B]
time = 0.61, size = 152, normalized size = 1.75

$$b^2\,c^2\,\ln\left(x\right)-\frac{b^2\,c^2\,\mathrm{atan}\left(c\,x^2\right)^2}{4}-\frac{b^2\,\mathrm{atan}\left(c\,x^2\right)^2}{4\,x^4}-\frac{b^2\,c^2\,\ln\left(c^2\,x^4+1\right)}{4}-\frac{a^2}{4\,x^4}-\frac{b^2\,c\,\mathrm{atan}\left(c\,x^2\right)}{2\,x^2}-\frac{a\,b\,c}{2\,x^2}-\frac{a\,b\,c^2\,\mathrm{atan}\left(\frac{a^2\,c\,x^2}{a^2+25\,b^2}+\frac{25\,b^2\,c\,x^2}{a^2+25\,b^2}\right)}{2}-\frac{a\,b\,\mathrm{atan}\left(c\,x^2\right)}{2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))^2/x^5,x)
```

```
[Out] b^2*c^2*log(x) - (b^2*c^2*atan(c*x^2)^2)/4 - (b^2*atan(c*x^2)^2)/(4*x^4) -
(b^2*c^2*log(c^2*x^4 + 1))/4 - a^2/(4*x^4) - (b^2*c*atan(c*x^2))/(2*x^2) -
(a*b*c)/(2*x^2) - (a*b*c^2*atan((a^2*c*x^2)/(a^2 + 25*b^2) + (25*b^2*c*x^2)
/(a^2 + 25*b^2)))/2 - (a*b*atan(c*x^2))/(2*x^4)
```

3.81 $\int x^2(a + b\mathbf{ArcTan}(cx^2))^2\, dx$

Optimal. Leaf size=1393

$$-\frac{4abx}{3c}+\frac{2}{9}iabx^3+\frac{4(-1)^{3/4}b^2\mathrm{ArcTan}\left((-1)^{3/4}\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{\sqrt[4]{-1}\,b^2\mathrm{ArcTan}\left((-1)^{3/4}\sqrt{c}\,x\right)^2}{3c^{3/2}}-\frac{2\sqrt[4]{-1}\,ab\tanh^{-1}\left((-\right.}{3c^{3/2}}$$

```
[Out] -1/9*b^2*x^3*ln(1-I*c*x^2)-1/12*b^2*x^3*ln(1+I*c*x^2)^2-1/3*(-1)^(1/4)*b*ar
ctan((-1)^(3/4)*x*c^(1/2))*(2*a+I*b*ln(1-I*c*x^2))/c^(3/2)+1/3*(-1)^(3/4)*b
^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)/c^(3/2)+1/3*(-1)^(3/4)*b^2*ar
ctanh((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)/c^(3/2)-2/3*(-1)^(3/4)*b^2*arctan
((-1)^(3/4)*x*c^(1/2))*ln(2/(1-(-1)^(1/4)*x*c^(1/2)))/c^(3/2)+2/3*(-1)^(3/4
)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(1/4)*x*c^(1/2)))/c^(3/2)-1
/3*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2^(1/2)*((-1)^(1/4)+x*c^(
1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(3/2)+2/3*(-1)^(3/4)*b^2*arctanh((-1)^(3/
4)*x*c^(1/2))*ln(2/(1-(-1)^(3/4)*x*c^(1/2)))/c^(3/2)-2/3*(-1)^(3/4)*b^2*arc
tanh((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(3/4)*x*c^(1/2)))/c^(3/2)+1/3*(-1)^
(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(-2^(1/2)*((-1)^(3/4)+x*c^(1/2))/
(1+(-1)^(3/4)*x*c^(1/2)))/c^(3/2)+1/3*(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c
^(1/2))*ln((1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))/c^(3/2)
-1/3*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln((1-I)*(1+(-1)^(3/4)*x*c
^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(3/2)-2/3*I*b^2*x*ln(1-I*c*x^2)/c-1/3*I
*a*b*x^3*ln(1+I*c*x^2)-2/3*(-1)^(1/4)*a*b*arctanh((-1)^(3/4)*x*c^(1/2))/c^(
3/2)+2/3*I*b^2*x*ln(1+I*c*x^2)/c-1/3*(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^
(1/2))*ln(1-I*c*x^2)/c^(3/2)+1/12*x^3*(2*a+I*b*ln(1-I*c*x^2))^2-4/3*a*b*x/c
-1/6*(-1)^(1/4)*b^2*polylog(2,1-2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4
)*x*c^(1/2)))/c^(3/2)+1/3*(-1)^(3/4)*b^2*polylog(2,1-2/(1-(-1)^(3/4)*x*c^(1
/2)))/c^(3/2)+1/3*(-1)^(3/4)*b^2*polylog(2,1-2/(1+(-1)^(3/4)*x*c^(1/2)))/c^
(3/2)-1/6*(-1)^(3/4)*b^2*polylog(2,1+2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1)
^(3/4)*x*c^(1/2)))/c^(3/2)-1/6*(-1)^(3/4)*b^2*polylog(2,1-(1+I)*(1+(-1)^(1/
4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))/c^(3/2)-1/6*(-1)^(1/4)*b^2*polylog(
2,1+(-1+I)*(1+(-1)^(3/4)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(3/2)-1/9*I
*b*x^3*(2*a+I*b*ln(1-I*c*x^2))+4/3*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/
2))/c^(3/2)+1/3*(-1)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))^2/c^(3/2)-4/3*(
-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))/c^(3/2)-1/3*(-1)^(3/4)*b^2*arct
anh((-1)^(3/4)*x*c^(1/2))^2/c^(3/2)+1/6*b^2*x^3*ln(1-I*c*x^2)*ln(1+I*c*x^2)
+1/3*(-1)^(1/4)*b^2*polylog(2,1-2/(1-(-1)^(1/4)*x*c^(1/2)))/c^(3/2)+1/3*(-1
)^(1/4)*b^2*polylog(2,1-2/(1+(-1)^(1/4)*x*c^(1/2)))/c^(3/2)+2/9*I*a*b*x^3
```

Rubi [A]
time = 1.85, antiderivative size = 1393, normalized size of antiderivative = 1.00, number of steps used = 86, number of rules used = 27, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.687$, Rules used = {4950, 2507, 2526, 2498, 327, 209, 2505, 308, 2520, 12, 5040, 4964, 2449, 2352,

6874, 212, 30, 2637, 211, 5048, 4966, 2497, 214, 6139, 6057, 6131, 6055}

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] (-4*a*b*x)/(3*c) + ((2*I)/9)*a*b*x^3 + (4*(-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*
Sqrt[c]*x])/(3*c^(3/2)) + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/(
3*c^(3/2)) - (2*(-1)^(1/4)*a*b*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/(3*c^(3/2)) -
 (4*(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/(3*c^(3/2)) - ((-1)^(3/4)
*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]^2)/(3*c^(3/2)) - (2*(-1)^(3/4)*b^2*ArcTa
n[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/(3*c^(3/2)) + (2
*(-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*
x)])/(3*c^(3/2)) - ((-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2
]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(3*c^(3/2)) + (2*(
-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x
)])/(3*c^(3/2)) - (2*(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1
+ (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) + ((-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*
Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]
*x))])/(3*c^(3/2)) + ((-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[((1
+ I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) -
 ((-1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1 + (-1)^(3/4)*
Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(3*c^(3/2)) - (((2*I)/3)*b^2*x*Log
[1 - I*c*x^2])/c - (b^2*x^3*Log[1 - I*c*x^2])/9 - ((-1)^(3/4)*b^2*ArcTanh[(
-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/(3*c^(3/2)) - (I/9)*b*x^3*(2*a + I*b
*Log[1 - I*c*x^2]) - ((-1)^(1/4)*b*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*
Log[1 - I*c*x^2]))/(3*c^(3/2)) + (x^3*(2*a + I*b*Log[1 - I*c*x^2])^2)/12 +
(((2*I)/3)*b^2*x*Log[1 + I*c*x^2])/c - (I/3)*a*b*x^3*Log[1 + I*c*x^2] + ((-
1)^(3/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/(3*c^(3/2)) + (
(-1)^(3/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/(3*c^(3/2))
+ (b^2*x^3*Log[1 - I*c*x^2]*Log[1 + I*c*x^2])/6 - (b^2*x^3*Log[1 + I*c*x^2]
^2)/12 + ((-1)^(1/4)*b^2*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/(3*c
^(3/2)) + ((-1)^(1/4)*b^2*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(3*
c^(3/2)) - ((-1)^(1/4)*b^2*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x)
)/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(6*c^(3/2)) + ((-1)^(3/4)*b^2*PolyLog[2, 1 -
 2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) + ((-1)^(3/4)*b^2*PolyLog[2, 1
- 2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(3*c^(3/2)) - ((-1)^(3/4)*b^2*PolyLog[2, 1
 + (Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(6*c^(3/
2)) - ((-1)^(3/4)*b^2*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(
1 + (-1)^(3/4)*Sqrt[c]*x)])/(6*c^(3/2)) - ((-1)^(1/4)*b^2*PolyLog[2, 1 - ((
1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(6*c^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
```

```
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
```

```
& IntegerQ[s]
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> I
nt[ExpandIntegrand[x^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n
])/2)^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Integ
erQ[m]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\,dx=\int\left(\frac{1}{4}x^2\left(2a+ib\log\left(1-icx^2\right)\right)^2+\frac{1}{2}bx^2\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(1+ic\right.\right.$$

$$=\frac{1}{4}\int x^2\left(2a+ib\log\left(1-icx^2\right)\right)^2\,dx+\frac{1}{2}b\int x^2\left(-2ia+b\log\left(1-icx^2\right)\right)\log\left(\right.$$

$$=\frac{1}{12}x^3\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{1}{12}b^2x^3\log^2\left(1+icx^2\right)+\frac{1}{2}b\int\left(-2iax^2\log\left(\right.\right.$$

$$=\frac{1}{12}x^3\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{1}{12}b^2x^3\log^2\left(1+icx^2\right)-(iab)\int x^2\log\left(1+\right.$$

$$=\frac{1}{12}x^3\left(2a+ib\log\left(1-icx^2\right)\right)^2-\frac{1}{3}iabx^3\log\left(1+icx^2\right)+\frac{1}{6}b^2x^3\log\left(1-icx^2\right)$$

$$=-\frac{2abx}{3c}-\frac{1}{9}ibx^3\left(2a+ib\log\left(1-icx^2\right)\right)-\frac{\sqrt[4]{-1}\,b\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)\left(2a+\right.}{3c^{3/2}}$$

$$=-\frac{4abx}{3c}-\frac{2ib^2x}{3c}+\frac{2}{9}iabx^3-\frac{ib^2x\log\left(1-icx^2\right)}{3c}-\frac{1}{9}ibx^3\left(2a+ib\log\left(1-icx^2\right.\right.$$

$$=-\frac{4abx}{3c}+\frac{2}{9}iabx^3-\frac{4b^2x^3}{27}+\frac{\sqrt[4]{-1}\,b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2}{3c^{3/2}}-\frac{2\sqrt[4]{-1}\,ab\tanh}{}$$

$$=-\frac{4abx}{3c}+\frac{2}{9}iabx^3-\frac{4b^2x^3}{27}+\frac{8(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{9c^{3/2}}+\frac{\sqrt[4]{-1}\,b^2\tan}{}$$

$$=-\frac{4abx}{3c}+\frac{2}{9}iabx^3-\frac{4b^2x^3}{27}+\frac{8(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{9c^{3/2}}+\frac{\sqrt[4]{-1}\,b^2\tan}{}$$

$$=-\frac{4abx}{3c}+\frac{2}{9}iabx^3+\frac{14(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{9c^{3/2}}+\frac{\sqrt[4]{-1}\,b^2\tan^{-1}\left((-1\right.}{3c^{3/2}}$$

$$=-\frac{4abx}{3c}+\frac{2}{9}iabx^3+\frac{4(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{\sqrt[4]{-1}\,b^2\tan^{-1}\left((-1)^{3}\right.}{3c^{3/2}}$$

$$=-\frac{4abx}{3c}+\frac{2}{9}iabx^3+\frac{4(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{\sqrt[4]{-1}\,b^2\tan^{-1}\left((-1)^{3}\right.}{3c^{3/2}}$$

$$=-\frac{4abx}{3c}+\frac{2}{9}iabx^3+\frac{4(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{\sqrt[4]{-1}\,b^2\tan^{-1}\left((-1)^{3}\right.}{3c^{3/2}}$$

$$=-\frac{4abx}{3c}+\frac{2}{9}iabx^3+\frac{4(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{3c^{3/2}}+\frac{\sqrt[4]{-1}\,b^2\tan^{-1}\left((-1)^{3}\right.}{3c^{3/2}}$$

Mathematica [F]
time = 4.14, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b\text{ArcTan}\left(cx^2\right)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x^2])^2,x]
[Out] Integrate[x^2*(a + b*ArcTan[c*x^2])^2, x]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \arctan\left(c\,x^2\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x^2))^2,x)
[Out] int(x^2*(a+b*arctan(c*x^2))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
[Out] 1/3*a^2*x^3 + 1/6*(4*x^3*arctan(c*x^2) - c*(8*x/c^2 - (2*sqrt(2)*arctan(1/2
*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + 2*sqrt(2)*arctan(1/2*
sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sqrt(c) + sqrt(2)*log(c*x^2 + sq
rt(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1)/s
qrt(c))/c^2))*a*b + 1/48*(4*x^3*arctan(c*x^2)^2 - x^3*log(c^2*x^4 + 1)^2 +
48*integrate(1/48*(8*c^2*x^6*log(c^2*x^4 + 1) - 16*c*x^4*arctan(c*x^2) + 36
*(c^2*x^6 + x^2)*arctan(c*x^2)^2 + 3*(c^2*x^6 + x^2)*log(c^2*x^4 + 1)^2)/(c
^2*x^4 + 1), x))*b^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^2*arctan(c*x^2)^2 + 2*a*b*x^2*arctan(c*x^2) + a^2*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \operatorname{atan}\left(cx^2\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x**2))**2,x)
[Out] Integral(x**2*(a + b*atan(c*x**2))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^2*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + b \operatorname{atan}\left(c\,x^2\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x^2))^2,x)
[Out] int(x^2*(a + b*atan(c*x^2))^2, x)
```

3.82 $\int \left(a+b\mathbf{ArcTan}(cx^2)\right)^2\,dx$

Optimal. Leaf size=1191

$$a^2x-\frac{2(-1)^{3/4}ab\text{ArcTan}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}}+\frac{(-1)^{3/4}b^2\text{ArcTan}\left((-1)^{3/4}\sqrt{c}\,x\right)^2}{\sqrt{c}}+\frac{2(-1)^{3/4}ab\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\right.}{\sqrt{c}}$$

```
[Out] -1/4*b^2*x*ln(1-I*c*x^2)^2-1/4*b^2*x*ln(1+I*c*x^2)^2+2*(-1)^(3/4)*a*b*arcta
nh((-1)^(3/4)*x*c^(1/2))/c^(1/2)+2*(-1)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^(1/
2))*ln(2/(1-(-1)^(1/4)*x*c^(1/2)))/c^(1/2)-2*(-1)^(1/4)*b^2*arctan((-1)^(3/
4)*x*c^(1/2))*ln(2/(1+(-1)^(1/4)*x*c^(1/2)))/c^(1/2)+2*(-1)^(1/4)*b^2*arcta
nh((-1)^(3/4)*x*c^(1/2))*ln(2/(1-(-1)^(3/4)*x*c^(1/2)))/c^(1/2)-2*(-1)^(1/4
)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(3/4)*x*c^(1/2)))/c^(1/2)-
I*a*b*x*ln(1+I*c*x^2)+I*a*b*x*ln(1-I*c*x^2)+(-1)^(1/4)*b^2*arctan((-1)^(3/4
)*x*c^(1/2))*ln(1-I*c*x^2)/c^(1/2)-(-1)^(1/4)*b^2*arctanh((-1)^(3/4)*x*c^(1
/2))*ln(1-I*c*x^2)/c^(1/2)-(-1)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(1
+I*c*x^2)/c^(1/2)+(-1)^(1/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2
)/c^(1/2)+(-1)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2^(1/2)*((-1)^(1/4
)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(1/2)+(-1)^(1/4)*b^2*arctanh((-1)^
(3/4)*x*c^(1/2))*ln(-2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)
))/c^(1/2)+(-1)^(1/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln((1+I)*(1+(-1)^(1
/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))/c^(1/2)+(-1)^(1/4)*b^2*arctan((-1)
^(3/4)*x*c^(1/2))*ln((1-I)*(1+(-1)^(3/4)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)
))/c^(1/2)-2*(-1)^(3/4)*a*b*arctan((-1)^(3/4)*x*c^(1/2))/c^(1/2)+a^2*x+(-1)
^(3/4)*b^2*polylog(2,1-2/(1-(-1)^(1/4)*x*c^(1/2)))/c^(1/2)+(-1)^(3/4)*b^2*p
olylog(2,1-2/(1+(-1)^(1/4)*x*c^(1/2)))/c^(1/2)+(-1)^(1/4)*b^2*polylog(2,1-2
/(1-(-1)^(3/4)*x*c^(1/2)))/c^(1/2)+(-1)^(1/4)*b^2*polylog(2,1-2/(1+(-1)^(3/
4)*x*c^(1/2)))/c^(1/2)+1/2*b^2*x*ln(1-I*c*x^2)*ln(1+I*c*x^2)-1/2*(-1)^(3/4)
*b^2*polylog(2,1-2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c
^(1/2)-1/2*(-1)^(1/4)*b^2*polylog(2,1+2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1
)^(3/4)*x*c^(1/2)))/c^(1/2)-1/2*(-1)^(1/4)*b^2*polylog(2,1-(1+I)*(1+(-1)^(1
/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))/c^(1/2)-1/2*(-1)^(3/4)*b^2*polylog
(2,1+(-1+I)*(1+(-1)^(3/4)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))/c^(1/2)+(-1)
^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))^2/c^(1/2)-(-1)^(1/4)*b^2*arctanh((-
1)^(3/4)*x*c^(1/2))^2/c^(1/2)
```

Rubi [A]
time = 1.23, antiderivative size = 1191, normalized size of antiderivative = 1.00, number of steps used = 69, number of rules used = 23, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.917$, Rules used = {4932, 2498, 327, 209, 2500, 2526, 2520, 12, 5040, 4964, 2449, 2352, 212, 2636, 211, 5048, 4966, 2497, 214, 6139, 6057, 6131, 6055}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] a^2*x - (2*(-1)^(3/4)*a*b*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/Sqrt[c] + ((-1)^(3/
4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/Sqrt[c] + (2*(-1)^(3/4)*a*b*ArcTanh[
(-1)^(3/4)*Sqrt[c]*x])/Sqrt[c] - ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]
*x]^2)/Sqrt[c] + (2*(-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 -
(-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] - (2*(-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[
c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTan[(
-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)
*Sqrt[c]*x)])/Sqrt[c] + (2*(-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log
[2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] - (2*(-1)^(1/4)*b^2*ArcTanh[(-1)^(3
/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2
*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1
+ (-1)^(3/4)*Sqrt[c]*x))])/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqr
t[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)
])/Sqrt[c] + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1 +
 (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + I*a*b*x*Log[
1 - I*c*x^2] + ((-1)^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2
])/Sqrt[c] - ((-1)^(1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2]
)/Sqrt[c] - (b^2*x*Log[1 - I*c*x^2]^2)/4 - I*a*b*x*Log[1 + I*c*x^2] - ((-1)
^(1/4)*b^2*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/Sqrt[c] + ((-1)^(
1/4)*b^2*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/Sqrt[c] + (b^2*x*L
og[1 - I*c*x^2]*Log[1 + I*c*x^2])/2 - (b^2*x*Log[1 + I*c*x^2]^2)/4 + ((-1)^
(3/4)*b^2*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] + ((-1)^(3/
4)*b^2*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/Sqrt[c] - ((-1)^(3/4)*
b^2*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[
c]*x)])/(2*Sqrt[c]) + ((-1)^(1/4)*b^2*PolyLog[2, 1 - 2/(1 - (-1)^(3/4)*Sqrt
[c]*x)])/Sqrt[c] + ((-1)^(1/4)*b^2*PolyLog[2, 1 - 2/(1 + (-1)^(3/4)*Sqrt[c]
*x)])/Sqrt[c] - ((-1)^(1/4)*b^2*PolyLog[2, 1 + (Sqrt[2]*((-1)^(3/4) + Sqrt[
c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(2*Sqrt[c]) - ((-1)^(1/4)*b^2*PolyLog[2
, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/(2*
Sqrt[c]) - ((-1)^(3/4)*b^2*PolyLog[2, 1 - ((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*
x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/(2*Sqrt[c])
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 209</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2498

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
```

```
e, n, p}, x]
```

Rule 2500

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[x^n*(
(a + b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2636

```
Int[Log[v_]*Log[w_], x_Symbol] :> Simp[x*Log[v]*Log[w], x] + (-Int[Simplify
Integrand[x*Log[w]*(D[v, x]/v), x], x] - Int[SimplifyIntegrand[x*Log[v]*(D[
w, x]/w), x], x]) /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w,
x]
```

Rule 4932

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[ExpandIn
tegrand[(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n])/2)^p, x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rubi steps

$$\int \left(a+b\tan^{-1}\left(cx^2\right)\right)^2\,dx = \int \Big(a^2 + iab\log\left(1-icx^2\right) - \frac{1}{4}b^2\log^2\left(1-icx^2\right) - iab\log\left(1+icx^2\right) + \frac{1}{2}b^2\,\mathrm{lo}$$

$$= a^2x + (iab)\int \log\left(1-icx^2\right)\,dx - (iab)\int \log\left(1+icx^2\right)\,dx - \frac{1}{4}b^2\int \log^2\left(1-\right.$$

$$= a^2x + iabx\log\left(1-icx^2\right) - \frac{1}{4}b^2x\log^2\left(1-icx^2\right) - iabx\log\left(1+icx^2\right) + \frac{1}{2}b^2x$$

$$= a^2x + iabx\log\left(1-icx^2\right) - \frac{1}{4}b^2x\log^2\left(1-icx^2\right) - iabx\log\left(1+icx^2\right) + \frac{1}{2}b^2x$$

$$= a^2x - \frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{2(-1)^{3/4}ab\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}}$$

$$= a^2x - \frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{2(-1)^{3/4}ab\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}}$$

$$= a^2x - 4b^2x - \frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{2(-1)^{3/4}ab\tanh^{-1}\left((-1)^{3/4}\right.}{\sqrt{c}}$$

$$= a^2x - \frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}} - \frac{2\sqrt[4]{-1}\,b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}} + ($$

$$= a^2x - \frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2}{\sqrt{c}} +$$

$$= a^2x - \frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2}{\sqrt{c}} +$$

$$= a^2x - \frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2}{\sqrt{c}} +$$

$$= a^2x - \frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2}{\sqrt{c}} +$$

$$= a^2x - \frac{2(-1)^{3/4}ab\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{(-1)^{3/4}b^2\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2}{\sqrt{c}} +$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 5293 vs. 2(1191) = 2382.

time = 30.76, size = 5293, normalized size = 4.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \left(a+b\arctan\left(c\,x^{2}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2,x)
```

```
[Out] int((a+b*arctan(c*x^2))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/c^
(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/c^(
3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log(c*x
^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) - 4*x*arctan(c*x^2))*a*b + 1/16*(4*x*a
rctan(c*x^2)^2 - x*log(c^2*x^4 + 1)^2 + 16*integrate(1/16*(8*c^2*x^4*log(c^
2*x^4 + 1) - 16*c*x^2*arctan(c*x^2) + 12*(c^2*x^4 + 1)*arctan(c*x^2)^2 + (c
^2*x^4 + 1)*log(c^2*x^4 + 1)^2)/(c^2*x^4 + 1), x))*b^2 + a^2*x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{atan}\left(cx^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2,x)
[Out] Integral((a + b*atan(c*x**2))**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))^2,x)
[Out] int((a + b*atan(c*x^2))^2, x)
```

3.83 $\int \frac{\left(a+b\mathbf{ArcTan}(cx^2)\right)^2}{x^2}\,dx$

Optimal. Leaf size=1164

$\sqrt[4]{-1}\,b^2\sqrt{c}\,\mathrm{ArcTan}\left((-1)^{3/4}\sqrt{c}\,x\right)^2-2\sqrt[4]{-1}\,ab\sqrt{c}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)-(-1)^{3/4}b^2\sqrt{c}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\right.$

```
[Out] 1/4*b^2*ln(1+I*c*x^2)^2/x+(-1)^(1/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))^2*c^(
1/2)-(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))^2*c^(1/2)-1/2*b^2*ln(1-I*
c*x^2)*ln(1+I*c*x^2)/x-1/2*(-1)^(1/4)*b^2*polylog(2,1-2^(1/2)*((-1)^(1/4)+x
*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))*c^(1/2)-1/2*(-1)^(3/4)*b^2*polylog(2,1+
2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))*c^(1/2)-1/2*(-1)^(
3/4)*b^2*polylog(2,1-(1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)
))*c^(1/2)-1/2*(-1)^(1/4)*b^2*polylog(2,1+(-1+I)*(1+(-1)^(3/4)*x*c^(1/2))/(
1+(-1)^(1/4)*x*c^(1/2)))*c^(1/2)-2*(-1)^(1/4)*a*b*arctanh((-1)^(3/4)*x*c^(1
/2))*c^(1/2)-2*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(1-(-1)^(1/
4)*x*c^(1/2)))*c^(1/2)+2*(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(
1+(-1)^(1/4)*x*c^(1/2)))*c^(1/2)+2*(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1
/2))*ln(2/(1-(-1)^(3/4)*x*c^(1/2)))*c^(1/2)-2*(-1)^(3/4)*b^2*arctanh((-1)^(
3/4)*x*c^(1/2))*ln(2/(1+(-1)^(3/4)*x*c^(1/2)))*c^(1/2)+I*a*b*ln(1+I*c*x^2)/
x-(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(1-I*c*x^2)*c^(1/2)-(-1)^(
1/4)*b*arctan((-1)^(3/4)*x*c^(1/2))*(2*a+I*b*ln(1-I*c*x^2))*c^(1/2)+(-1)^(3
/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)*c^(1/2)+(-1)^(3/4)*b^2*a
rctanh((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)*c^(1/2)-(-1)^(3/4)*b^2*arctan((-
1)^(3/4)*x*c^(1/2))*ln(2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/
2)))*c^(1/2)+(-1)^(3/4)*b^2*arctanh((-1)^(3/4)*x*c^(1/2))*ln(-2^(1/2)*((-1)
^(3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))*c^(1/2)+(-1)^(3/4)*b^2*arctanh(
(-1)^(3/4)*x*c^(1/2))*ln((1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(
1/2)))*c^(1/2)-(-1)^(3/4)*b^2*arctan((-1)^(3/4)*x*c^(1/2))*ln((1-I)*(1+(-1)
^(3/4)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))*c^(1/2)-1/4*(2*a+I*b*ln(1-I*c*x
^2))^2/x+(-1)^(1/4)*b^2*polylog(2,1-2/(1-(-1)^(1/4)*x*c^(1/2)))*c^(1/2)+(-1
)^(1/4)*b^2*polylog(2,1-2/(1+(-1)^(1/4)*x*c^(1/2)))*c^(1/2)+(-1)^(3/4)*b^2*
polylog(2,1-2/(1-(-1)^(3/4)*x*c^(1/2)))*c^(1/2)+(-1)^(3/4)*b^2*polylog(2,1-
2/(1+(-1)^(3/4)*x*c^(1/2)))*c^(1/2)
```

Rubi [A]
time = 1.15, antiderivative size = 1164, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 23, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.438$, Rules used = {4950, 2507, 209, 2520, 12, 5040, 4964, 2449, 2352, 2505, 6874, 212, 30, 2637, 211, 5048, 4966, 2497, 214, 6139, 6057, 6131, 6055}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x^2,x]
```

```
[Out] (-1)^(1/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2 - 2*(-1)^(1/4)*a*b*Sq
rt[c]*ArcTanh[(-1)^(3/4)*Sqrt[c]*x] - (-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^(
3/4)*Sqrt[c]*x]^2 - 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*L
og[2/(1 - (-1)^(1/4)*Sqrt[c]*x)] + 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/
4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)] - (-1)^(3/4)*b^2*Sqrt[c]*Ar
cTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)
^(1/4)*Sqrt[c]*x)] + 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]
*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x)] - 2*(-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^
(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)] + (-1)^(3/4)*b^2*Sqrt[c]
*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1
+ (-1)^(3/4)*Sqrt[c]*x))] + (-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-1)^(3/4)*Sqrt[
c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)]
- (-1)^(3/4)*b^2*Sqrt[c]*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1 + (-1
)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)] - (-1)^(3/4)*b^2*Sqrt[c]*Ar
cTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2] - (-1)^(1/4)*b*Sqrt[c]*ArcTan[
(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*Log[1 - I*c*x^2]) - (2*a + I*b*Log[1 - I*c
*x^2])^2/(4*x) + (I*a*b*Log[1 + I*c*x^2])/x + (-1)^(3/4)*b^2*Sqrt[c]*ArcTan
[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2] + (-1)^(3/4)*b^2*Sqrt[c]*ArcTanh[(-
1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2] - (b^2*Log[1 - I*c*x^2]*Log[1 + I*c*x^
2])/(2*x) + (b^2*Log[1 + I*c*x^2]^2)/(4*x) + (-1)^(1/4)*b^2*Sqrt[c]*PolyLog
[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)] + (-1)^(1/4)*b^2*Sqrt[c]*PolyLog[2, 1
 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)] - ((-1)^(1/4)*b^2*Sqrt[c]*PolyLog[2, 1 - (
Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/2 + (-1)^(3/
4)*b^2*Sqrt[c]*PolyLog[2, 1 - 2/(1 - (-1)^(3/4)*Sqrt[c]*x)] + (-1)^(3/4)*b^
2*Sqrt[c]*PolyLog[2, 1 - 2/(1 + (-1)^(3/4)*Sqrt[c]*x)] - ((-1)^(3/4)*b^2*Sq
rt[c]*PolyLog[2, 1 + (Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqr
t[c]*x)])/2 - ((-1)^(3/4)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/
4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/2 - ((-1)^(1/4)*b^2*Sqrt[c]*Pol
yLog[2, 1 - ((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)
])/2
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

<u>Rule 30</u>

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

<u>Rule 209</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
```

```
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> I
nt[ExpandIntegrand[x^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n
])/2)^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Integ
erQ[m]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx^2))^2}{x^2}\,dx = \int \left(\frac{(2a+ib\log(1-icx^2))^2}{4x^2} + \frac{b(-2ia+b\log(1-icx^2))\log(1+icx^2)}{2x^2} - \frac{b^2\,\mathrm{l}\cdots}{\cdots}\right.$$

$$= \frac{1}{4}\int \frac{(2a+ib\log(1-icx^2))^2}{x^2}\,dx + \frac{1}{2}b\int \frac{(-2ia+b\log(1-icx^2))\log(1+icx^2\cdots}{x^2}$$

$$= -\frac{(2a+ib\log(1-icx^2))^2}{4x} + \frac{b^2\log^2(1+icx^2)}{4x} + \frac{1}{2}b\int\left(-\frac{2ia\log(1+icx^2)}{x^2} + \cdots\right.$$

$$= -\sqrt[4]{-1}\,b\sqrt{c}\,\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)\left(2a+ib\log(1-icx^2)\right) - \frac{(2a+ib\log(1-\cdots}{4x}$$

$$= -\sqrt[4]{-1}\,b\sqrt{c}\,\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)\left(2a+ib\log(1-icx^2)\right) - \frac{(2a+ib\log(1-\cdots}{4x}$$

$$= \sqrt[4]{-1}\,b^2\sqrt{c}\,\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2 - 2\sqrt[4]{-1}\,ab\sqrt{c}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) - (\cdots$$

$$= \sqrt[4]{-1}\,b^2\sqrt{c}\,\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2 - 2\sqrt[4]{-1}\,ab\sqrt{c}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) - (\cdots$$

$$= \sqrt[4]{-1}\,b^2\sqrt{c}\,\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2 - 2\sqrt[4]{-1}\,ab\sqrt{c}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) - (\cdots$$

$$= \sqrt[4]{-1}\,b^2\sqrt{c}\,\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2 - 2\sqrt[4]{-1}\,ab\sqrt{c}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) - (\cdots$$

$$= \sqrt[4]{-1}\,b^2\sqrt{c}\,\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2 - 2\sqrt[4]{-1}\,ab\sqrt{c}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) - (\cdots$$

$$= \sqrt[4]{-1}\,b^2\sqrt{c}\,\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2 - 2\sqrt[4]{-1}\,ab\sqrt{c}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) - (\cdots$$

$$= \sqrt[4]{-1}\,b^2\sqrt{c}\,\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2 - 2\sqrt[4]{-1}\,ab\sqrt{c}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) - (\cdots$$

$$= \sqrt[4]{-1}\,b^2\sqrt{c}\,\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)^2 - 2\sqrt[4]{-1}\,ab\sqrt{c}\,\tanh^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) - (\cdots$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 5293 vs. 2(1164) = 2328.
time = 30.55, size = 5293, normalized size = 4.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x^2,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\arctan \left(c\,x^{2}\right)\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2/x^2,x)
```

```
[Out] int((a+b*arctan(c*x^2))^2/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))/sqr
t(c) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/sqrt
(c) + sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/sqrt(c) - sqrt(2)*log(c*x^
2 - sqrt(2)*sqrt(c)*x + 1)/sqrt(c)) - 4*arctan(c*x^2)/x)*a*b - 1/16*(4*arct
an(c*x^2)^2 - 16*x*integrate(-1/16*(8*c^2*x^4*log(c^2*x^4 + 1) - 16*c*x^2*a
rctan(c*x^2) - 12*(c^2*x^4 + 1)*arctan(c*x^2)^2 - (c^2*x^4 + 1)*log(c^2*x^4
 + 1)^2)/(c^2*x^6 + x^2), x) - log(c^2*x^4 + 1)^2)*b^2/x - a^2/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\operatorname{atan}\left(cx^{2}\right)\right)^{2}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2/x**2,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)}^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))^2/x^2,x)
```

```
[Out] int((a + b*atan(c*x^2))^2/x^2, x)
```

3.84 $\int \frac{\left(a+b\,\mathbf{ArcTan}(cx^2)\right)^2}{x^4}\,dx$

Optimal. Leaf size=1360

$$-\frac{2abc}{3x}-\frac{4}{3}\sqrt[4]{-1}\,b^2c^{3/2}\text{ArcTan}\left((-1)^{3/4}\sqrt{c}\,x\right)+\frac{1}{3}(-1)^{3/4}b^2c^{3/2}\text{ArcTan}\left((-1)^{3/4}\sqrt{c}\,x\right)^2+\frac{2}{3}(-1)^{3/4}abc^{3/2}\tanh^{-}$$

```
[Out] -1/12*(2*a+I*b*ln(1-I*c*x^2))^2/x^3+1/12*b^2*ln(1+I*c*x^2)^2/x^3-4/3*(-1)^(
1/4)*b^2*c^(3/2)*arctan((-1)^(3/4)*x*c^(1/2))+1/3*(-1)^(3/4)*b^2*c^(3/2)*ar
ctan((-1)^(3/4)*x*c^(1/2))^2-4/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*
x*c^(1/2))-1/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*x*c^(1/2))^2-1/3*b
*c*(2*a+I*b*ln(1-I*c*x^2))/x-1/6*b^2*ln(1-I*c*x^2)*ln(1+I*c*x^2)/x^3+1/3*(-
1)^(3/4)*b^2*c^(3/2)*polylog(2,1-2/(1-(-1)^(1/4)*x*c^(1/2)))+1/3*(-1)^(3/4)
*b^2*c^(3/2)*polylog(2,1-2/(1+(-1)^(1/4)*x*c^(1/2)))-1/6*(-1)^(3/4)*b^2*c^(
3/2)*polylog(2,1-2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))+1
/3*(-1)^(1/4)*b^2*c^(3/2)*polylog(2,1-2/(1-(-1)^(3/4)*x*c^(1/2)))+1/3*(-1)^
(1/4)*b^2*c^(3/2)*polylog(2,1-2/(1+(-1)^(3/4)*x*c^(1/2)))-1/6*(-1)^(1/4)*b^
2*c^(3/2)*polylog(2,1+2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2
)))-1/6*(-1)^(1/4)*b^2*c^(3/2)*polylog(2,1-(1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(
1+(-1)^(3/4)*x*c^(1/2)))-1/6*(-1)^(3/4)*b^2*c^(3/2)*polylog(2,1+(-1+I)*(1+(
-1)^(3/4)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))+1/3*(-1)^(1/4)*b^2*c^(3/2)*a
rctanh((-1)^(3/4)*x*c^(1/2))*ln(1+I*c*x^2)+2/3*(-1)^(1/4)*b^2*c^(3/2)*arcta
n((-1)^(3/4)*x*c^(1/2))*ln(2/(1-(-1)^(1/4)*x*c^(1/2)))-2/3*(-1)^(1/4)*b^2*c
^(3/2)*arctan((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(1/4)*x*c^(1/2)))+1/3*(-1)
^(1/4)*b^2*c^(3/2)*arctan((-1)^(3/4)*x*c^(1/2))*ln(2^(1/2)*((-1)^(1/4)+x*c^
(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))+2/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3
/4)*x*c^(1/2))*ln(2/(1-(-1)^(3/4)*x*c^(1/2)))-2/3*(-1)^(1/4)*b^2*c^(3/2)*ar
ctanh((-1)^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(3/4)*x*c^(1/2)))+1/3*(-1)^(1/4)*b
^2*c^(3/2)*arctanh((-1)^(3/4)*x*c^(1/2))*ln(-2^(1/2)*((-1)^(3/4)+x*c^(1/2))
/(1+(-1)^(3/4)*x*c^(1/2)))+1/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*x*
c^(1/2))*ln((1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))+1/3*(-
1)^(1/4)*b^2*c^(3/2)*arctan((-1)^(3/4)*x*c^(1/2))*ln((1-I)*(1+(-1)^(3/4)*x*
c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))-1/3*I*b^2*c*ln(1-I*c*x^2)/x+1/3*I*a*b*ln
(1+I*c*x^2)/x^3+2/3*I*b^2*c*ln(1+I*c*x^2)/x+2/3*(-1)^(3/4)*a*b*c^(3/2)*arct
anh((-1)^(3/4)*x*c^(1/2))-1/3*(-1)^(1/4)*b^2*c^(3/2)*arctanh((-1)^(3/4)*x*c
^(1/2))*ln(1-I*c*x^2)-1/3*(-1)^(3/4)*b*c^(3/2)*arctan((-1)^(3/4)*x*c^(1/2))
*(2*a+I*b*ln(1-I*c*x^2))-1/3*(-1)^(1/4)*b^2*c^(3/2)*arctan((-1)^(3/4)*x*c^(
1/2))*ln(1+I*c*x^2)-2/3*a*b*c/x
```

Rubi [A]

time = 1.55, antiderivative size = 1360, normalized size of antiderivative = 1.00, number of steps used = 64, number of rules used = 25, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.562$, Rules used = {4950, 2507, 2526, 2505, 209, 211, 2520, 12, 5040, 4964, 2449, 2352, 331, 6874, 212, 30, 2637, 5048, 4966, 2497, 214, 6139, 6057, 6131, 6055}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x^4,x]
```

```
[Out] (-2*a*b*c)/(3*x) - (4*(-1)^(1/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/
3 + ((-1)^(3/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/3 + (2*(-1)^(3/
4)*a*b*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/3 - (4*(-1)^(1/4)*b^2*c^(3/2)
*ArcTanh[(-1)^(3/4)*Sqrt[c]*x])/3 - ((-1)^(1/4)*b^2*c^(3/2)*ArcTanh[(-1)^(3
/4)*Sqrt[c]*x]^2)/3 + (2*(-1)^(1/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x
]*Log[2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/3 - (2*(-1)^(1/4)*b^2*c^(3/2)*ArcTan[(
-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/3 + ((-1)^(1/4)*b^2
*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x)
)/(1 + (-1)^(1/4)*Sqrt[c]*x)])/3 + (2*(-1)^(1/4)*b^2*c^(3/2)*ArcTanh[(-1)^(
3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(3/4)*Sqrt[c]*x)])/3 - (2*(-1)^(1/4)*b^2*c^
(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/3 +
((-1)^(1/4)*b^2*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^
(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x))])/3 + ((-1)^(1/4)*b^2*c^(3/
2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(
1 + (-1)^(3/4)*Sqrt[c]*x)])/3 + ((-1)^(1/4)*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*S
qrt[c]*x]*Log[((1 - I)*(1 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*
x)])/3 - ((I/3)*b^2*c*Log[1 - I*c*x^2])/x - ((-1)^(1/4)*b^2*c^(3/2)*ArcTanh
[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/3 - (b*c*(2*a + I*b*Log[1 - I*c*x^
2]))/(3*x) - ((-1)^(3/4)*b*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*
Log[1 - I*c*x^2]))/3 - (2*a + I*b*Log[1 - I*c*x^2])^2/(12*x^3) + ((I/3)*a*b
*Log[1 + I*c*x^2])/x^3 + (((2*I)/3)*b^2*c*Log[1 + I*c*x^2])/x - ((-1)^(1/4)
*b^2*c^(3/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/3 + ((-1)^(1/4)
*b^2*c^(3/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/3 - (b^2*Log[1
 - I*c*x^2]*Log[1 + I*c*x^2])/(6*x^3) + (b^2*Log[1 + I*c*x^2]^2)/(12*x^3) +
 ((-1)^(3/4)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/3 +
((-1)^(3/4)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/3 - (
(-1)^(3/4)*b^2*c^(3/2)*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1
 + (-1)^(1/4)*Sqrt[c]*x)])/6 + ((-1)^(1/4)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1
- (-1)^(3/4)*Sqrt[c]*x)])/3 + ((-1)^(1/4)*b^2*c^(3/2)*PolyLog[2, 1 - 2/(1 +
 (-1)^(3/4)*Sqrt[c]*x)])/3 - ((-1)^(1/4)*b^2*c^(3/2)*PolyLog[2, 1 + (Sqrt[2
]*((-1)^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/6 - ((-1)^(1/4)*b^
2*c^(3/2)*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/
4)*Sqrt[c]*x)])/6 - ((-1)^(3/4)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 - I)*(1 + (-
1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/6
```

<u>Rule 12</u>

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> I
nt[ExpandIntegrand[x^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n
```

```
])/2)^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Integ
erQ[m]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
```

```
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx^2))^2}{x^4}\,dx = \int \left(\frac{(2a+ib\log(1-icx^2))^2}{4x^4} + \frac{b(-2ia+b\log(1-icx^2))\log(1+icx^2)}{2x^4} - \frac{b^2\log$$

$$= \frac{1}{4}\int \frac{(2a+ib\log(1-icx^2))^2}{x^4}\,dx + \frac{1}{2}b\int \frac{(-2ia+b\log(1-icx^2))\log(1+icx^2)}{x^4}$$

$$= -\frac{(2a+ib\log(1-icx^2))^2}{12x^3} + \frac{b^2\log^2(1+icx^2)}{12x^3} + \frac{1}{2}b\int \left(-\frac{2ia\log(1+icx^2)}{x^4} + \right.$$

$$= -\frac{(2a+ib\log(1-icx^2))^2}{12x^3} + \frac{b^2\log^2(1+icx^2)}{12x^3} - (iab)\int \frac{\log(1+icx^2)}{x^4}\,dx + \frac{1}{2}b$$

$$= -\frac{(2a+ib\log(1-icx^2))^2}{12x^3} + \frac{iab\log(1+icx^2)}{3x^3} - \frac{b^2\log(1-icx^2)\log(1+icx^2)}{6x^3}$$

$$= -\frac{2abc}{3x} - \frac{bc(2a+ib\log(1-icx^2))}{3x} - \frac{1}{3}(-1)^{3/4}bc^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right)(2a+$$

$$= -\frac{2abc}{3x} - \frac{2}{3}\sqrt[4]{-1}\,b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) + \frac{2}{3}(-1)^{3/4}abc^{3/2}\tanh^{-1}\left((-1)^{3/4}\right.$$

$$= -\frac{2abc}{3x} - \frac{2}{3}\sqrt[4]{-1}\,b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{\right.$$

$$= -\frac{2abc}{3x} - \frac{2}{3}\sqrt[4]{-1}\,b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{\right.$$

$$= -\frac{2abc}{3x} - \frac{4}{3}\sqrt[4]{-1}\,b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{\right.$$

$$= -\frac{2abc}{3x} - \frac{4}{3}\sqrt[4]{-1}\,b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{\right.$$

$$= -\frac{2abc}{3x} - \frac{4}{3}\sqrt[4]{-1}\,b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{\right.$$

$$= -\frac{2abc}{3x} - \frac{4}{3}\sqrt[4]{-1}\,b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{\right.$$

$$= -\frac{2abc}{3x} - \frac{4}{3}\sqrt[4]{-1}\,b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{\right.$$

$$= -\frac{2abc}{3x} - \frac{4}{3}\sqrt[4]{-1}\,b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) + \frac{1}{3}(-1)^{3/4}b^2c^{3/2}\tan^{-1}\left((-1)^{3/4}\sqrt{\right.$$

Mathematica [F]

time = 2.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\text{ArcTan}(cx^2))^2}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x^4,x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x^2])^2/x^4, x]
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(a + b\arctan(c\,x^2))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2/x^4,x)
```

```
[Out] int((a+b*arctan(c*x^2))^2/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt(c))
/c^(3/2) + 2*sqrt(2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(c))/
c^(3/2) - sqrt(2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1)/c^(3/2) + sqrt(2)*log(
c*x^2 - sqrt(2)*sqrt(c)*x + 1)/c^(3/2)) + 8/x)*c + 4*arctan(c*x^2)/x^3)*a*b
 + 1/48*(48*x^3*integrate(-1/48*(8*c^2*x^4*log(c^2*x^4 + 1) - 16*c*x^2*arct
an(c*x^2) - 36*(c^2*x^4 + 1)*arctan(c*x^2)^2 - 3*(c^2*x^4 + 1)*log(c^2*x^4
+ 1)^2)/(c^2*x^8 + x^4), x) - 4*arctan(c*x^2)^2 + log(c^2*x^4 + 1)^2)*b^2/x
^3 - 1/3*a^2/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^4, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}\left(cx^{2}\right)\right)^{2}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2/x**4,x)
[Out] Integral((a + b*atan(c*x**2))**2/x**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^4,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^2/x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)}^2}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))^2/x^4,x)
[Out] int((a + b*atan(c*x^2))^2/x^4, x)
```

3.85 $\int \frac{\left(a+b\mathbf{ArcTan}\left(cx^2\right)\right)^2}{x^6}\,dx$

Optimal. Leaf size=1444

$$-\frac{2abc}{15x^3}+\frac{2iabc^2}{5x}-\frac{8b^2c^2}{15x}-\frac{4}{15}(-1)^{3/4}b^2c^{5/2}\text{ArcTan}\left((-1)^{3/4}\sqrt{c}\,x\right)-\frac{1}{5}\sqrt[4]{-1}\,b^2c^{5/2}\text{ArcTan}\left((-1)^{3/4}\sqrt{c}\,x\right)^2+\frac{2}{5}\sqrt[4]{}$$

```
[Out] 1/20*b^2*ln(1+I*c*x^2)^2/x^5-8/15*b^2*c^2/x-1/20*(2*a+I*b*ln(1-I*c*x^2))^2/
x^5-1/5*(-1)^(1/4)*b^2*c^(5/2)*arctan((-1)^(3/4)*x*c^(1/2))^2+4/15*(-1)^(3/
4)*b^2*c^(5/2)*arctanh((-1)^(3/4)*x*c^(1/2))+1/5*(-1)^(3/4)*b^2*c^(5/2)*arc
tanh((-1)^(3/4)*x*c^(1/2))^2-1/5*b^2*c^2*ln(1-I*c*x^2)/x-1/15*b*c*(2*a+I*b*
ln(1-I*c*x^2))/x^3-1/10*b^2*ln(1-I*c*x^2)*ln(1+I*c*x^2)/x^5-1/5*(-1)^(1/4)*
b^2*c^(5/2)*polylog(2,1-2/(1-(-1)^(1/4)*x*c^(1/2)))-1/5*(-1)^(1/4)*b^2*c^(5
/2)*polylog(2,1-2/(1+(-1)^(1/4)*x*c^(1/2)))+1/10*(-1)^(1/4)*b^2*c^(5/2)*pol
ylog(2,1-2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))-1/5*(-1)^
(3/4)*b^2*c^(5/2)*polylog(2,1-2/(1-(-1)^(3/4)*x*c^(1/2)))-1/5*(-1)^(3/4)*b^
2*c^(5/2)*polylog(2,1-2/(1+(-1)^(3/4)*x*c^(1/2)))+1/10*(-1)^(3/4)*b^2*c^(5/
2)*polylog(2,1+2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))+1/1
0*(-1)^(3/4)*b^2*c^(5/2)*polylog(2,1-(1+I)*(1+(-1)^(1/4)*x*c^(1/2))/(1+(-1)
^(3/4)*x*c^(1/2)))+1/10*(-1)^(1/4)*b^2*c^(5/2)*polylog(2,1+(-1+I)*(1+(-1)^(
3/4)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))-4/15*(-1)^(3/4)*b^2*c^(5/2)*arcta
n((-1)^(3/4)*x*c^(1/2))-1/5*(-1)^(3/4)*b^2*c^(5/2)*arctanh((-1)^(3/4)*x*c^(
1/2))*ln(-2^(1/2)*((-1)^(3/4)+x*c^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))-1/5*(-1)
^(3/4)*b^2*c^(5/2)*arctanh((-1)^(3/4)*x*c^(1/2))*ln((1+I)*(1+(-1)^(1/4)*x*c
^(1/2))/(1+(-1)^(3/4)*x*c^(1/2)))+1/5*(-1)^(3/4)*b^2*c^(5/2)*arctan((-1)^(3
/4)*x*c^(1/2))*ln((1-I)*(1+(-1)^(3/4)*x*c^(1/2))/(1+(-1)^(1/4)*x*c^(1/2)))-
1/15*I*b^2*c*ln(1-I*c*x^2)/x^3-1/5*I*b*c^2*(2*a+I*b*ln(1-I*c*x^2))/x+2/5*(-
1)^(1/4)*a*b*c^(5/2)*arctanh((-1)^(3/4)*x*c^(1/2))+1/5*(-1)^(3/4)*b^2*c^(5/
2)*arctanh((-1)^(3/4)*x*c^(1/2))*ln(1-I*c*x^2)+1/5*(-1)^(1/4)*b*c^(5/2)*arc
tan((-1)^(3/4)*x*c^(1/2))*(2*a+I*b*ln(1-I*c*x^2))+1/5*I*a*b*ln(1+I*c*x^2)/x
^5+2/15*I*b^2*c*ln(1+I*c*x^2)/x^3-1/5*(-1)^(3/4)*b^2*c^(5/2)*arctan((-1)^(3
/4)*x*c^(1/2))*ln(1+I*c*x^2)-1/5*(-1)^(3/4)*b^2*c^(5/2)*arctanh((-1)^(3/4)*
x*c^(1/2))*ln(1+I*c*x^2)+2/5*(-1)^(3/4)*b^2*c^(5/2)*arctan((-1)^(3/4)*x*c^(
1/2))*ln(2/(1-(-1)^(1/4)*x*c^(1/2)))-2/5*(-1)^(3/4)*b^2*c^(5/2)*arctan((-1)
^(3/4)*x*c^(1/2))*ln(2/(1+(-1)^(1/4)*x*c^(1/2)))+1/5*(-1)^(3/4)*b^2*c^(5/2)
*arctan((-1)^(3/4)*x*c^(1/2))*ln(2^(1/2)*((-1)^(1/4)+x*c^(1/2))/(1+(-1)^(1/
4)*x*c^(1/2)))-2/5*(-1)^(3/4)*b^2*c^(5/2)*arctanh((-1)^(3/4)*x*c^(1/2))*ln(
2/(1-(-1)^(3/4)*x*c^(1/2)))+2/5*(-1)^(3/4)*b^2*c^(5/2)*arctanh((-1)^(3/4)*x
*c^(1/2))*ln(2/(1+(-1)^(3/4)*x*c^(1/2)))-2/15*a*b*c/x^3+2/5*I*a*b*c^2/x
```

Rubi [A]

time = 1.69, antiderivative size = 1444, normalized size of antiderivative = 1.00, number of steps used = 77, number of rules used = 25, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.562$, Rules used = {4950, 2507, 2526, 2505, 331, 209, 211, 2520, 12, 5040, 4964, 2449, 2352, 6874, 212, 30, 2637, 5048, 4966, 2497, 214, 6139, 6057, 6131, 6055}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^2/x^6,x]
```

```
[Out] (-2*a*b*c)/(15*x^3) + (((2*I)/5)*a*b*c^2)/x - (8*b^2*c^2)/(15*x) - (4*(-1)^
(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x])/15 - ((-1)^(1/4)*b^2*c^(5/2
)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]^2)/5 + (2*(-1)^(1/4)*a*b*c^(5/2)*ArcTanh[(-1
)^(3/4)*Sqrt[c]*x])/5 + (4*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c
]*x])/15 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]^2)/5 + (2*
(-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1 - (-1)^(1/4)*S
qrt[c]*x)])/5 - (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[
2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/5 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4
)*Sqrt[c]*x]*Log[(Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]
*x)])/5 - (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[2/(1
- (-1)^(3/4)*Sqrt[c]*x)])/5 + (2*(-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*
Sqrt[c]*x]*Log[2/(1 + (-1)^(3/4)*Sqrt[c]*x)])/5 - ((-1)^(3/4)*b^2*c^(5/2)*A
rcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[-((Sqrt[2]*((-1)^(3/4) + Sqrt[c]*x))/(1 +
(-1)^(3/4)*Sqrt[c]*x))])/5 - ((-1)^(3/4)*b^2*c^(5/2)*ArcTanh[(-1)^(3/4)*Sqr
t[c]*x]*Log[((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)
])/5 + ((-1)^(3/4)*b^2*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[((1 - I)*(1
 + (-1)^(3/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/5 - ((I/15)*b^2*c*Lo
g[1 - I*c*x^2])/x^3 - (b^2*c^2*Log[1 - I*c*x^2])/(5*x) + ((-1)^(3/4)*b^2*c^
(5/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 - I*c*x^2])/5 - (b*c*(2*a + I*b*L
og[1 - I*c*x^2]))/(15*x^3) - ((I/5)*b*c^2*(2*a + I*b*Log[1 - I*c*x^2]))/x +
 ((-1)^(1/4)*b*c^(5/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*(2*a + I*b*Log[1 - I*c*
x^2]))/5 - (2*a + I*b*Log[1 - I*c*x^2])^2/(20*x^5) + ((I/5)*a*b*Log[1 + I*c
*x^2])/x^5 + (((2*I)/15)*b^2*c*Log[1 + I*c*x^2])/x^3 - ((-1)^(3/4)*b^2*c^(5
/2)*ArcTan[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/5 - ((-1)^(3/4)*b^2*c^(5
/2)*ArcTanh[(-1)^(3/4)*Sqrt[c]*x]*Log[1 + I*c*x^2])/5 - (b^2*Log[1 - I*c*x^
2]*Log[1 + I*c*x^2])/(10*x^5) + (b^2*Log[1 + I*c*x^2]^2)/(20*x^5) - ((-1)^(
1/4)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 - (-1)^(1/4)*Sqrt[c]*x)])/5 - ((-1)^(1
/4)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 + (-1)^(1/4)*Sqrt[c]*x)])/5 + ((-1)^(1/
4)*b^2*c^(5/2)*PolyLog[2, 1 - (Sqrt[2]*((-1)^(1/4) + Sqrt[c]*x))/(1 + (-1)^
(1/4)*Sqrt[c]*x)])/10 - ((-1)^(3/4)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 - (-1)^
(3/4)*Sqrt[c]*x)])/5 - ((-1)^(3/4)*b^2*c^(5/2)*PolyLog[2, 1 - 2/(1 + (-1)^(
3/4)*Sqrt[c]*x)])/5 + ((-1)^(3/4)*b^2*c^(5/2)*PolyLog[2, 1 + (Sqrt[2]*((-1)
^(3/4) + Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqrt[c]*x)])/10 + ((-1)^(3/4)*b^2*c^(5
/2)*PolyLog[2, 1 - ((1 + I)*(1 + (-1)^(1/4)*Sqrt[c]*x))/(1 + (-1)^(3/4)*Sqr
t[c]*x)])/10 + ((-1)^(1/4)*b^2*c^(5/2)*PolyLog[2, 1 - ((1 - I)*(1 + (-1)^(3
/4)*Sqrt[c]*x))/(1 + (-1)^(1/4)*Sqrt[c]*x)])/10
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
```

```
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 2505

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])/(f*(m
+ 1))), x] - Dist[b*e*n*(p/(f*(m + 1))), Int[x^(n - 1)*((f*x)^(m + 1)/(d +
e*x^n)), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 2507

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_)*((f_.)*(
x_))^(m_.), x_Symbol] :> Simp[(f*x)^(m + 1)*((a + b*Log[c*(d + e*x^n)^p])^q
/(f*(m + 1))), x] - Dist[b*e*n*p*(q/(f^n*(m + 1))), Int[(f*x)^(m + n)*((a +
 b*Log[c*(d + e*x^n)^p])^(q - 1)/(d + e*x^n)), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && IGtQ[q, 1] && IntegerQ[n] && NeQ[m, -1]
```

Rule 2520

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[u*(x^(n - 1)/(d + e*x^n)), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 2526

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2637

```
Int[Log[v_]*Log[w_]*(u_), x_Symbol] :> With[{z = IntHide[u, x]}, Dist[Log[v
]*Log[w], z, x] + (-Int[SimplifyIntegrand[z*Log[w]*(D[v, x]/v), x], x] - In
t[SimplifyIntegrand[z*Log[v]*(D[w, x]/w), x], x]) /; InverseFunctionFreeQ[z
, x]] /; InverseFunctionFreeQ[v, x] && InverseFunctionFreeQ[w, x]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> I
nt[ExpandIntegrand[x^m*(a + (I*b*Log[1 - I*c*x^n])/2 - (I*b*Log[1 + I*c*x^n
])/2)^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Integ
erQ[m]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 4966

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e
*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x] + Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x]) /; FreeQ[{a,
b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5048

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

Rule 6055

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c
*(p/e), Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^2
)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2,
 0]
```

Rule 6057

```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> S
imp[(-(a + b*ArcTanh[c*x]))*(Log[2/(1 + c*x)]/e), x] + (Dist[b*(c/e), Int[L
og[2/(1 + c*x)]/(1 - c^2*x^2), x], x] - Dist[b*(c/e), Int[Log[2*c*((d + e*x
)/((c*d + e)*(1 + c*x)))]/(1 - c^2*x^2), x], x] + Simp[(a + b*ArcTanh[c*x])
*(Log[2*c*((d + e*x)/((c*d + e)*(1 + c*x)))]/e), x]) /; FreeQ[{a, b, c, d,
e}, x] && NeQ[c^2*d^2 - e^2, 0]
```

Rule 6131

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Dist[1/
(c*d), Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 6139

```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTanh[c*x], x^m/(d + e*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0]
)
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}{x^6}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{4x^6} + \frac{b(-2ia+b\log\left(1-icx^2\right))\log\left(1+icx^2\right)}{2x^6} - \frac{b^2 l}{}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{x^6}\,dx + \frac{1}{2}b\int \frac{(-2ia+b\log\left(1-icx^2\right))\log\left(1+icx^2\right)}{x^6}$$

$$= -\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{20x^5} + \frac{b^2\log^2\left(1+icx^2\right)}{20x^5} + \frac{1}{2}b\int\left(-\frac{2ia\log\left(1+icx^2\right)}{x^6} + \right.$$

$$= -\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{20x^5} + \frac{b^2\log^2\left(1+icx^2\right)}{20x^5} - (iab)\int \frac{\log\left(1+icx^2\right)}{x^6}\,dx + \frac{1}{2}$$

$$= -\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^2}{20x^5} + \frac{iab\log\left(1+icx^2\right)}{5x^5} - \frac{b^2\log\left(1-icx^2\right)\log\left(1+icx^2\right)}{10x^5}$$

$$= -\frac{2abc}{15x^3} - \frac{bc(2a+ib\log\left(1-icx^2\right))}{15x^3} - \frac{ibc^2(2a+ib\log\left(1-icx^2\right))}{5x} + \frac{1}{5}\sqrt[4]{-1}\,bc^5$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{4b^2c^2}{15x} - \frac{2}{5}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) + \frac{2}{5}(-1)^{3/4}b^2$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{4b^2c^2}{15x} - \frac{8}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) - \frac{1}{5}\sqrt[4]{-1}\,b^2$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{4b^2c^2}{15x} - \frac{8}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) - \frac{1}{5}\sqrt[4]{-1}\,b^2$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{8b^2c^2}{15x} - \frac{2}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) - \frac{1}{5}\sqrt[4]{-1}\,b^2$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{8b^2c^2}{15x} - \frac{4}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) - \frac{1}{5}\sqrt[4]{-1}\,b^2$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{8b^2c^2}{15x} - \frac{4}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) - \frac{1}{5}\sqrt[4]{-1}\,b^2$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{8b^2c^2}{15x} - \frac{4}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) - \frac{1}{5}\sqrt[4]{-1}\,b^2$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{8b^2c^2}{15x} - \frac{4}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) - \frac{1}{5}\sqrt[4]{-1}\,b^2$$

$$= -\frac{2abc}{15x^3} + \frac{2iabc^2}{5x} - \frac{8b^2c^2}{15x} - \frac{4}{15}(-1)^{3/4}b^2c^{5/2}\tan^{-1}\left((-1)^{3/4}\sqrt{c}\,x\right) - \frac{1}{5}\sqrt[4]{-1}\,b^2$$

Mathematica [F]

time = 2.03, size = 0, normalized size = 0.00

$$\int \frac{(a + b\text{ArcTan}(cx^2))^2}{x^6}\, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^2/x^6,x]
```

```
[Out] Integrate[(a + b*ArcTan[c*x^2])^2/x^6, x]
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(a + b\arctan(c\,x^2))^2}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^2/x^6,x)
```

```
[Out] int((a+b*arctan(c*x^2))^2/x^6,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^6,x, algorithm="maxima")
```

```
[Out] -1/30*((6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*c*x + sqrt(2)*sqrt(c))/sqrt
(c)) + 6*sqrt(2)*c^(3/2)*arctan(1/2*sqrt(2)*(2*c*x - sqrt(2)*sqrt(c))/sqrt(
c)) + 3*sqrt(2)*c^(3/2)*log(c*x^2 + sqrt(2)*sqrt(c)*x + 1) - 3*sqrt(2)*c^(3
/2)*log(c*x^2 - sqrt(2)*sqrt(c)*x + 1) + 8/x^3)*c + 12*arctan(c*x^2)/x^5)*a
*b + 1/80*(80*x^5*integrate(-1/80*(8*c^2*x^4*log(c^2*x^4 + 1) - 16*c*x^2*ar
ctan(c*x^2) - 60*(c^2*x^4 + 1)*arctan(c*x^2)^2 - 5*(c^2*x^4 + 1)*log(c^2*x^
4 + 1)^2)/(c^2*x^10 + x^6), x) - 4*arctan(c*x^2)^2 + log(c^2*x^4 + 1)^2)*b^
2/x^5 - 1/5*a^2/x^5
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^6,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)/x^6, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atan}\,(cx^2))^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**2/x**6,x)
[Out] Integral((a + b*atan(c*x**2))**2/x**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^2/x^6,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^2) + a)^2/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{atan}(c\,x^2))^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))^2/x^6,x)
[Out] int((a + b*atan(c*x^2))^2/x^6, x)
```

3.86 $\int x^3(a+b\textbf{ArcTan}(cx^2))^3\,dx$

Optimal. Leaf size=149

$$-\frac{3ib(a+b\text{ArcTan}(cx^2))^2}{4c^2}-\frac{3bx^2(a+b\text{ArcTan}(cx^2))^2}{4c}+\frac{(a+b\text{ArcTan}(cx^2))^3}{4c^2}+\frac{1}{4}x^4(a+b\text{ArcTan}(cx^2))^3-\frac{3b}{}$$

```
[Out] -3/4*I*b*(a+b*arctan(c*x^2))^2/c^2-3/4*b*x^2*(a+b*arctan(c*x^2))^2/c+1/4*(a
+b*arctan(c*x^2))^3/c^2+1/4*x^4*(a+b*arctan(c*x^2))^3-3/2*b^2*(a+b*arctan(c
*x^2))*ln(2/(1+I*c*x^2))/c^2-3/4*I*b^3*polylog(2,1-2/(1+I*c*x^2))/c^2
```

Rubi [A]
time = 0.21, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {4948, 4946, 5036, 4930, 5040, 4964, 2449, 2352, 5004}

$$-\frac{3b^2\log\left(\frac{2}{1+icx^2}\right)(a+b\text{ArcTan}(cx^2))}{2c^2}+\frac{(a+b\text{ArcTan}(cx^2))^3}{4c^2}-\frac{3ib(a+b\text{ArcTan}(cx^2))^2}{4c^2}-\frac{3bx^2(a+b\text{ArcTan}(cx^2))^2}{4c}+\frac{1}{4}x^4(a+b\text{ArcTan}(cx^2))^3-\frac{3ib^3\text{Li}_2\left(1-\frac{2}{icx^2+1}\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] (((-3*I)/4)*b*(a + b*ArcTan[c*x^2])^2)/c^2 - (3*b*x^2*(a + b*ArcTan[c*x^2])
^2)/(4*c) + (a + b*ArcTan[c*x^2])^3/(4*c^2) + (x^4*(a + b*ArcTan[c*x^2])^3)
/4 - (3*b^2*(a + b*ArcTan[c*x^2])*Log[2/(1 + I*c*x^2)])/(2*c^2) - (((3*I)/4
)*b^3*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c^2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(a+b\tan^{-1}\left(cx^2\right)\right)^3\,dx &= \int\left(\frac{1}{8}x^3\left(2a+ib\log\left(1-icx^2\right)\right)^3+\frac{3}{8}ibx^3\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log\left(1+\right.\right.\\
&= \frac{1}{8}\int x^3\left(2a+ib\log\left(1-icx^2\right)\right)^3\,dx+\frac{1}{8}(3ib)\int x^3\left(-2ia+b\log\left(1-icx^2\right)\right)^2\\
&= \frac{1}{16}\text{Subst}\left(\int x(2a+ib\log(1-icx))^3\,dx,x,x^2\right)+\frac{1}{16}(3ib)\text{Subst}\left(\int x(-2ia\right.\\
&= \frac{3}{32}ibx^4\left(2ia-b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)+\frac{3}{32}ib^2x^4\left(2ia-b\log\left(1-icx\right.\right.\\
&= \frac{3}{32}ibx^4\left(2ia-b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)+\frac{3}{32}ib^2x^4\left(2ia-b\log\left(1-icx\right.\right.\\
&= \frac{3}{32}ibx^4\left(2ia-b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)+\frac{3}{32}ib^2x^4\left(2ia-b\log\left(1-icx\right.\right.\\
&= \frac{\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^3}{16c^2}-\frac{\left(1-icx^2\right)^2\left(2a+ib\log\left(1-icx^2\right)\right)^3}{32c^2}+\\
&= \frac{3ib(1-icx^2)\left(2ia-b\log\left(1-icx^2\right)\right)^2}{32c^2}-\frac{3ib(1-icx^2)\left(2a+ib\log\left(1-icx^2\right)\right)}{16c^2}\\
&= \frac{9iab^2x^2}{8c}+\frac{9b^3x^2}{16c}-\frac{3ib^3(1-icx^2)^2}{128c^2}+\frac{3ib^3(1+icx^2)^2}{128c^2}+\frac{3ib(1-icx^2)\left(2ia-\right.}{32c^2}\\
&= \frac{9iab^2x^2}{8c}+\frac{9b^3x^2}{8c}-\frac{3ib^3(1-icx^2)^2}{128c^2}+\frac{3ib^3(1+icx^2)^2}{128c^2}-\frac{9ib^3(1-icx^2)\log\left(1\right.}{16c^2}\\
&= \frac{3iab^2x^2}{4c}+\frac{15b^3x^2}{16c}-\frac{9ib^3(1-icx^2)\log\left(1-icx^2\right)}{16c^2}+\frac{3ib^2(1-icx^2)^2\left(2ia-b\right.}{64c^2}\\
&= \frac{3iab^2x^2}{4c}+\frac{3b^3x^2}{4c}-\frac{3ib^3(1-icx^2)\log\left(1-icx^2\right)}{8c^2}+\frac{3ib^2(1-icx^2)^2\left(2ia-b\log\right.}{64c^2}
\end{aligned}
$$

Mathematica [A]

time = 0.30, size = 170, normalized size = 1.14

$$\frac{3b^2(a+ac^2x^4+b(i-cx^2))\,\text{ArcTan}(cx^2)^2+b^3(1+c^2x^4)\,\text{ArcTan}(cx^2)^3+3b\text{ArcTan}(cx^2)\left(a(a-2bcx^2+ac^2x^4)-2b^2\log\left(1+e^{2i\text{ArcTan}(cx^2)}\right)\right)+a(acx^2(-3b+acx^2)+3b^2\log(1+c^2x^4))+3ib^3\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx^2)}\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] (3*b^2*(a + a*c^2*x^4 + b*(I - c*x^2))*ArcTan[c*x^2]^2 + b^3*(1 + c^2*x^4)*
ArcTan[c*x^2]^3 + 3*b*ArcTan[c*x^2]*(a*(a - 2*b*c*x^2 + a*c^2*x^4) - 2*b^2*
Log[1 + E^((2*I)*ArcTan[c*x^2])]) + a*(a*c*x^2*(-3*b + a*c*x^2) + 3*b^2*Log
[1 + c^2*x^4]) + (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x^2])])/(4*c^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.47, size = 758, normalized size = 5.09

method	result
risch	$\frac{x^4 a^3}{4} + \frac{3ib^2 \left(\sum_{\_\alpha=\mathrm{RootOf}\left(c\_Z^2-\mathrm{RootOf}\left(\_Z^2+1, index=1\right)\right)} \frac{\left(\ln(x-\_\alpha)\ln\left(-icx^2+1\right)+2c\left(-\frac{\ln(x-\_\alpha)\left(\ln\left(\frac{\left(-\frac{1}{2}-\frac{i}{2}\right)\left(i\sqrt{\frac{i}{c}}-\sqrt{\frac{i}{c}}\right.}{\sqrt{\frac{i}{c}}}\right.\right.}{} \right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x^2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^4*a^3+3/16*I/c^2*b^3*ln(c^2*x^4+1)-3/4*I/c*b^2*a*x^2*ln(1-I*c*x^2)+3/
8*I*b*a^2*x^4*ln(1-I*c*x^2)-1/32*I*b^3/c^2*ln(1-I*c*x^2)^3+3/4*I/c*b^2*Sum(
(ln(x-_alpha)*ln(1-I*c*x^2)+2*c*(-1/2*ln(x-_alpha)*(ln((-1/2-1/2*I)*(I*(I/c
)^(1/2)-(I/c)^(1/2)-x+_alpha)/(I/c)^(1/2))+ln((1/2-1/2*I)*(I*(I/c)^(1/2)+(I
/c)^(1/2)+x-_alpha)/(I/c)^(1/2)))/c-1/2*(dilog((-1/2-1/2*I)*(I*(I/c)^(1/2)-
(I/c)^(1/2)-x+_alpha)/(I/c)^(1/2))+dilog((1/2-1/2*I)*(I*(I/c)^(1/2)+(I/c)^(
1/2)+x-_alpha)/(I/c)^(1/2)))/c))*b/c,_alpha=RootOf(c*_Z^2-RootOf(_Z^2+1,ind
ex=1)))+3/16*b^3/c*x^2*ln(1-I*c*x^2)^2-3/16*b^2*a*x^4*ln(1-I*c*x^2)^2+3/4/c
^2*b^2*a*ln(c^2*x^4+1)-3/16/c^2*b^2*a*ln(1-I*c*x^2)^2+3/16*I/c^2*b^3*ln(1-I
*c*x^2)^2-1/32*I*b^3*x^4*ln(1-I*c*x^2)^3+3/4*b*a^2/c^2*arctan(c*x^2)-3/32*b
^2*(I*b*c^2*x^4*ln(1-I*c*x^2)+2*a*c^2*x^4-2*b*c*x^2+I*b*ln(1-I*c*x^2)+2*I*b
+2*a)/c^2*ln(1+I*c*x^2)^2-3/4*b*a^2/c*x^2+1/32*I*b^3*(c^2*x^4+1)/c^2*ln(1+I
*c*x^2)^3-3/8*b^3/c^2*arctan(c*x^2)+(3/32*I*b^3*(c^2*x^4+1)/c^2*ln(1-I*c*x^
2)^2+3/32*b^2*(2*a*c*x^2-b)^2/a/c^2*ln(1-I*c*x^2)-3/32*b*(4*I*a^3*c^2*x^4-8
*I*a^2*b*c*x^2+4*I*ln(1-I*c*x^2)*a*b^2+4*I*a*b^2-4*ln(1-I*c*x^2)*a^2*b+ln(1
-I*c*x^2)*b^3)/a/c^2)*ln(1+I*c*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^3,x, algorithm="maxima")
```

```
[Out] 3/4*a*b^2*x^4*arctan(c*x^2)^2 + 1/4*a^3*x^4 + 3/4*(x^4*arctan(c*x^2) - c*(x
^2/c^2 - arctan(c*x^2)/c^3))*a^2*b - 3/4*(2*c*(x^2/c^2 - arctan(c*x^2)/c^3)
*arctan(c*x^2) + (arctan(c*x^2)^2 - log(4*c^5*x^4 + 4*c^3))/c^2)*a*b^2 + 1/
128*(4*x^4*arctan(c*x^2)^3 - 3*x^4*arctan(c*x^2)*log(c^2*x^4 + 1)^2 + 128*i
ntegrate(1/64*(12*c^2*x^7*arctan(c*x^2)*log(c^2*x^4 + 1) - 12*c*x^5*arctan(
c*x^2)^2 + 56*(c^2*x^7 + x^3)*arctan(c*x^2)^3 + 3*(c*x^5 + 2*(c^2*x^7 + x^3
)*arctan(c*x^2))*log(c^2*x^4 + 1)^2)/(c^2*x^4 + 1), x))*b^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctan(c*x^2)^3 + 3*a*b^2*x^3*arctan(c*x^2)^2 + 3*a^2*b*x^
3*arctan(c*x^2) + a^3*x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\left(a+b\operatorname{atan}\left(cx^2\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x**2))**3,x)
```

```
[Out] Integral(x**3*(a + b*atan(c*x**2))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^3*x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x^2))^3,x)
```

```
[Out] int(x^3*(a + b*atan(c*x^2))^3, x)
```

3.87 $\int x(a+b\mathbf{ArcTan}(cx^2))^3\,dx$

Optimal. Leaf size=144

$$\frac{i(a+b\text{ArcTan}(cx^2))^3}{2c}+\frac{1}{2}x^2(a+b\text{ArcTan}(cx^2))^3+\frac{3b(a+b\text{ArcTan}(cx^2))^2\log\left(\frac{2}{1+icx^2}\right)}{2c}+\frac{3ib^2(a+b\text{ArcTan}(c\cdots}{}$$

```
[Out] 1/2*I*(a+b*arctan(c*x^2))^3/c+1/2*x^2*(a+b*arctan(c*x^2))^3+3/2*b*(a+b*arct
an(c*x^2))^2*ln(2/(1+I*c*x^2))/c+3/2*I*b^2*(a+b*arctan(c*x^2))*polylog(2,1-
2/(1+I*c*x^2))/c+3/4*b^3*polylog(3,1-2/(1+I*c*x^2))/c
```

Rubi [A]
time = 0.18, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4948, 4930, 5040, 4964, 5004, 5114, 6745}

$$\frac{3ib^2\text{Li}_2\left(1-\frac{2}{icx^2+1}\right)(a+b\text{ArcTan}(cx^2))}{2c}+\frac{1}{2}x^2(a+b\text{ArcTan}(cx^2))^3+\frac{i(a+b\text{ArcTan}(cx^2))^3}{2c}+\frac{3b\log\left(\frac{2}{1+icx^2}\right)(a+b\text{ArcTan}(cx^2))^2}{2c}+\frac{3b^3\text{Li}_3\left(1-\frac{2}{icx^2+1}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] ((I/2)*(a + b*ArcTan[c*x^2])^3)/c + (x^2*(a + b*ArcTan[c*x^2])^3)/2 + (3*b*
(a + b*ArcTan[c*x^2])^2*Log[2/(1 + I*c*x^2)])/(2*c) + (((3*I)/2)*b^2*(a + b
*ArcTan[c*x^2])*PolyLog[2, 1 - 2/(1 + I*c*x^2)])/c + (3*b^3*PolyLog[3, 1 -
2/(1 + I*c*x^2)])/(4*c)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int x\left(a+b\tan^{-1}\left(cx^2\right)\right)^3\,dx = \int \left(\frac{1}{8}x\left(2a+ib\log\left(1-icx^2\right)\right)^3+\frac{3}{8}ibx\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log\left(1+icx\right.\right.$$

$$= \frac{1}{8}\int x\left(2a+ib\log\left(1-icx^2\right)\right)^3\,dx+\frac{1}{8}(3ib)\int x\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log$$

$$= \frac{1}{16}\text{Subst}\left(\int(2a+ib\log(1-icx))^3\,dx,x,x^2\right)+\frac{1}{16}(3ib)\text{Subst}\left(\int(-2ia+b\log\right.$$

$$= \frac{3}{16}ibx^2\left(2ia-b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)+\frac{3}{16}ib^2x^2\left(2ia-b\log\left(1-icx^2\right.\right.$$

$$= \frac{i(1-icx^2)\left(2a+ib\log\left(1-icx^2\right)\right)^3}{16c}+\frac{3}{16}ibx^2\left(2ia-b\log\left(1-icx^2\right)\right)^2\log\left(1+\right.$$

$$= \frac{3b(1-icx^2)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16c}+\frac{i(1-icx^2)\left(2a+ib\log\left(1-icx^2\right)\right)^3}{16c}+$$

$$= -\frac{3}{4}ab^2x^2-\frac{3}{8}ib^3x^2+\frac{3b(1-icx^2)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16c}+\frac{i(1-icx^2)\left(2a+\right.}{}$$

$$= -\frac{3}{4}ab^2x^2+\frac{3b^3(1-icx^2)\log\left(1-icx^2\right)}{8c}+\frac{3b(1-icx^2)\left(2ia-b\log\left(1-icx^2\right)\right)}{16c}$$

$$= \frac{3}{8}ib^3x^2+\frac{3b^3(1-icx^2)\log\left(1-icx^2\right)}{8c}+\frac{3b(1-icx^2)\left(2ia-b\log\left(1-icx^2\right)\right)^2}{16c}$$

$$= \frac{3b(1-icx^2)\left(2ia-b\log\left(1-icx^2\right)\right)^2}{16c}+\frac{3b(1-icx^2)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16c}$$

$$= \frac{3b(1-icx^2)\left(2ia-b\log\left(1-icx^2\right)\right)^2}{16c}+\frac{3b(1-icx^2)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{16c}$$

Mathematica [A]
time = 0.04, size = 224, normalized size = 1.56

$$\frac{2a^3cx^2+6a^2bcx^2\text{ArcTan}(cx^2)-6iab^2\text{ArcTan}(cx^2)^2+6ab^2cx^2\text{ArcTan}(cx^2)^2-2ib^3\text{ArcTan}(cx^2)^3+2b^3cx^2\text{ArcTan}(cx^2)^3+12ab^2\text{ArcTan}(cx^2)\log\left(1+e^{2i\text{ArcTan}(cx^2)}\right)+6b^3\text{ArcTan}(cx^2)^2\log\left(1+e^{2i\text{ArcTan}(cx^2)}\right)-3a^2b\log\left(1+c^2x^4\right)-6ib^2(a+b\text{ArcTan}(cx^2))\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx^2)}\right)+3b^3\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(cx^2)}\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] (2*a^3*c*x^2 + 6*a^2*b*c*x^2*ArcTan[c*x^2] - (6*I)*a*b^2*ArcTan[c*x^2]^2 +
6*a*b^2*c*x^2*ArcTan[c*x^2]^2 - (2*I)*b^3*ArcTan[c*x^2]^3 + 2*b^3*c*x^2*Arc
Tan[c*x^2]^3 + 12*a*b^2*ArcTan[c*x^2]*Log[1 + E^((2*I)*ArcTan[c*x^2])] + 6*
b^3*ArcTan[c*x^2]^2*Log[1 + E^((2*I)*ArcTan[c*x^2])] - 3*a^2*b*Log[1 + c^2*
```

```
x^4] - (6*I)*b^2*(a + b*ArcTan[c*x^2])*PolyLog[2, -E^((2*I)*ArcTan[c*x^2])]
 + 3*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x^2])])/(4*c)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(129) = 258$.
time = 0.41, size = 289, normalized size = 2.01

method	result
derivativedivides	$\frac{a^3 c x^2 - i b^3 \arctan(c x^2)^3 + b^3 \arctan(c x^2)^3 c x^2 + 3 b^3 \arctan(c x^2)^2 \ln\left(1 + \frac{(i c x^2+1)^2}{c^2 x^4+1}\right) - 3 i b^3 \arctan(c x^2) \operatorname{polylog}\left(2, -\right.}{}$
default	$\frac{a^3 c x^2 - i b^3 \arctan(c x^2)^3 + b^3 \arctan(c x^2)^3 c x^2 + 3 b^3 \arctan(c x^2)^2 \ln\left(1 + \frac{(i c x^2+1)^2}{c^2 x^4+1}\right) - 3 i b^3 \arctan(c x^2) \operatorname{polylog}\left(2, -\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x^2))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c*(a^3*c*x^2-I*b^3*arctan(c*x^2)^3+b^3*arctan(c*x^2)^3*c*x^2+3*b^3*arct
an(c*x^2)^2*ln(1+(1+I*c*x^2)^2/(c^2*x^4+1))-3*I*b^3*arctan(c*x^2)*polylog(2
,-(1+I*c*x^2)^2/(c^2*x^4+1))+3/2*b^3*polylog(3,-(1+I*c*x^2)^2/(c^2*x^4+1))+
3*a^2*b*c*x^2*arctan(c*x^2)-3/2*a^2*b*ln(c^2*x^4+1)-3*I*arctan(c*x^2)^2*a*b
^2+3*arctan(c*x^2)^2*a*b^2*c*x^2-3*I*polylog(2,-(1+I*c*x^2)^2/(c^2*x^4+1))*
a*b^2+6*arctan(c*x^2)*ln(1+(1+I*c*x^2)^2/(c^2*x^4+1))*a*b^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2))^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*x^2*arctan(c*x^2)^3 - 3/64*b^3*x^2*arctan(c*x^2)*log(c^2*x^4 + 1)^
2 + 7/64*b^3*arctan(c*x^2)^4/c + 28*b^3*c^2*integrate(1/32*x^5*arctan(c*x^2
)^3/(c^2*x^4 + 1), x) + 3*b^3*c^2*integrate(1/32*x^5*arctan(c*x^2)*log(c^2*
x^4 + 1)^2/(c^2*x^4 + 1), x) + 96*a*b^2*c^2*integrate(1/32*x^5*arctan(c*x^2
)^2/(c^2*x^4 + 1), x) + 12*b^3*c^2*integrate(1/32*x^5*arctan(c*x^2)*log(c^2
*x^4 + 1)/(c^2*x^4 + 1), x) + 1/2*a^3*x^2 + 1/2*a*b^2*arctan(c*x^2)^3/c - 1
2*b^3*c*integrate(1/32*x^3*arctan(c*x^2)^2/(c^2*x^4 + 1), x) + 3*b^3*c*inte
grate(1/32*x^3*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 3*b^3*integrate(1/32*
x*arctan(c*x^2)*log(c^2*x^4 + 1)^2/(c^2*x^4 + 1), x) + 3/4*(2*c*x^2*arctan(
c*x^2) - log(c^2*x^4 + 1))*a^2*b/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arctan(c*x^2)^3 + 3*a*b^2*x*arctan(c*x^2)^2 + 3*a^2*b*x*arct
an(c*x^2) + a^3*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + b\,\mathrm{atan}\,(cx^2))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x**2))**3,x)
```

```
[Out] Integral(x*(a + b*atan(c*x**2))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^3*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a + b\,\mathrm{atan}\left(c\,x^2\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x^2))^3,x)
```

```
[Out] int(x*(a + b*atan(c*x^2))^3, x)
```

3.88 $\int \frac{\left(a+b\mathbf{ArcTan}(cx^2)\right)^3}{x}\,dx$

Optimal. Leaf size=229

$$\left(a+b\text{ArcTan}(cx^2)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)-\frac{3}{4}ib\left(a+b\text{ArcTan}(cx^2)\right)^2\text{PolyLog}\left(2,1-\frac{2}{1+icx^2}\right)+\frac{3}{4}ib(a$$

```
[Out] -(a+b*arctan(c*x^2))^3*arctanh(-1+2/(1+I*c*x^2))-3/4*I*b*(a+b*arctan(c*x^2)
)^2*polylog(2,1-2/(1+I*c*x^2))+3/4*I*b*(a+b*arctan(c*x^2))^2*polylog(2,-1+2
/(1+I*c*x^2))-3/4*b^2*(a+b*arctan(c*x^2))*polylog(3,1-2/(1+I*c*x^2))+3/4*b^
2*(a+b*arctan(c*x^2))*polylog(3,-1+2/(1+I*c*x^2))+3/8*I*b^3*polylog(4,1-2/(
1+I*c*x^2))-3/8*I*b^3*polylog(4,-1+2/(1+I*c*x^2))
```

Rubi [A]

time = 0.34, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {4944, 4942, 5108, 5004, 5114, 5118, 6745}

$$-\frac{3}{4}b^2\text{Li}_3\left(1-\frac{2}{icx^2+1}\right)\left(a+b\text{ArcTan}(cx^2)\right)+\frac{3}{4}b^2\text{Li}_3\left(\frac{2}{icx^2+1}-1\right)\left(a+b\text{ArcTan}(cx^2)\right)-\frac{3}{4}ib\text{Li}_2\left(1-\frac{2}{icx^2+1}\right)\left(a+b\text{ArcTan}(cx^2)\right)^2+\frac{3}{4}ib\text{Li}_2\left(\frac{2}{icx^2+1}-1\right)\left(a+b\text{ArcTan}(cx^2)\right)^2+\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right)\left(a+b\text{ArcTan}(cx^2)\right)^3+\frac{3}{8}ib^3\text{Li}_4\left(1-\frac{2}{icx^2+1}\right)-\frac{3}{8}ib^3\text{Li}_4\left(\frac{2}{icx^2+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^3/x,x]
```

```
[Out] (a + b*ArcTan[c*x^2])^3*ArcTanh[1 - 2/(1 + I*c*x^2)] - ((3*I)/4)*b*(a + b*A
rcTan[c*x^2])^2*PolyLog[2, 1 - 2/(1 + I*c*x^2)] + ((3*I)/4)*b*(a + b*ArcTan
[c*x^2])^2*PolyLog[2, -1 + 2/(1 + I*c*x^2)] - (3*b^2*(a + b*ArcTan[c*x^2])*
PolyLog[3, 1 - 2/(1 + I*c*x^2)])/4 + (3*b^2*(a + b*ArcTan[c*x^2])*PolyLog[3
, -1 + 2/(1 + I*c*x^2)])/4 + ((3*I)/8)*b^3*PolyLog[4, 1 - 2/(1 + I*c*x^2)]
- ((3*I)/8)*b^3*PolyLog[4, -1 + 2/(1 + I*c*x^2)]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\tan^{-1}(cx^2))^3}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a+b\tan^{-1}(cx))^3}{x}\,dx, x, x^2\right)\\
&= \left(a+b\tan^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right) - (3bc)\text{Subst}\left(\int \frac{(a+b\tan^{-1}(cx\\
&= \left(a+b\tan^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right) + \frac{1}{2}(3bc)\text{Subst}\left(\int \frac{(a+b\tan^{-1}(\\
&= \left(a+b\tan^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right) - \frac{3}{4}ib\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\text{Li}_2\left(1\\
&= \left(a+b\tan^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right) - \frac{3}{4}ib\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\text{Li}_2\left(1\\
&= \left(a+b\tan^{-1}\left(cx^2\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^2}\right) - \frac{3}{4}ib\left(a+b\tan^{-1}\left(cx^2\right)\right)^2\text{Li}_2\left(1
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 245, normalized size = 1.07

$$
(a+b\text{ArcTan}(cx^2))^3\tanh^{-1}\left(1+\frac{2i}{-i+cx^2}\right)+\frac{3}{8}ib\left(2(a+b\text{ArcTan}(cx^2))^2\text{PolyLog}\left(2,\frac{i+cx^2}{i-cx^2}\right)-2(a+b\text{ArcTan}(cx^2))^2\text{PolyLog}\left(2,\frac{i+cx^2}{-i+cx^2}\right)+b\left(-2i(a+b\text{ArcTan}(cx^2))\text{PolyLog}\left(3,\frac{i+cx^2}{i-cx^2}\right)+2i(a+b\text{ArcTan}(cx^2))\text{PolyLog}\left(3,\frac{i+cx^2}{-i+cx^2}\right)+b\left(-\text{PolyLog}\left(4,\frac{i+cx^2}{i-cx^2}\right)+\text{PolyLog}\left(4,\frac{i+cx^2}{-i+cx^2}\right)\right)\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^3/x,x]
```

```
[Out] (a + b*ArcTan[c*x^2])^3*ArcTanh[1 + (2*I)/(-I + c*x^2)] + ((3*I)/8)*b*(2*(a
 + b*ArcTan[c*x^2])^2*PolyLog[2, (I + c*x^2)/(I - c*x^2)] - 2*(a + b*ArcTan
[c*x^2])^2*PolyLog[2, (I + c*x^2)/(-I + c*x^2)] + b*((-2*I)*(a + b*ArcTan[c
*x^2])*PolyLog[3, (I + c*x^2)/(I - c*x^2)] + (2*I)*(a + b*ArcTan[c*x^2])*Po
lyLog[3, (I + c*x^2)/(-I + c*x^2)] + b*(-PolyLog[4, (I + c*x^2)/(I - c*x^2)
] + PolyLog[4, (I + c*x^2)/(-I + c*x^2)])))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$
\int \frac{\left(a+b\arctan\left(c\,x^2\right)\right)^3}{x}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^3/x,x)
```

```
[Out] int((a+b*arctan(c*x^2))^3/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + 1/32*integrate((28*b^3*arctan(c*x^2)^3 + 3*b^3*arctan(c*x^2)*l
og(c^2*x^4 + 1)^2 + 96*a*b^2*arctan(c*x^2)^2 + 96*a^2*b*arctan(c*x^2))/x, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c*
x^2) + a^3)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}{\left(c x^{2} \right)}\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**3/x,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)}^3}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))^3/x,x)
```

```
[Out] int((a + b*atan(c*x^2))^3/x, x)
```

3.89 $\int \frac{\left(a+b\text{ArcTan}\left(cx^2\right)\right)^3}{x^3} \, dx$

Optimal. Leaf size=138

$$-\frac{1}{2}ic\left(a+b\text{ArcTan}\left(cx^2\right)\right)^3-\frac{\left(a+b\text{ArcTan}\left(cx^2\right)\right)^3}{2x^2}+\frac{3}{2}bc\left(a+b\text{ArcTan}\left(cx^2\right)\right)^2\log\left(2-\frac{2}{1-icx^2}\right)-\frac{3}{2}ib^2c(a+$$

```
[Out] -1/2*I*c*(a+b*arctan(c*x^2))^3-1/2*(a+b*arctan(c*x^2))^3/x^2+3/2*b*c*(a+b*a
rctan(c*x^2))^2*ln(2-2/(1-I*c*x^2))-3/2*I*b^2*c*(a+b*arctan(c*x^2))*polylog
(2,-1+2/(1-I*c*x^2))+3/4*b^3*c*polylog(3,-1+2/(1-I*c*x^2))
```

Rubi [A]
time = 0.22, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {4948, 4946, 5044, 4988, 5004, 5112, 6745}

$$-\frac{3}{2}ib^2c\text{Li}_2\left(\frac{2}{1-icx^2}-1\right)\left(a+b\text{ArcTan}\left(cx^2\right)\right)-\frac{1}{2}ic\left(a+b\text{ArcTan}\left(cx^2\right)\right)^3-\frac{\left(a+b\text{ArcTan}\left(cx^2\right)\right)^3}{2x^2}+\frac{3}{2}bc\log\left(2-\frac{2}{1-icx^2}\right)\left(a+b\text{ArcTan}\left(cx^2\right)\right)^2+\frac{3}{4}b^3c\text{Li}_3\left(\frac{2}{1-icx^2}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^3/x^3,x]
```

```
[Out] (-1/2*I)*c*(a + b*ArcTan[c*x^2])^3 - (a + b*ArcTan[c*x^2])^3/(2*x^2) + (3*b
*c*(a + b*ArcTan[c*x^2])^2*Log[2 - 2/(1 - I*c*x^2)])/2 - ((3*I)/2)*b^2*c*(a
 + b*ArcTan[c*x^2])*PolyLog[2, -1 + 2/(1 - I*c*x^2)] + (3*b^3*c*PolyLog[3,
-1 + 2/(1 - I*c*x^2)])/4
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
```

```
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx^2))^3}{x^3}\,dx = \int \left(\frac{(2a+ib\log(1-icx^2))^3}{8x^3} + \frac{3ib(-2ia+b\log(1-icx^2))^2\log(1+icx^2)}{8x^3} - \frac{3i}{}\right.$$

$$= \frac{1}{8}\int \frac{(2a+ib\log(1-icx^2))^3}{x^3}\,dx + \frac{1}{8}(3ib)\int \frac{(-2ia+b\log(1-icx^2))^2\log(1+}{x^3}$$

$$= \frac{1}{16}\text{Subst}\left(\int \frac{(2a+ib\log(1-icx))^3}{x^2}\,dx, x, x^2\right) + \frac{1}{16}(3ib)\text{Subst}\left(\int \frac{(-2ia+b\log}{}\right.$$

$$= -\frac{(1-icx^2)(2a+ib\log(1-icx^2))^3}{16x^2} - \frac{ib^3(1+icx^2)\log^3(1+icx^2)}{16x^2} + \frac{1}{16}(3ib)\text{S}$$

$$= \frac{3}{16}bc\log(icx^2)(2a+ib\log(1-icx^2))^2 - \frac{(1-icx^2)(2a+ib\log(1-icx^2))^3}{16x^2} -$$

$$= \frac{3}{16}bc\log(icx^2)(2a+ib\log(1-icx^2))^2 - \frac{(1-icx^2)(2a+ib\log(1-icx^2))^3}{16x^2} -$$

$$= \frac{3}{16}bc\log(icx^2)(2a+ib\log(1-icx^2))^2 - \frac{(1-icx^2)(2a+ib\log(1-icx^2))^3}{16x^2} -$$

$$= \frac{3}{16}bc\log(icx^2)(2a+ib\log(1-icx^2))^2 - \frac{(1-icx^2)(2a+ib\log(1-icx^2))^3}{16x^2} -$$

Mathematica [A]
time = 0.29, size = 239, normalized size = 1.73

$$\frac{1}{4}\left(-\frac{2a^3}{x^2} - \frac{6a^2b\text{ArcTan}(cx^2)}{x^2} + 12a^2bc\log(x) - 3a^2bc\log(1+c^2x^4) + 6ab^2c\left(\text{ArcTan}(cx^2)\left(\left(-i-\frac{1}{cx^2}\right)\text{ArcTan}(cx^2) + 2\log\left(1-e^{2i\text{ArcTan}(cx^2)}\right)\right) - i\text{PolyLog}\left(2, e^{2i\text{ArcTan}(cx^2)}\right)\right) + 2b^3c\left(-\frac{i\pi^3}{8} + i\text{ArcTan}(cx^2)^3 - \frac{\text{ArcTan}(cx^2)^3}{cx^2} + 3\text{ArcTan}(cx^2)^2\log\left(1-e^{-2i\text{ArcTan}(cx^2)}\right) + 3i\text{ArcTan}(cx^2)\text{PolyLog}\left(2, e^{-2i\text{ArcTan}(cx^2)}\right) + \frac{3}{2}\text{PolyLog}\left(3, e^{-2i\text{ArcTan}(cx^2)}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^3/x^3,x]
```

```
[Out] ((-2*a^3)/x^2 - (6*a^2*b*ArcTan[c*x^2])/x^2 + 12*a^2*b*c*Log[x] - 3*a^2*b*c
*Log[1 + c^2*x^4] + 6*a*b^2*c*(ArcTan[c*x^2]*((-I - 1/(c*x^2))*ArcTan[c*x^2
] + 2*Log[1 - E^((2*I)*ArcTan[c*x^2])]) - I*PolyLog[2, E^((2*I)*ArcTan[c*x^
2])]) + 2*b^3*c*((-1/8*I)*Pi^3 + I*ArcTan[c*x^2]^3 - ArcTan[c*x^2]^3/(c*x^2
) + 3*ArcTan[c*x^2]^2*Log[1 - E^((-2*I)*ArcTan[c*x^2])] + (3*I)*ArcTan[c*x^
2]*PolyLog[2, E^((-2*I)*ArcTan[c*x^2])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c*
x^2])])/2))/4
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\arctan(c\,x^2))^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^3/x^3,x)
```

```
[Out] int((a+b*arctan(c*x^2))^3/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x^3,x, algorithm="maxima")
```

```
[Out] -3/4*(c*(log(c^2*x^4 + 1) - log(x^4)) + 2*arctan(c*x^2)/x^2)*a^2*b - 1/2*a^
3/x^2 - 1/64*(4*b^3*arctan(c*x^2)^3 - 3*b^3*arctan(c*x^2)*log(c^2*x^4 + 1)^
2 - 64*x^2*integrate(-1/32*(12*b^3*c^2*x^4*arctan(c*x^2)*log(c^2*x^4 + 1) -
 28*(b^3*c^2*x^4 + b^3)*arctan(c*x^2)^3 - 12*(8*a*b^2*c^2*x^4 + b^3*c*x^2 +
 8*a*b^2)*arctan(c*x^2)^2 + 3*(b^3*c*x^2 - (b^3*c^2*x^4 + b^3)*arctan(c*x^2
))*log(c^2*x^4 + 1)^2)/(c^2*x^7 + x^3), x))/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c*
x^2) + a^3)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}{\left(c x^{2} \right)}\right)^{3}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**3/x**3,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**3/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^3/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))^3/x^3,x)
```

```
[Out] int((a + b*atan(c*x^2))^3/x^3, x)
```

3.90 $\int \frac{\left(a+b\mathbf{ArcTan}(cx^2)\right)^3}{x^5}\,dx$

Optimal. Leaf size=149

$$-\frac{3}{4}ibc^2\left(a+b\text{ArcTan}\left(cx^2\right)\right)^2-\frac{3bc\left(a+b\text{ArcTan}\left(cx^2\right)\right)^2}{4x^2}-\frac{1}{4}c^2\left(a+b\text{ArcTan}\left(cx^2\right)\right)^3-\frac{\left(a+b\text{ArcTan}\left(cx^2\right)\right)^3}{4x^4}+$$

```
[Out] -3/4*I*b*c^2*(a+b*arctan(c*x^2))^2-3/4*b*c*(a+b*arctan(c*x^2))^2/x^2-1/4*c^
2*(a+b*arctan(c*x^2))^3-1/4*(a+b*arctan(c*x^2))^3/x^4+3/2*b^2*c^2*(a+b*arct
an(c*x^2))*ln(2-2/(1-I*c*x^2))-3/4*I*b^3*c^2*polylog(2,-1+2/(1-I*c*x^2))
```

Rubi [A]
time = 0.23, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {4948, 4946, 5038, 5044, 4988, 2497, 5004}

$$\frac{3}{2}b^2c^2\log\left(2-\frac{2}{1-icx^2}\right)\left(a+b\text{ArcTan}\left(cx^2\right)\right)-\frac{3}{4}ibc^2\left(a+b\text{ArcTan}\left(cx^2\right)\right)^2-\frac{1}{4}c^2\left(a+b\text{ArcTan}\left(cx^2\right)\right)^3-\frac{3bc\left(a+b\text{ArcTan}\left(cx^2\right)\right)^2}{4x^2}-\frac{\left(a+b\text{ArcTan}\left(cx^2\right)\right)^3}{4x^4}-\frac{3}{4}ib^3c^2\text{Li}_2\left(\frac{2}{1-icx^2}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^2])^3/x^5,x]
```

```
[Out] ((-3*I)/4)*b*c^2*(a + b*ArcTan[c*x^2])^2 - (3*b*c*(a + b*ArcTan[c*x^2])^2)/
(4*x^2) - (c^2*(a + b*ArcTan[c*x^2])^3)/4 - (a + b*ArcTan[c*x^2])^3/(4*x^4)
 + (3*b^2*c^2*(a + b*ArcTan[c*x^2])*Log[2 - 2/(1 - I*c*x^2)])/2 - ((3*I)/4)
*b^3*c^2*PolyLog[2, -1 + 2/(1 - I*c*x^2)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
```

```
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^2\right)\right)^3}{x^5}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{8x^5}+\frac{3ib\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log\left(1+icx^2\right)}{8x^5}-\right.$$

$$=\frac{1}{8}\int \frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{x^5}\,dx+\frac{1}{8}(3ib)\int \frac{\left(-2ia+b\log\left(1-icx^2\right)\right)^2\log\left(1+\right.}{x^5}$$

$$=\frac{1}{16}\mathrm{Subst}\left(\int \frac{\left(2a+ib\log(1-icx)\right)^3}{x^3}\,dx,x,x^2\right)+\frac{1}{16}(3ib)\mathrm{Subst}\left(\int \frac{\left(-2ia+b\right.}{}\right.$$

$$=-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{32x^4}-\frac{ib^3\log^3\left(1+icx^2\right)}{32x^4}+\frac{1}{16}(3ib)\mathrm{Subst}\left(\int \frac{\left(-2ia+b\right.}{}\right.$$

$$=-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{32x^4}-\frac{ib^3\log^3\left(1+icx^2\right)}{32x^4}+\frac{1}{32}(3ib)\mathrm{Subst}\left(\int \frac{\left(2a+ib\,\mathrm{lo}\right.}{x\left(-\frac{i}{c}+\right.}\right.$$

$$=-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{32x^4}-\frac{ib^3\log^3\left(1+icx^2\right)}{32x^4}+\frac{1}{32}(3ib)\mathrm{Subst}\left(\int \frac{\left(2a+ib\,\mathrm{lo}\right.}{\left(-\frac{i}{c}+\right.}\right.$$

$$=-\frac{3bc\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{32x^2}-\frac{\left(2a+ib\log\left(1-icx^2\right)\right)^3}{32x^4}+\frac{3b^3c(1+}{}$$

$$=\frac{3}{4}ab^2c^2\log(x)-\frac{3bc\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{32x^2}+\frac{3}{32}ibc^2\log\left(icx^2\right)(2a$$

$$=\frac{3}{4}ab^2c^2\log(x)-\frac{3bc\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{32x^2}+\frac{3}{32}ibc^2\log\left(icx^2\right)(2a$$

$$=\frac{3}{4}ab^2c^2\log(x)-\frac{3bc\left(1-icx^2\right)\left(2a+ib\log\left(1-icx^2\right)\right)^2}{32x^2}+\frac{3}{32}ibc^2\log\left(icx^2\right)(2a$$

Mathematica [**A**]
time = 0.22, size = 196, normalized size = 1.32

$$-\frac{3b^2\left(a+ac^2x^4+bcx^2(1+icx^2)\right)\mathrm{ArcTan}(cx^2)^2+b^3(1+c^2x^4)\mathrm{ArcTan}(cx^2)^3+3b\mathrm{ArcTan}(cx^2)\left(a(a+2bcx^2+ac^2x^4)-2b^2c^2x^4\log\left(1-e^{2i\mathrm{ArcTan}(cx^2)}\right)\right)+a\left(a(a+3bcx^2)-6b^2c^2x^4\log\left(\frac{cx^2}{\sqrt{1+c^2x^4}}\right)\right)+3ib^3c^2x^4\mathrm{PolyLog}\left(2,e^{2i\mathrm{ArcTan}(cx^2)}\right)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^2])^3/x^5,x]
```

```
[Out] -1/4*(3*b^2*(a + a*c^2*x^4 + b*c*x^2*(1 + I*c*x^2))*ArcTan[c*x^2]^2 + b^3*(
1 + c^2*x^4)*ArcTan[c*x^2]^3 + 3*b*ArcTan[c*x^2]*(a*(a + 2*b*c*x^2 + a*c^2*
x^4) - 2*b^2*c^2*x^4*Log[1 - E^((2*I)*ArcTan[c*x^2])]) + a*(a*(a + 3*b*c*x^
2) - 6*b^2*c^2*x^4*Log[(c*x^2)/Sqrt[1 + c^2*x^4]]) + (3*I)*b^3*c^2*x^4*Poly
Log[2, E^((2*I)*ArcTan[c*x^2])])/x^4
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\arctan\left(c\,x^2\right)\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^2))^3/x^5,x)
```

```
[Out] int((a+b*arctan(c*x^2))^3/x^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x^5,x, algorithm="maxima")
```

```
[Out] -3/4*((c*arctan(c*x^2) + 1/x^2)*c + arctan(c*x^2)/x^4)*a^2*b + 3/4*((arctan
(c*x^2)^2 - log(c^2*x^4 + 1) + 4*log(x))*c^2 - 2*(c*arctan(c*x^2) + 1/x^2)*
c*arctan(c*x^2))*a*b^2 - 3/4*a*b^2*arctan(c*x^2)^2/x^4 + 1/128*(128*x^4*int
egrate(-1/64*(12*c^2*x^4*arctan(c*x^2)*log(c^2*x^4 + 1) - 12*c*x^2*arctan(c
*x^2)^2 - 56*(c^2*x^4 + 1)*arctan(c*x^2)^3 + 3*(c*x^2 - 2*(c^2*x^4 + 1)*arc
tan(c*x^2))*log(c^2*x^4 + 1)^2)/(c^2*x^9 + x^5), x) - 4*arctan(c*x^2)^3 + 3
*arctan(c*x^2)*log(c^2*x^4 + 1)^2)*b^3/x^4 - 1/4*a^3/x^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x^5,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c*
x^2) + a^3)/x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}{\left(c x^{2} \right)}\right)^{3}}{x^{5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**2))**3/x**5,x)
```

```
[Out] Integral((a + b*atan(c*x**2))**3/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^2))^3/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^3/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atan}(c\,x^2))^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^2))^3/x^5,x)
```

```
[Out] int((a + b*atan(c*x^2))^3/x^5, x)
```

3.91 $\int (dx)^m \left(a+b\mathbf{ArcTan}(cx^2)\right)^3 \, dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m \left(a+b\text{ArcTan}\left(cx^2\right)\right)^3,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x^2))^3,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\text{ArcTan}\left(cx^2\right)\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x^2])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^3 \, dx = \int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^3 \, dx$$

Mathematica [A]
time = 1.27, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\text{ArcTan}\left(cx^2\right)\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^3,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^3, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\arctan\left(c\,x^2\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x^2))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x^2))^3,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^3,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/32*(4*b^3*d^m*x*x^m*arctan(c*x^2)^3 - 3*b
^3*d^m*x*x^m*arctan(c*x^2)*log(c^2*x^4 + 1)^2 + 32*(m + 1)*integrate(1/32*(
24*b^3*c^2*d^m*x^4*x^m*arctan(c*x^2)*log(c^2*x^4 + 1) + 28*(b^3*d^m*m + (b^
3*c^2*d^m*m + b^3*c^2*d^m)*x^4 + b^3*d^m)*x^m*arctan(c*x^2)^3 - 24*(b^3*c*d
^m*x^2 - 4*a*b^2*d^m*m - 4*(a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^4 - 4*a*b^2*
d^m)*x^m*arctan(c*x^2)^2 + 96*(a^2*b*d^m*m + (a^2*b*c^2*d^m*m + a^2*b*c^2*d
^m)*x^4 + a^2*b*d^m)*x^m*arctan(c*x^2) + 3*(2*b^3*c*d^m*x^2*x^m + (b^3*d^m*
m + (b^3*c^2*d^m*m + b^3*c^2*d^m)*x^4 + b^3*d^m)*x^m*arctan(c*x^2))*log(c^2
*x^4 + 1)^2)/((c^2*m + c^2)*x^4 + m + 1), x))/(m + 1)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^2)^3 + 3*a*b^2*arctan(c*x^2)^2 + 3*a^2*b*arctan(c*
x^2) + a^3)*(d*x)^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x**2))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^3*(d*x)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atan(c*x^2))^3,x)
```

```
[Out] int((d*x)^m*(a + b*atan(c*x^2))^3, x)
```

3.92 $\int (dx)^m \left(a+b\textbf{ArcTan}(cx^2)\right)^2 \, dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m \left(a+b\text{ArcTan}\left(cx^2\right)\right)^2,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x^2))^2,x)
```

Rubi [A]

time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\text{ArcTan}\left(cx^2\right)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^2])^2,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x^2])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^2 \, dx = \int (dx)^m \left(a+b\tan^{-1}\left(cx^2\right)\right)^2 \, dx$$

Mathematica [A]

time = 0.83, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\text{ArcTan}\left(cx^2\right)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x^2])^2, x]
```

Maple [A]

time = 0.01, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\arctan\left(c\,x^2\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x^2))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x^2))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/16*(4*b^2*d^m*x*x^m*arctan(c*x^2)^2 - b^2
*d^m*x*x^m*log(c^2*x^4 + 1)^2 + 16*(m + 1)*integrate(1/16*(8*b^2*c^2*d^m*x^
4*x^m*log(c^2*x^4 + 1) + 12*((b^2*c^2*d^m*m + b^2*c^2*d^m)*x^4 + b^2*d^m*m
+ b^2*d^m)*x^m*arctan(c*x^2)^2 + ((b^2*c^2*d^m*m + b^2*c^2*d^m)*x^4 + b^2*d
^m*m + b^2*d^m)*x^m*log(c^2*x^4 + 1)^2 - 16*(b^2*c*d^m*x^2 - 2*(a*b*c^2*d^m
*m + a*b*c^2*d^m)*x^4 - 2*a*b*d^m*m - 2*a*b*d^m)*x^m*arctan(c*x^2))/((c^2*m
 + c^2)*x^4 + m + 1), x))/(m + 1)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2)*(d*x)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a + b \operatorname{atan}\left(cx^2\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x**2))**2,x)
```

```
[Out] Integral((d*x)**m*(a + b*atan(c*x**2))**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)^2*(d*x)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,{\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atan(c*x^2))^2,x)
```

```
[Out] int((d*x)^m*(a + b*atan(c*x^2))^2, x)
```

3.93 $\int (dx)^m \left(a+b\mathbf{ArcTan}(cx^2)\right) dx$

Optimal. Leaf size=75

$$\frac{(dx)^{1+m}\left(a+b\text{ArcTan}(cx^2)\right)}{d(1+m)}-\frac{2bc(dx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{4};\frac{7+m}{4};-c^2x^4\right)}{d^3(1+m)(3+m)}$$

```
[Out] (d*x)^(1+m)*(a+b*arctan(c*x^2))/d/(1+m)-2*b*c*(d*x)^(3+m)*hypergeom([1, 3/4
+1/4*m],[7/4+1/4*m],-c^2*x^4)/d^3/(1+m)/(3+m)
```

Rubi [A]

time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4958, 371}

$$\frac{(dx)^{m+1}\left(a+b\text{ArcTan}(cx^2)\right)}{d(m+1)}-\frac{2bc(dx)^{m+3}\,{}_2F_1\left(1,\frac{m+3}{4};\frac{m+7}{4};-c^2x^4\right)}{d^3(m+1)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTan[c*x^2]))/(d*(1 + m)) - (2*b*c*(d*x)^(3 + m)*Hy
pergeometric2F1[1, (3 + m)/4, (7 + m)/4, -(c^2*x^4)])/(d^3*(1 + m)*(3 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :>
 Simp[(d*x)^(m + 1)*((a + b*ArcTan[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n/(
d^n*(m + 1))), Int[(d*x)^(m + n)/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{a, b,
c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (dx)^m\left(a+b\tan^{-1}\left(cx^2\right)\right)dx &= \frac{(dx)^{1+m}\left(a+b\tan^{-1}\left(cx^2\right)\right)}{d(1+m)}-\frac{(2bc)\int\frac{x(dx)^{1+m}}{1+c^2x^4}\,dx}{d(1+m)}\\ &= \frac{(dx)^{1+m}\left(a+b\tan^{-1}\left(cx^2\right)\right)}{d(1+m)}-\frac{(2bc)\int\frac{(dx)^{2+m}}{1+c^2x^4}\,dx}{d^2(1+m)}\\ &= \frac{(dx)^{1+m}\left(a+b\tan^{-1}\left(cx^2\right)\right)}{d(1+m)}-\frac{2bc(dx)^{3+m}\,{}_2F_1\left(1,\frac{3+m}{4};\frac{7+m}{4};-c^2x^4\right)}{d^3(1+m)(3+m)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 65, normalized size = 0.87

$$-\frac{x(dx)^m\left(-((3+m)\left(a+b\text{ArcTan}(cx^2)\right))+2bcx^2\,{}_2F_1\left(1,\frac{3+m}{4};\frac{7+m}{4};-c^2x^4\right)\right)}{(1+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^2]),x]
```

```
[Out] -((x*(d*x)^m*(-((3 + m)*(a + b*ArcTan[c*x^2])) + 2*b*c*x^2*Hypergeometric2F
1[1, (3 + m)/4, (7 + m)/4, -(c^2*x^4)]))/((1 + m)*(3 + m)))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (dx)^m\left(a+b\arctan\left(c\,x^2\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x^2)),x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x^2)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2)),x, algorithm="maxima")
```

```
[Out] (d^m*x*x^m*arctan(c*x^2) - 2*(c*d^m*m + c*d^m)*integrate(x^2*x^m/((c^2*m +
c^2)*x^4 + m + 1), x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2)),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x^2) + a)*(d*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m\left(a+b\,\text{atan}\left(cx^2\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x**2)),x)
```

```
[Out] Integral((d*x)**m*(a + b*atan(c*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^2)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^2) + a)*(d*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atan(c*x^2)),x)
```

```
[Out] int((d*x)^m*(a + b*atan(c*x^2)), x)
```

3.94 $\int \frac{(dx)^m}{a+b\mathbf{ArcTan}(cx^2)}\,dx$

Optimal. Leaf size=21

$$\operatorname{Int}\left(\frac{(dx)^m}{a+b\operatorname{ArcTan}(cx^2)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctan(c*x^2)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(dx)^m}{a+b\operatorname{ArcTan}(cx^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTan[c*x^2]),x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTan[c*x^2]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tan^{-1}(cx^2)}\,dx = \int \frac{(dx)^m}{a+b\tan^{-1}(cx^2)}\,dx$$

Mathematica [A]
time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\operatorname{ArcTan}(cx^2)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTan[c*x^2]),x]
[Out] Integrate[(d*x)^m/(a + b*ArcTan[c*x^2]), x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\arctan(c\,x^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctan(c*x^2)),x)
[Out] int((d*x)^m/(a+b*arctan(c*x^2)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arctan(c*x^2) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arctan(c*x^2) + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atan(c*x**2)),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2)),x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arctan(c*x^2) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{a+b\,\mathrm{atan}\left(c\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atan(c*x^2)),x)
[Out] int((d*x)^m/(a + b*atan(c*x^2)), x)
```

3.95 $\int \frac{(dx)^m}{\left(a+b\mathbf{ArcTan}\left(cx^2\right)\right)^2}\,dx$

Optimal. Leaf size=21

$$\mathrm{Int}\left(\frac{(dx)^m}{\left(a+b\mathrm{ArcTan}\left(cx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctan(c*x^2))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\mathrm{ArcTan}\left(cx^2\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTan[c*x^2])^2,x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTan[c*x^2])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^2\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\mathrm{ArcTan}\left(cx^2\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTan[c*x^2])^2,x]
[Out] Integrate[(d*x)^m/(a + b*ArcTan[c*x^2])^2, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\arctan\left(c\,x^2\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctan(c*x^2))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arctan(c*x^2))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2))^2,x, algorithm="maxima")
```

```
[Out] -1/2*((c^2*d^m*x^4 + d^m)*x^m - 2*(b^2*c*x*arctan(c*x^2) + a*b*c*x)*integra
te(1/2*((c^2*d^m*m + 3*c^2*d^m)*x^4 + d^m*m - d^m)*x^m/(b^2*c*x^2*arctan(c*
x^2) + a*b*c*x^2), x))/(b^2*c*x*arctan(c*x^2) + a*b*c*x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*arctan(c*x^2)^2 + 2*a*b*arctan(c*x^2) + a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atan(c*x**2))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^2))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctan(c*x^2) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{\left(a+b\,\mathrm{atan}\left(c\,x^2\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atan(c*x^2))^2,x)
[Out] int((d*x)^m/(a + b*atan(c*x^2))^2, x)
```

3.96 $\int x^{11}(a+b\mathbf{ArcTan}(cx^3))\,dx$

Optimal. Leaf size=54

$$\frac{bx^3}{12c^3}-\frac{bx^9}{36c}-\frac{b\text{ArcTan}(cx^3)}{12c^4}+\frac{1}{12}x^{12}\left(a+b\text{ArcTan}\left(cx^3\right)\right)$$

```
[Out] 1/12*b*x^3/c^3-1/36*b*x^9/c-1/12*b*arctan(c*x^3)/c^4+1/12*x^12*(a+b*arctan(
c*x^3))
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {4946, 281, 308, 209}

$$\frac{1}{12}x^{12}\left(a+b\text{ArcTan}\left(cx^3\right)\right)-\frac{b\text{ArcTan}(cx^3)}{12c^4}+\frac{bx^3}{12c^3}-\frac{bx^9}{36c}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (b*x^3)/(12*c^3) - (b*x^9)/(36*c) - (b*ArcTan[c*x^3])/(12*c^4) + (x^12*(a +
 b*ArcTan[c*x^3]))/12
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
```

```
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^{11}\left(a+b\tan^{-1}\left(cx^3\right)\right)\,dx &= \frac{1}{12}x^{12}\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{4}(bc)\int\frac{x^{14}}{1+c^2x^6}\,dx\\
&= \frac{1}{12}x^{12}\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{12}(bc)\text{Subst}\left(\int\frac{x^4}{1+c^2x^2}\,dx,x,x^3\right)\\
&= \frac{1}{12}x^{12}\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{12}(bc)\text{Subst}\left(\int\left(-\frac{1}{c^4}+\frac{x^2}{c^2}+\frac{1}{c^4\left(1+c^2x^2\right)}\right)\right)\\
&= \frac{bx^3}{12c^3}-\frac{bx^9}{36c}+\frac{1}{12}x^{12}\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{b\text{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,x^3\right)}{12c^3}\\
&= \frac{bx^3}{12c^3}-\frac{bx^9}{36c}-\frac{b\tan^{-1}\left(cx^3\right)}{12c^4}+\frac{1}{12}x^{12}\left(a+b\tan^{-1}\left(cx^3\right)\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 1.09

$$\frac{bx^3}{12c^3}-\frac{bx^9}{36c}+\frac{ax^{12}}{12}-\frac{b\text{ArcTan}(cx^3)}{12c^4}+\frac{1}{12}bx^{12}\text{ArcTan}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*ArcTan[c*x^3]),x]
[Out] (b*x^3)/(12*c^3) - (b*x^9)/(36*c) + (a*x^12)/12 - (b*ArcTan[c*x^3])/(12*c^4
) + (b*x^12*ArcTan[c*x^3])/12
```

Maple [A]
time = 0.05, size = 50, normalized size = 0.93

method	result	size
default	$\frac{x^{12}a}{12}+\frac{b\,x^{12}\arctan\left(c\,x^3\right)}{12}-\frac{b\,x^9}{36c}+\frac{b\,x^3}{12c^3}-\frac{b\arctan\left(c\,x^3\right)}{12c^4}$	50
risch	$-\frac{ix^{12}b\ln\left(ic\,x^3+1\right)}{24}+\frac{ix^{12}b\ln\left(-ic\,x^3+1\right)}{24}+\frac{x^{12}a}{12}-\frac{b\,x^9}{36c}+\frac{b\,x^3}{12c^3}-\frac{b\arctan\left(c\,x^3\right)}{12c^4}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
[Out] 1/12*x^12*a+1/12*b*x^12*arctan(c*x^3)-1/36*b*x^9/c+1/12*b*x^3/c^3-1/12*b*ar
ctan(c*x^3)/c^4
```

Maxima [A]
time = 0.49, size = 54, normalized size = 1.00

$$\frac{1}{12}\,ax^{12}+\frac{1}{36}\,\left(3\,x^{12}\arctan\left(cx^{3}\right)-c\left(\frac{c^{2}x^{9}-3\,x^{3}}{c^{4}}+\frac{3\,\arctan\left(cx^{3}\right)}{c^{5}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/12*a*x^12 + 1/36*(3*x^12*arctan(c*x^3) - c*((c^2*x^9 - 3*x^3)/c^4 + 3*arc
tan(c*x^3)/c^5))*b
```

Fricas [A]
time = 1.47, size = 51, normalized size = 0.94

$$\frac{3\,ac^{4}x^{12}-bc^{3}x^{9}+3\,bcx^{3}+3\,\left(bc^{4}x^{12}-b\right)\arctan\left(cx^{3}\right)}{36\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/36*(3*a*c^4*x^12 - b*c^3*x^9 + 3*b*c*x^3 + 3*(b*c^4*x^12 - b)*arctan(c*x^
3))/c^4
```

Sympy [A]
time = 125.82, size = 58, normalized size = 1.07

$$\begin{cases}\frac{ax^{12}}{12}+\frac{bx^{12}\operatorname{atan}\left(cx^{3}\right)}{12}-\frac{bx^{9}}{36c}+\frac{bx^{3}}{12c^{3}}-\frac{b\operatorname{atan}\left(cx^{3}\right)}{12c^{4}} & \text{for}\: c\neq 0\\\frac{ax^{12}}{12} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**12/12 + b*x**12*atan(c*x**3)/12 - b*x**9/(36*c) + b*x**3/(1
2*c**3) - b*atan(c*x**3)/(12*c**4), Ne(c, 0)), (a*x**12/12, True))
```

Giac [A]
time = 0.42, size = 60, normalized size = 1.11

$$\frac{3\,acx^{12}+\left(3\,cx^{12}\arctan\left(cx^{3}\right)-\frac{3\,\arctan\left(cx^{3}\right)}{c^{3}}-\frac{c^{9}x^{9}-3\,c^{7}x^{3}}{c^{9}}\right)b}{36\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/36*(3*a*c*x^12 + (3*c*x^12*arctan(c*x^3) - 3*arctan(c*x^3)/c^3 - (c^9*x^9
 - 3*c^7*x^3)/c^9)*b)/c
```

Mupad [B]
time = 0.45, size = 49, normalized size = 0.91

$$\frac{a\,x^{12}}{12}+\frac{b\,x^3}{12\,c^3}-\frac{b\,x^9}{36\,c}-\frac{b\,\mathrm{atan}(c\,x^3)}{12\,c^4}+\frac{b\,x^{12}\,\mathrm{atan}(c\,x^3)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*atan(c*x^3)),x)
```

```
[Out] (a*x^12)/12 + (b*x^3)/(12*c^3) - (b*x^9)/(36*c) - (b*atan(c*x^3))/(12*c^4)
+ (b*x^12*atan(c*x^3))/12
```

3.97 $\int x^8(a+b\mathbf{ArcTan}(cx^3))\,dx$

Optimal. Leaf size=47

$$-\frac{bx^6}{18c}+\frac{1}{9}x^9\left(a+b\text{ArcTan}\left(cx^3\right)\right)+\frac{b\log\left(1+c^2x^6\right)}{18c^3}$$

```
[Out] -1/18*b*x^6/c+1/9*x^9*(a+b*arctan(c*x^3))+1/18*b*ln(c^2*x^6+1)/c^3
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4946, 272, 45}

$$\frac{1}{9}x^9\left(a+b\text{ArcTan}\left(cx^3\right)\right)+\frac{b\log\left(c^2x^6+1\right)}{18c^3}-\frac{bx^6}{18c}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -1/18*(b*x^6)/c + (x^9*(a + b*ArcTan[c*x^3]))/9 + (b*Log[1 + c^2*x^6])/(18*
c^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)\,dx=\frac{1}{9}x^9\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{3}(bc)\int\frac{x^{11}}{1+c^2x^6}\,dx$$
$$=\frac{1}{9}x^9\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{18}(bc)\text{Subst}\left(\int\frac{x}{1+c^2x}\,dx,x,x^6\right)$$
$$=\frac{1}{9}x^9\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{18}(bc)\text{Subst}\left(\int\left(\frac{1}{c^2}-\frac{1}{c^2\left(1+c^2x\right)}\right)\,dx,x,x^6\right)$$
$$=-\frac{bx^6}{18c}+\frac{1}{9}x^9\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1+c^2x^6\right)}{18c^3}$$

Mathematica [A]
time = 0.01, size = 52, normalized size = 1.11

$$-\frac{bx^6}{18c}+\frac{ax^9}{9}+\frac{1}{9}bx^9\text{ArcTan}\left(cx^3\right)+\frac{b\log\left(1+c^2x^6\right)}{18c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -1/18*(b*x^6)/c + (a*x^9)/9 + (b*x^9*ArcTan[c*x^3])/9 + (b*Log[1 + c^2*x^6]
)/(18*c^3)
```

Maple [A]
time = 0.04, size = 45, normalized size = 0.96

method	result	size
default	$\frac{x^9 a}{9}+\frac{b\,x^9\arctan\left(c\,x^3\right)}{9}-\frac{b\,x^6}{18c}+\frac{b\ln\left(c^2x^6+1\right)}{18c^3}$	45
risch	$-\frac{ix^9b\ln\left(ic\,x^3+1\right)}{18}+\frac{ix^9b\ln\left(-ic\,x^3+1\right)}{18}+\frac{x^9a}{9}-\frac{b\,x^6}{18c}+\frac{b\ln\left(-c^2x^6-1\right)}{18c^3}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*x^9*a+1/9*b*x^9*arctan(c*x^3)-1/18*b*x^6/c+1/18*b*ln(c^2*x^6+1)/c^3
```

Maxima [A]
time = 0.26, size = 48, normalized size = 1.02

$$\frac{1}{9}\,ax^9+\frac{1}{18}\left(2\,x^9\arctan\left(cx^3\right)-\left(\frac{x^6}{c^2}-\frac{\log\left(c^2x^6+1\right)}{c^4}\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/9*a*x^9 + 1/18*(2*x^9*arctan(c*x^3) - (x^6/c^2 - log(c^2*x^6 + 1)/c^4)*c)
*b
```

Fricas [A]
time = 1.08, size = 51, normalized size = 1.09

$$\frac{2\,bc^3x^9\arctan\left(cx^3\right)+2\,ac^3x^9-bc^2x^6+b\log\left(c^2x^6+1\right)}{18\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/18*(2*b*c^3*x^9*arctan(c*x^3) + 2*a*c^3*x^9 - b*c^2*x^6 + b*log(c^2*x^6 +
 1))/c^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(39) = 78$.
time = 69.18, size = 117, normalized size = 2.49

$$\begin{cases} \frac{ax^9}{9}+\frac{bx^9\operatorname{atan}\left(cx^3\right)}{9}-\frac{bx^6}{18c}-\frac{b\sqrt{-\frac{1}{c^2}}\operatorname{atan}\left(cx^3\right)}{9c^2}+\frac{b\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{9c^3}+\frac{b\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{9c^3} & \text{for}\ c\neq 0 \\ \frac{ax^9}{9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**9/9 + b*x**9*atan(c*x**3)/9 - b*x**6/(18*c) - b*sqrt(-1/c**
2)*atan(c*x**3)/(9*c**2) + b*log(x - (-1/c**2)**(1/6))/(9*c**3) + b*log(4*x
**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(9*c**3), Ne(c, 0)), (a*x*
*9/9, True))
```

Giac [A]
time = 0.44, size = 47, normalized size = 1.00

$$\frac{2\,acx^9+\left(2\,cx^9\arctan\left(cx^3\right)-x^6+\frac{\log\left(c^2x^6+1\right)}{c^2}\right)b}{18\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/18*(2*a*c*x^9 + (2*c*x^9*arctan(c*x^3) - x^6 + log(c^2*x^6 + 1)/c^2)*b)/c
```

Mupad [B]
time = 0.38, size = 44, normalized size = 0.94

$$\frac{a\,x^9}{9}+\frac{b\,\ln\left(c^2\,x^6+1\right)}{18\,c^3}-\frac{b\,x^6}{18\,c}+\frac{b\,x^9\,\mathrm{atan}\left(c\,x^3\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*atan(c*x^3)),x)
```

```
[Out] (a*x^9)/9 + (b*log(c^2*x^6 + 1))/(18*c^3) - (b*x^6)/(18*c) + (b*x^9*atan(c*
x^3))/9
```

3.98 $\int x^5(a+b\mathbf{ArcTan}(cx^3))\,dx$

Optimal. Leaf size=43

$$-\frac{bx^3}{6c}+\frac{b\text{ArcTan}(cx^3)}{6c^2}+\frac{1}{6}x^6\left(a+b\text{ArcTan}\left(cx^3\right)\right)$$

```
[Out] -1/6*b*x^3/c+1/6*b*arctan(c*x^3)/c^2+1/6*x^6*(a+b*arctan(c*x^3))
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {4946, 281, 327, 209}

$$\frac{1}{6}x^6\left(a+b\text{ArcTan}\left(cx^3\right)\right)+\frac{b\text{ArcTan}(cx^3)}{6c^2}-\frac{bx^3}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -1/6*(b*x^3)/c + (b*ArcTan[c*x^3])/(6*c^2) + (x^6*(a + b*ArcTan[c*x^3]))/6
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
```

```
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)\,dx &= \frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{2}(bc)\int\frac{x^8}{1+c^2x^6}\,dx\\ &= \frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{6}(bc)\text{Subst}\left(\int\frac{x^2}{1+c^2x^2}\,dx,x,x^3\right)\\ &= -\frac{bx^3}{6c}+\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\text{Subst}\left(\int\frac{1}{1+c^2x^2}\,dx,x,x^3\right)}{6c}\\ &= -\frac{bx^3}{6c}+\frac{b\tan^{-1}\left(cx^3\right)}{6c^2}+\frac{1}{6}x^6\left(a+b\tan^{-1}\left(cx^3\right)\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 1.12

$$-\frac{bx^3}{6c}+\frac{ax^6}{6}+\frac{b\text{ArcTan}\left(cx^3\right)}{6c^2}+\frac{1}{6}bx^6\text{ArcTan}\left(cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x^3]),x]
[Out] -1/6*(b*x^3)/c + (a*x^6)/6 + (b*ArcTan[c*x^3])/(6*c^2) + (b*x^6*ArcTan[c*x^
3])/6
```

Maple [A]
time = 0.06, size = 41, normalized size = 0.95

method	result	size
default	$\frac{a\,x^6}{6}+\frac{b\,x^6\arctan\left(c\,x^3\right)}{6}-\frac{b\,x^3}{6c}+\frac{b\arctan\left(c\,x^3\right)}{6c^2}$	41
risch	$-\frac{ix^6b\ln\left(ic\,x^3+1\right)}{12}+\frac{ix^6b\ln\left(-ic\,x^3+1\right)}{12}+\frac{a\,x^6}{6}-\frac{b\,x^3}{6c}+\frac{b\arctan\left(c\,x^3\right)}{6c^2}+\frac{b^2}{24a\,c^2}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
[Out] 1/6*a*x^6+1/6*b*x^6*arctan(c*x^3)-1/6*b*x^3/c+1/6*b*arctan(c*x^3)/c^2
```

Maxima [A]
time = 0.47, size = 43, normalized size = 1.00

$$\frac{1}{6}\,ax^6+\frac{1}{6}\left(x^6\arctan\left(cx^3\right)-c\left(\frac{x^3}{c^2}-\frac{\arctan\left(cx^3\right)}{c^3}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/6*(x^6*arctan(c*x^3) - c*(x^3/c^2 - arctan(c*x^3)/c^3))*b
```

Fricas [A]
time = 1.41, size = 38, normalized size = 0.88

$$\frac{ac^2x^6 - bcx^3 + (bc^2x^6 + b)\arctan(cx^3)}{6c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/6*(a*c^2*x^6 - b*c*x^3 + (b*c^2*x^6 + b)*arctan(c*x^3))/c^2
```

Sympy [A]
time = 35.58, size = 48, normalized size = 1.12

$$\begin{cases} \frac{ax^6}{6} + \frac{bx^6 \operatorname{atan}(cx^3)}{6} - \frac{bx^3}{6c} + \frac{b \operatorname{atan}(cx^3)}{6c^2} & \text{for } c \neq 0 \\ \frac{ax^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**6/6 + b*x**6*atan(c*x**3)/6 - b*x**3/(6*c) + b*atan(c*x**3)
/(6*c**2), Ne(c, 0)), (a*x**6/6, True))
```

Giac [A]
time = 0.44, size = 43, normalized size = 1.00

$$\frac{acx^6 + \frac{(c^2x^6\arctan(cx^3) - cx^3 + \arctan(cx^3))b}{c}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/6*(a*c*x^6 + (c^2*x^6*arctan(c*x^3) - c*x^3 + arctan(c*x^3))*b/c)/c
```

Mupad [B]
time = 0.40, size = 40, normalized size = 0.93

$$\frac{a\,x^6}{6} - \frac{b\,x^3}{6\,c} + \frac{b\,\mathrm{atan}(c\,x^3)}{6\,c^2} + \frac{b\,x^6\,\mathrm{atan}(c\,x^3)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atan(c*x^3)),x)
```

```
[Out] (a*x^6)/6 - (b*x^3)/(6*c) + (b*atan(c*x^3))/(6*c^2) + (b*x^6*atan(c*x^3))/6
```

3.99 $\int x^2(a+b\mathbf{ArcTan}(cx^3))\,dx$

Optimal. Leaf size=36

$$\frac{1}{3}x^3\left(a+b\text{ArcTan}\left(cx^3\right)\right)-\frac{b\log\left(1+c^2x^6\right)}{6c}$$

```
[Out] 1/3*x^3*(a+b*arctan(c*x^3))-1/6*b*ln(c^2*x^6+1)/c
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4946, 266}

$$\frac{1}{3}x^3\left(a+b\text{ArcTan}\left(cx^3\right)\right)-\frac{b\log\left(c^2x^6+1\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (x^3*(a + b*ArcTan[c*x^3]))/3 - (b*Log[1 + c^2*x^6])/(6*c)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)\,dx&=\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^3\right)\right)-(bc)\int\frac{x^5}{1+c^2x^6}\,dx\\&=\frac{1}{3}x^3\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{b\log\left(1+c^2x^6\right)}{6c}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.14

$$\frac{ax^3}{3}+\frac{1}{3}bx^3\text{ArcTan}\left(cx^3\right)-\frac{b\log\left(1+c^2x^6\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x^3]),x]
[Out] (a*x^3)/3 + (b*x^3*ArcTan[c*x^3])/3 - (b*Log[1 + c^2*x^6])/(6*c)
```

Maple [A]
time = 0.03, size = 38, normalized size = 1.06

method	result	size
derivativedivides	$\frac{ac\,x^3+bc\,x^3\arctan(c\,x^3)-\frac{b\ln\left(c^2x^6+1\right)}{2}}{3c}$	38
default	$\frac{ac\,x^3+bc\,x^3\arctan(c\,x^3)-\frac{b\ln\left(c^2x^6+1\right)}{2}}{3c}$	38
risch	$-\frac{ix^3b\ln(ic\,x^3+1)}{6}+\frac{ib\,x^3\ln(-ic\,x^3+1)}{6}+\frac{a\,x^3}{3}-\frac{b\ln(-c^2x^6-1)}{6c}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
[Out] 1/3/c*(a*c*x^3+b*c*x^3*arctan(c*x^3)-1/2*b*ln(c^2*x^6+1))
```

Maxima [A]
time = 0.26, size = 38, normalized size = 1.06

$$\frac{1}{3}\,ax^3+\frac{(2\,cx^3\arctan\left(cx^3\right)-\log\left(c^2x^6+1\right))b}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3)),x, algorithm="maxima")
[Out] 1/3*a*x^3 + 1/6*(2*c*x^3*arctan(c*x^3) - log(c^2*x^6 + 1))*b/c
```

Fricas [A]
time = 1.29, size = 39, normalized size = 1.08

$$\frac{2\,bcx^3\arctan\left(cx^3\right)+2\,acx^3-b\log\left(c^2x^6+1\right)}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3)),x, algorithm="fricas")
[Out] 1/6*(2*b*c*x^3*arctan(c*x^3) + 2*a*c*x^3 - b*log(c^2*x^6 + 1))/c
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(29) = 58$.
time = 19.45, size = 102, normalized size = 2.83

$$\begin{cases} \frac{ax^3}{3}+\frac{bx^3\operatorname{atan}\left(cx^3\right)}{3}+\frac{b\sqrt{-\frac{1}{c^2}}\operatorname{atan}\left(cx^3\right)}{3}-\frac{b\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{3c}-\frac{b\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{3c} & \text{for}\ c\neq 0 \\ \frac{ax^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**3/3 + b*x**3*atan(c*x**3)/3 + b*sqrt(-1/c**2)*atan(c*x**3)/
3 - b*log(x - (-1/c**2)**(1/6))/(3*c) - b*log(4*x**2 + 4*x*(-1/c**2)**(1/6)
 + 4*(-1/c**2)**(1/3))/(3*c), Ne(c, 0)), (a*x**3/3, True))
```

Giac [**A**]
time = 0.40, size = 40, normalized size = 1.11

$$\frac{2\,acx^3 + (2\,cx^3\arctan\left(cx^3\right) - \log\left(c^2x^6+1\right))b}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/6*(2*a*c*x^3 + (2*c*x^3*arctan(c*x^3) - log(c^2*x^6 + 1))*b)/c
```

Mupad [**B**]
time = 0.10, size = 35, normalized size = 0.97

$$\frac{a\,x^3}{3} - \frac{b\,\ln\left(c^2\,x^6+1\right)}{6\,c} + \frac{b\,x^3\,\mathrm{atan}(c\,x^3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x^3)),x)
```

```
[Out] (a*x^3)/3 - (b*log(c^2*x^6 + 1))/(6*c) + (b*x^3*atan(c*x^3))/3
```

3.100 $\int \frac{a+b\text{ArcTan}(cx^3)}{x}\,dx$

Optimal. Leaf size=39

$$a\log(x)+\frac{1}{6}ib\text{PolyLog}(2,-icx^3)-\frac{1}{6}ib\text{PolyLog}(2,icx^3)$$

```
[Out] a*ln(x)+1/6*I*b*polylog(2,-I*c*x^3)-1/6*I*b*polylog(2,I*c*x^3)
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.214$, Rules used = {4944, 4940, 2438}

$$a\log(x)+\frac{1}{6}ib\text{Li}_2(-icx^3)-\frac{1}{6}ib\text{Li}_2(icx^3)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x,x]
```

```
[Out] a*Log[x] + (I/6)*b*PolyLog[2, (-I)*c*x^3] - (I/6)*b*PolyLog[2, I*c*x^3]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}(cx^3)}{x}\,dx&=\frac{1}{3}\text{Subst}\left(\int \frac{a+b\tan^{-1}(cx)}{x}\,dx,x,x^3\right)\\&=a\log(x)+\frac{1}{6}(ib)\text{Subst}\left(\int \frac{\log(1-icx)}{x}\,dx,x,x^3\right)-\frac{1}{6}(ib)\text{Subst}\left(\int \frac{\log(1+icx)}{x}\,d\right.\\&=a\log(x)+\frac{1}{6}ib\text{Li}_2(-icx^3)-\frac{1}{6}ib\text{Li}_2(icx^3)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.00

$$a\log(x)+\frac{1}{6}ib\text{PolyLog}\left(2,-icx^3\right)-\frac{1}{6}ib\text{PolyLog}\left(2,icx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x,x]
[Out] a*Log[x] + (I/6)*b*PolyLog[2, (-I)*c*x^3] - (I/6)*b*PolyLog[2, I*c*x^3]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.04, size = 63, normalized size = 1.62

method	result
default	$a\ln(x)+b\ln(x)\arctan(c\,x^3)-\frac{b\left(\sum_{\_R1=\text{RootOf}\left(c^2\_Z^6+1\right)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^3}\right)}{2c}$
risch	$-\frac{i\left(\sum_{\_R1=\text{RootOf}\left(c\_Z^3+\text{RootOf}\left(\_Z^2+1,index=1\right)\right)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)b}{2}+\frac{i\ln(x)\ln\left(-icx^3+1\right)b}{2}+a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x,x,method=_RETURNVERBOSE)
[Out] a*ln(x)+b*ln(x)*arctan(c*x^3)-1/2*b/c*sum(1/_R1^3*(ln(x)*ln((_R1-x)/_R1)+di
log((_R1-x)/_R1)),_R1=RootOf(_Z^6*c^2+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x,x, algorithm="maxima")
[Out] b*integrate(arctan(c*x^3)/x, x) + a*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x^3) + a)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{atan}{\left(cx^{3} \right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x,x)
[Out] Integral((a + b*atan(c*x**3))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)/x, x)
```

Mupad [B]
time = 0.35, size = 32, normalized size = 0.82

$$a\,\ln\left(x\right)-\frac{b\,\left(\mathrm{Li_2}\left(1-c\,x^3\,1{}\mathrm{i}\right)-\mathrm{Li_2}\left(1{}\mathrm{i}\,c\,x^3+1\right)\right)\,1{}\mathrm{i}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))/x,x)
[Out] a*log(x) - (b*(dilog(1 - c*x^3*1i) - dilog(c*x^3*1i + 1))*1i)/6
```

3.101 $\int \frac{a+b\text{ArcTan}(cx^3)}{x^4}\,dx$

Optimal. Leaf size=39

$$-\frac{a+b\text{ArcTan}(cx^3)}{3x^3}+bc\log(x)-\frac{1}{6}bc\log\left(1+c^2x^6\right)$$

```
[Out] 1/3*(-a-b*arctan(c*x^3))/x^3+b*c*ln(x)-1/6*b*c*ln(c^2*x^6+1)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4946, 272, 36, 29, 31}

$$-\frac{a+b\text{ArcTan}(cx^3)}{3x^3}-\frac{1}{6}bc\log\left(c^2x^6+1\right)+bc\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x^4,x]
```

```
[Out] -1/3*(a + b*ArcTan[c*x^3])/x^3 + b*c*Log[x] - (b*c*Log[1 + c^2*x^6])/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}(cx^3)}{x^4}\,dx &= -\frac{a+b\tan^{-1}(cx^3)}{3x^3}+(bc)\int \frac{1}{x(1+c^2x^6)}\,dx\\ &= -\frac{a+b\tan^{-1}(cx^3)}{3x^3}+\frac{1}{6}(bc)\text{Subst}\left(\int \frac{1}{x(1+c^2x)}\,dx,x,x^6\right)\\ &= -\frac{a+b\tan^{-1}(cx^3)}{3x^3}+\frac{1}{6}(bc)\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^6\right)-\frac{1}{6}(bc^3)\,\text{Subst}\left(\int \frac{1}{1+c^2x}\,dx,\right.\\ &= -\frac{a+b\tan^{-1}(cx^3)}{3x^3}+bc\log(x)-\frac{1}{6}bc\log\left(1+c^2x^6\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 1.13

$$-\frac{a}{3x^3}-\frac{b\text{ArcTan}(cx^3)}{3x^3}+bc\log(x)-\frac{1}{6}bc\log\left(1+c^2x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^4,x]
[Out] -1/3*a/x^3 - (b*ArcTan[c*x^3])/(3*x^3) + b*c*Log[x] - (b*c*Log[1 + c^2*x^6]
)/6
```

Maple [A]
time = 0.04, size = 39, normalized size = 1.00

method	result	size
default	$-\frac{a}{3x^3}-\frac{b\arctan(c\,x^3)}{3x^3}+bc\ln(x)-\frac{bc\ln(c^2x^6+1)}{6}$	39
risch	$\frac{ib\ln(ic\,x^3+1)}{6x^3}-\frac{-6bc\ln(x)x^3+bc\ln(-c^2x^6-1)x^3+ib\ln(-ic\,x^3+1)+2a}{6x^3}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^4,x,method=_RETURNVERBOSE)
[Out] -1/3*a/x^3-1/3*b/x^3*arctan(c*x^3)+b*c*ln(x)-1/6*b*c*ln(c^2*x^6+1)
```

Maxima [A]
time = 0.27, size = 41, normalized size = 1.05

$$-\frac{1}{6}\left(c\left(\log\left(c^2x^6+1\right)-\log\left(x^6\right)\right)+\frac{2\,\arctan\left(cx^3\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(c*(log(c^2*x^6 + 1) - log(x^6)) + 2*arctan(c*x^3)/x^3)*b - 1/3*a/x^3
```

Fricas [A]
time = 1.07, size = 43, normalized size = 1.10

$$-\frac{bcx^3\log\left(c^2x^6+1\right)-6\,bcx^3\log\left(x\right)+2\,b\arctan\left(cx^3\right)+2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(b*c*x^3*log(c^2*x^6 + 1) - 6*b*c*x^3*log(x) + 2*b*arctan(c*x^3) + 2*a
)/x^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(39) = 78.
time = 38.42, size = 110, normalized size = 2.82

$$\begin{cases} -\frac{a}{3x^3}+bc\log\left(x\right)-\frac{bc\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{3}-\frac{bc\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{3}-\frac{b\operatorname{atan}\left(cx^3\right)}{3\sqrt{-\frac{1}{c^2}}}-\frac{b\operatorname{atan}\left(cx^3\right)}{3x^3} & \text{for}\ c\neq 0\\ -\frac{a}{3x^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3) + b*c*log(x) - b*c*log(x - (-1/c**2)**(1/6))/3 - b*c
*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/3 - b*atan(c*x**3)
/(3*sqrt(-1/c**2)) - b*atan(c*x**3)/(3*x**3), Ne(c, 0)), (-a/(3*x**3), True
))
```

Giac [A]
time = 0.40, size = 60, normalized size = 1.54

$$-\frac{bc^3x^3\log\left(c^2x^6+1\right)-2\,bc^3x^3\log\left(cx^3\right)+2\,bc^2\arctan\left(cx^3\right)+2\,ac^2}{6\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(b*c^3*x^3*log(c^2*x^6 + 1) - 2*b*c^3*x^3*log(c*x^3) + 2*b*c^2*arctan(
c*x^3) + 2*a*c^2)/(c^2*x^3)
```

Mupad [B]
time = 0.38, size = 38, normalized size = 0.97

$$b\,c\,\ln\left(x\right)-\frac{a}{3\,x^3}-\frac{b\,\mathrm{atan}(c\,x^3)}{3\,x^3}-\frac{b\,c\,\ln\left(c^2\,x^6+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))/x^4,x)
[Out] b*c*log(x) - a/(3*x^3) - (b*atan(c*x^3))/(3*x^3) - (b*c*log(c^2*x^6 + 1))/6
```

3.102 $\int \frac{a+b\text{ArcTan}(cx^3)}{x^7}\,dx$

Optimal. Leaf size=41

$$-\frac{bc}{6x^3}-\frac{1}{6}bc^2\text{ArcTan}(cx^3)-\frac{a+b\text{ArcTan}(cx^3)}{6x^6}$$

```
[Out] -1/6*b*c/x^3-1/6*b*c^2*arctan(c*x^3)+1/6*(-a-b*arctan(c*x^3))/x^6
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4946, 281, 331, 209}

$$-\frac{a+b\text{ArcTan}(cx^3)}{6x^6}-\frac{1}{6}bc^2\text{ArcTan}(cx^3)-\frac{bc}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x^7,x]
```

```
[Out] -1/6*(b*c)/x^3 - (b*c^2*ArcTan[c*x^3])/6 - (a + b*ArcTan[c*x^3])/(6*x^6)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
```

```
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}(cx^3)}{x^7}\,dx &= -\frac{a+b\tan^{-1}(cx^3)}{6x^6}+\frac{1}{2}(bc)\int \frac{1}{x^4(1+c^2x^6)}\,dx\\ &= -\frac{a+b\tan^{-1}(cx^3)}{6x^6}+\frac{1}{6}(bc)\text{Subst}\left(\int \frac{1}{x^2(1+c^2x^2)}\,dx,x,x^3\right)\\ &= -\frac{bc}{6x^3}-\frac{a+b\tan^{-1}(cx^3)}{6x^6}-\frac{1}{6}\left(bc^3\right)\text{Subst}\left(\int \frac{1}{1+c^2x^2}\,dx,x,x^3\right)\\ &= -\frac{bc}{6x^3}-\frac{1}{6}bc^2\tan^{-1}\left(cx^3\right)-\frac{a+b\tan^{-1}(cx^3)}{6x^6}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 48, normalized size = 1.17

$$-\frac{a}{6x^6}-\frac{b\text{ArcTan}(cx^3)}{6x^6}-\frac{bc\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-c^2x^6\right)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^7,x]
[Out] -1/6*a/x^6 - (b*ArcTan[c*x^3])/(6*x^6) - (b*c*Hypergeometric2F1[-1/2, 1, 1/
2, -(c^2*x^6)])/(6*x^3)
```

Maple [A]
time = 0.06, size = 39, normalized size = 0.95

method	result	size
default	$-\frac{a}{6x^6}-\frac{b\arctan(cx^3)}{6x^6}-\frac{bc}{6x^3}-\frac{bc^2\arctan(cx^3)}{6}$	39
risch	$\frac{ib\ln(icx^3+1)}{12x^6}-\frac{ic^2b\ln(cx^3+i)x^6-ic^2b\ln(cx^3-i)x^6+2bcx^3+ib\ln(-icx^3+1)+2a}{12x^6}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^7,x,method=_RETURNVERBOSE)
[Out] -1/6*a/x^6-1/6*b/x^6*arctan(c*x^3)-1/6*b*c/x^3-1/6*b*c^2*arctan(c*x^3)
```

Maxima [A]
time = 0.47, size = 35, normalized size = 0.85

$$-\frac{1}{6}\left(\left(c\arctan\left(cx^3\right)+\frac{1}{x^3}\right)c+\frac{\arctan(cx^3)}{x^6}\right)b-\frac{a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^7,x, algorithm="maxima")
[Out] -1/6*((c*arctan(c*x^3) + 1/x^3)*c + arctan(c*x^3)/x^6)*b - 1/6*a/x^6
```

Fricas [A]
time = 1.51, size = 30, normalized size = 0.73

$$-\frac{bcx^3 + (bc^2x^6 + b)\arctan(cx^3) + a}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^7,x, algorithm="fricas")
[Out] -1/6*(b*c*x^3 + (b*c^2*x^6 + b)*arctan(c*x^3) + a)/x^6
```

Sympy [A]
time = 35.41, size = 42, normalized size = 1.02

$$-\frac{a}{6x^6} - \frac{bc^2\operatorname{atan}(cx^3)}{6} - \frac{bc}{6x^3} - \frac{b\operatorname{atan}(cx^3)}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**7,x)
[Out] -a/(6*x**6) - b*c**2*atan(c*x**3)/6 - b*c/(6*x**3) - b*atan(c*x**3)/(6*x**6
)
```

Giac [C] Result contains complex when optimal does not.
time = 0.43, size = 72, normalized size = 1.76

$$\frac{i\,bc^5x^6\log(i\,cx^3+1) - i\,bc^5x^6\log(-i\,cx^3+1) - 2\,bc^4x^3 - 2\,bc^3\arctan(cx^3) - 2\,ac^3}{12\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^7,x, algorithm="giac")
[Out] 1/12*(I*b*c^5*x^6*log(I*c*x^3 + 1) - I*b*c^5*x^6*log(-I*c*x^3 + 1) - 2*b*c^
4*x^3 - 2*b*c^3*arctan(c*x^3) - 2*a*c^3)/(c^3*x^6)
```

Mupad [B]
time = 0.39, size = 41, normalized size = 1.00

$$-\frac{\frac{b\,c\,x^3}{3} + \frac{a}{3}}{2\,x^6} - \frac{b\,c^2\operatorname{atan}(c\,x^3)}{6} - \frac{b\operatorname{atan}(c\,x^3)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))/x^7,x)
[Out] - (a/3 + (b*c*x^3)/3)/(2*x^6) - (b*c^2*atan(c*x^3))/6 - (b*atan(c*x^3))/(6*
x^6)
```

3.103 $\int \frac{a+b\mathbf{ArcTan}(cx^3)}{x^{10}}\,dx$

Optimal. Leaf size=55

$$-\frac{bc}{18x^6}-\frac{a+b\text{ArcTan}(cx^3)}{9x^9}-\frac{1}{3}bc^3\log(x)+\frac{1}{18}bc^3\log\left(1+c^2x^6\right)$$

```
[Out] -1/18*b*c/x^6+1/9*(-a-b*arctan(c*x^3))/x^9-1/3*b*c^3*ln(x)+1/18*b*c^3*ln(c^
2*x^6+1)
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.214$, Rules used = {4946, 272, 46}

$$-\frac{a+b\text{ArcTan}(cx^3)}{9x^9}-\frac{1}{3}bc^3\log(x)+\frac{1}{18}bc^3\log\left(c^2x^6+1\right)-\frac{bc}{18x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x^10,x]
```

```
[Out] -1/18*(b*c)/x^6 - (a + b*ArcTan[c*x^3])/(9*x^9) - (b*c^3*Log[x])/3 + (b*c^3
*Log[1 + c^2*x^6])/18
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx^3)}{x^{10}}\,dx = -\frac{a+b\tan^{-1}(cx^3)}{9x^9}+\frac{1}{3}(bc)\int \frac{1}{x^7(1+c^2x^6)}\,dx$$
$$= -\frac{a+b\tan^{-1}(cx^3)}{9x^9}+\frac{1}{18}(bc)\text{Subst}\left(\int \frac{1}{x^2(1+c^2x)}\,dx, x, x^6\right)$$
$$= -\frac{a+b\tan^{-1}(cx^3)}{9x^9}+\frac{1}{18}(bc)\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{c^2}{x}+\frac{c^4}{1+c^2x}\right)dx, x, x^6\right)$$
$$= -\frac{bc}{18x^6}-\frac{a+b\tan^{-1}(cx^3)}{9x^9}-\frac{1}{3}bc^3\log(x)+\frac{1}{18}bc^3\log(1+c^2x^6)$$

Mathematica [A]
time = 0.01, size = 60, normalized size = 1.09

$$-\frac{a}{9x^9}-\frac{bc}{18x^6}-\frac{b\text{ArcTan}(cx^3)}{9x^9}-\frac{1}{3}bc^3\log(x)+\frac{1}{18}bc^3\log(1+c^2x^6)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^10,x]
```

```
[Out] -1/9*a/x^9 - (b*c)/(18*x^6) - (b*ArcTan[c*x^3])/(9*x^9) - (b*c^3*Log[x])/3
+ (b*c^3*Log[1 + c^2*x^6])/18
```

Maple [A]
time = 0.05, size = 51, normalized size = 0.93

method	result	size
default	$-\frac{a}{9x^9}-\frac{b\arctan(c\,x^3)}{9x^9}-\frac{bc}{18x^6}-\frac{b\,c^3\ln(x)}{3}+\frac{b\,c^3\ln(c^2x^6+1)}{18}$	51
risch	$\frac{ib\ln(ic\,x^3+1)}{18x^9}-\frac{6b\,c^3\ln(x)x^9-b\,c^3\ln(c^2x^6+1)x^9+bc\,x^3+ib\ln(-ic\,x^3+1)+2a}{18x^9}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^10,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9*a/x^9-1/9*b/x^9*arctan(c*x^3)-1/18*b*c/x^6-1/3*b*c^3*ln(x)+1/18*b*c^3*
ln(c^2*x^6+1)
```

Maxima [A]
time = 0.26, size = 53, normalized size = 0.96

$$\frac{1}{18}\left(\left(c^2\log(c^2x^6+1)-c^2\log(x^6)-\frac{1}{x^6}\right)c-\frac{2\arctan(cx^3)}{x^9}\right)b-\frac{a}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^10,x, algorithm="maxima")
```

```
[Out] 1/18*((c^2*log(c^2*x^6 + 1) - c^2*log(x^6) - 1/x^6)*c - 2*arctan(c*x^3)/x^9
)*b - 1/9*a/x^9
```

Fricas [A]
time = 1.46, size = 54, normalized size = 0.98

$$\frac{bc^3x^9\log\left(c^2x^6+1\right)-6\,bc^3x^9\log\left(x\right)-bcx^3-2\,b\arctan\left(cx^3\right)-2\,a}{18\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^10,x, algorithm="fricas")
```

```
[Out] 1/18*(b*c^3*x^9*log(c^2*x^6 + 1) - 6*b*c^3*x^9*log(x) - b*c*x^3 - 2*b*arcta
n(c*x^3) - 2*a)/x^9
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(53) = 106$.
time = 131.39, size = 129, normalized size = 2.35

$$\begin{cases} -\frac{a}{9x^9} - \frac{bc^4\sqrt{-\frac{1}{c^2}}\operatorname{atan}\left(cx^3\right)}{9} - \frac{bc^3\log\left(x\right)}{3} + \frac{bc^3\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{9} + \frac{bc^3\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{9} - \frac{bc}{18x^6} - \frac{b\operatorname{atan}\left(cx^3\right)}{9x^9} & \text{for}\: c \neq 0 \\ -\frac{a}{9x^9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**10,x)
```

```
[Out] Piecewise((-a/(9*x**9) - b*c**4*sqrt(-1/c**2)*atan(c*x**3)/9 - b*c**3*log(x
)/3 + b*c**3*log(x - (-1/c**2)**(1/6))/9 + b*c**3*log(4*x**2 + 4*x*(-1/c**2
)**(1/6) + 4*(-1/c**2)**(1/3))/9 - b*c/(18*x**6) - b*atan(c*x**3)/(9*x**9),
 Ne(c, 0)), (-a/(9*x**9), True))
```

Giac [A]
time = 0.43, size = 69, normalized size = 1.25

$$\frac{bc^7x^9\log\left(c^2x^6+1\right)-2\,bc^7x^9\log\left(cx^3\right)-bc^5x^3-2\,bc^4\arctan\left(cx^3\right)-2\,ac^4}{18\,c^4x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^10,x, algorithm="giac")
```

```
[Out] 1/18*(b*c^7*x^9*log(c^2*x^6 + 1) - 2*b*c^7*x^9*log(c*x^3) - b*c^5*x^3 - 2*b
*c^4*arctan(c*x^3) - 2*a*c^4)/(c^4*x^9)
```

Mupad [B]
time = 0.40, size = 50, normalized size = 0.91

$$\frac{b\,c^3\,\ln\left(c^2\,x^6+1\right)}{18}-\frac{a}{9\,x^9}-\frac{b\,c^3\,\ln\left(x\right)}{3}-\frac{b\,\mathrm{atan}\left(c\,x^3\right)}{9\,x^9}-\frac{b\,c}{18\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))/x^10,x)
```

```
[Out] (b*c^3*log(c^2*x^6 + 1))/18 - a/(9*x^9) - (b*c^3*log(x))/3 - (b*atan(c*x^3)
)/(9*x^9) - (b*c)/(18*x^6)
```

3.104 $\int x^3(a+b\mathbf{ArcTan}(cx^3))\,dx$

Optimal. Leaf size=174

$$-\frac{3bx}{4c}+\frac{b\text{ArcTan}\left(\sqrt[3]{c}\,x\right)}{4c^{4/3}}+\frac{1}{4}x^4\left(a+b\text{ArcTan}\left(cx^3\right)\right)-\frac{b\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)}{8c^{4/3}}+\frac{b\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{c}\,x\right)}{8c^{4/3}}-$$

```
[Out] -3/4*b*x/c+1/4*b*arctan(c^(1/3)*x)/c^(4/3)+1/4*x^4*(a+b*arctan(c*x^3))+1/8*
b*arctan(2*c^(1/3)*x-3^(1/2))/c^(4/3)+1/8*b*arctan(2*c^(1/3)*x+3^(1/2))/c^(
4/3)-1/16*b*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)/c^(4/3)+1/16*b*ln(1
+c^(2/3)*x^2+c^(1/3)*x*3^(1/2))*3^(1/2)/c^(4/3)
```

Rubi [A]
time = 0.23, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4946, 327, 215, 648, 632, 210, 642, 209}

$$\frac{1}{4}x^4\left(a+b\text{ArcTan}\left(cx^3\right)\right)+\frac{b\text{ArcTan}\left(\sqrt[3]{c}\,x\right)}{4c^{4/3}}-\frac{b\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)}{8c^{4/3}}+\frac{b\text{ArcTan}\left(2\sqrt[3]{c}\,x+\sqrt{3}\right)}{8c^{4/3}}-\frac{\sqrt{3}\,b\log\left(c^{2/3}x^2-\sqrt{3}\,\sqrt[3]{c}\,x+1\right)}{16c^{4/3}}+\frac{\sqrt{3}\,b\log\left(c^{2/3}x^2+\sqrt{3}\,\sqrt[3]{c}\,x+1\right)}{16c^{4/3}}-\frac{3bx}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x)/(4*c) + (b*ArcTan[c^(1/3)*x])/(4*c^(4/3)) + (x^4*(a + b*ArcTan[c*x
^3]))/4 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(8*c^(4/3)) + (b*ArcTan[Sqrt[3]
 + 2*c^(1/3)*x])/(8*c^(4/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/
3)*x^2])/(16*c^(4/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2]
)/(16*c^(4/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
```

```
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(a+b\tan^{-1}\left(cx^3\right)\right)\,dx &= \frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{4}(3bc)\int\frac{x^6}{1+c^2x^6}\,dx\\
&= -\frac{3bx}{4c}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{(3b)\int\frac{1}{1+c^2x^6}\,dx}{4c}\\
&= -\frac{3bx}{4c}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\int\frac{1}{1+c^{2/3}x^2}\,dx}{4c}+\frac{b\int\frac{1-\frac{1}{2}\sqrt{3}\sqrt[3]{c}\,x}{1-\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx}{4c}\\
&= -\frac{3bx}{4c}+\frac{b\tan^{-1}\left(\sqrt[3]{c}\,x\right)}{4c^{4/3}}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\left(\sqrt{3}\,b\right)\int\frac{-\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2}}{16c^{4/3}}\\
&= -\frac{3bx}{4c}+\frac{b\tan^{-1}\left(\sqrt[3]{c}\,x\right)}{4c^{4/3}}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\sqrt{3}\,b\log\left(1-\sqrt{3}\sqrt[3]{c}\,x\right.}{16c^{4/3}}\\
&= -\frac{3bx}{4c}+\frac{b\tan^{-1}\left(\sqrt[3]{c}\,x\right)}{4c^{4/3}}+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)}{8c^{4/3}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 179, normalized size = 1.03

$$
-\frac{3bx}{4c}+\frac{ax^4}{4}+\frac{b\text{ArcTan}\left(\sqrt[3]{c}\,x\right)}{4c^{4/3}}+\frac{1}{4}bx^4\text{ArcTan}\left(cx^3\right)-\frac{b\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)}{8c^{4/3}}+\frac{b\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{c}\,x\right)}{8c^{4/3}}-\frac{\sqrt{3}\,b\log\left(1-\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2\right)}{16c^{4/3}}+\frac{\sqrt{3}\,b\log\left(1+\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2\right)}{16c^{4/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x)/(4*c) + (a*x^4)/4 + (b*ArcTan[c^(1/3)*x])/(4*c^(4/3)) + (b*x^4*Arc
Tan[c*x^3])/4 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(8*c^(4/3)) + (b*ArcTan[S
qrt[3] + 2*c^(1/3)*x])/(8*c^(4/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x +
 c^(2/3)*x^2])/(16*c^(4/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3
)*x^2])/(16*c^(4/3))
```

Maple [A]
time = 0.09, size = 165, normalized size = 0.95

method	result
default	$\frac{a\,x^4}{4}+\frac{b\,x^4\arctan\left(c\,x^3\right)}{4}-\frac{3bx}{4c}-\frac{b\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\ln\left(x^2-\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{16c}+\frac{b\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{8c}+\frac{b\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a*x^4+1/4*b*x^4*arctan(c*x^3)-3/4*b*x/c-1/16*b/c*3^(1/2)*(1/c^2)^(1/6)*
ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/8*b/c*(1/c^2)^(1/6)*arctan(
2*x/(1/c^2)^(1/6)-3^(1/2))+1/16*b/c*3^(1/2)*(1/c^2)^(1/6)*ln(x^2+3^(1/2)*(1
/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/8*b/c*(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)
+3^(1/2))+1/4*b/c*(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6))
```

Maxima [A]
time = 0.47, size = 148, normalized size = 0.85

$$\frac{1}{4}\,ax^4+\frac{1}{16}\left(4\,x^4\arctan\left(cx^3\right)+c\left(\frac{\frac{\sqrt{3}\,\log\left(c^{\frac{2}{3}}x^2+\sqrt{3}\,c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}-\frac{\sqrt{3}\,\log\left(c^{\frac{2}{3}}x^2-\sqrt{3}\,c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}+\frac{4\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{1}{3}}}+\frac{2\arctan\left(\frac{2\,c^{\frac{2}{3}}x+\sqrt{3}\,c^{\frac{1}{3}}}{c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}+\frac{2\arctan\left(\frac{2\,c^{\frac{2}{3}}x-\sqrt{3}\,c^{\frac{1}{3}}}{c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}}{c^2}-\frac{12\,x}{c^2}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/16*(4*x^4*arctan(c*x^3) + c*((sqrt(3)*log(c^(2/3)*x^2 + sqrt(
3)*c^(1/3)*x + 1)/c^(1/3) - sqrt(3)*log(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1
)/c^(1/3) + 4*arctan(c^(1/3)*x)/c^(1/3) + 2*arctan((2*c^(2/3)*x + sqrt(3)*c
^(1/3))/c^(1/3))/c^(1/3) + 2*arctan((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/3)
)/c^(1/3))/c^2 - 12*x/c^2))*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(126) = 252$.
time = 1.67, size = 399, normalized size = 2.29

$$\frac{4\,bcx^4\arctan\left(cx^3\right)+4\,acx^4+\sqrt{3}\,c\left(\frac{b^6}{c^8}\right)^{\frac{1}{6}}\log\left(b^2x^2+\sqrt{3}\,bcx\left(\frac{b^6}{c^8}\right)^{\frac{1}{6}}+c^2\left(\frac{b^6}{c^8}\right)^{\frac{1}{3}}\right)-\sqrt{3}\,c\left(\frac{b^6}{c^8}\right)^{\frac{1}{6}}\log\left(b^2x^2-\sqrt{3}\,bcx\left(\frac{b^6}{c^8}\right)^{\frac{1}{6}}+c^2\left(\frac{b^6}{c^8}\right)^{\frac{1}{3}}\right)-4\,c\left(\frac{b^6}{c^8}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\,bc^7x\left(\frac{b^6}{c^8}\right)^{\frac{5}{6}}-2\sqrt{b^2x^2+\sqrt{3}\,bcx\left(\frac{b^6}{c^8}\right)^{\frac{1}{6}}+c^2\left(\frac{b^6}{c^8}\right)^{\frac{1}{3}}}\,c^7\left(\frac{b^6}{c^8}\right)^{\frac{5}{6}}+\sqrt{3}\,b^6}{b^6}\right)-4\,c\left(\frac{b^6}{c^8}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\,bc^7x\left(\frac{b^6}{c^8}\right)^{\frac{5}{6}}-2\sqrt{b^2x^2-\sqrt{3}\,bcx\left(\frac{b^6}{c^8}\right)^{\frac{1}{6}}+c^2\left(\frac{b^6}{c^8}\right)^{\frac{1}{3}}}\,c^7\left(\frac{b^6}{c^8}\right)^{\frac{5}{6}}-\sqrt{3}\,b^6}{b^6}\right)-8\,c\left(\frac{b^6}{c^8}\right)^{\frac{1}{6}}\arctan\left(-\frac{bc^7x\left(\frac{b^6}{c^8}\right)^{\frac{5}{6}}-\sqrt{b^2x^2+c^2\left(\frac{b^6}{c^8}\right)^{\frac{1}{3}}}\,c^7\left(\frac{b^6}{c^8}\right)^{\frac{5}{6}}}{b^6}\right)-12\,bx}{16\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/16*(4*b*c*x^4*arctan(c*x^3) + 4*a*c*x^4 + sqrt(3)*c*(b^6/c^8)^(1/6)*log(b
^2*x^2 + sqrt(3)*b*c*x*(b^6/c^8)^(1/6) + c^2*(b^6/c^8)^(1/3)) - sqrt(3)*c*(
b^6/c^8)^(1/6)*log(b^2*x^2 - sqrt(3)*b*c*x*(b^6/c^8)^(1/6) + c^2*(b^6/c^8)^
(1/3)) - 4*c*(b^6/c^8)^(1/6)*arctan(-(2*b*c^7*x*(b^6/c^8)^(5/6) - 2*sqrt(b^
2*x^2 + sqrt(3)*b*c*x*(b^6/c^8)^(1/6) + c^2*(b^6/c^8)^(1/3))*c^7*(b^6/c^8)^
(5/6) + sqrt(3)*b^6)/b^6) - 4*c*(b^6/c^8)^(1/6)*arctan(-(2*b*c^7*x*(b^6/c^8
)^(5/6) - 2*sqrt(b^2*x^2 - sqrt(3)*b*c*x*(b^6/c^8)^(1/6) + c^2*(b^6/c^8)^(1
/3))*c^7*(b^6/c^8)^(5/6) - sqrt(3)*b^6)/b^6) - 8*c*(b^6/c^8)^(1/6)*arctan(-
(b*c^7*x*(b^6/c^8)^(5/6) - sqrt(b^2*x^2 + c^2*(b^6/c^8)^(1/3))*c^7*(b^6/c^8
)^(5/6))/b^6) - 12*b*x)/c
```

Sympy [A]
time = 23.27, size = 255, normalized size = 1.47

$$\begin{cases}\frac{ax^4}{4}+\frac{bx^4\operatorname{atan}\left(cx^3\right)}{4}-\frac{3bx}{4c}-\frac{3b\sqrt[6]{-\frac{1}{c^2}}\log\left(4x^2-4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{16c}+\frac{3b\sqrt[6]{-\frac{1}{c^2}}\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{16c}+\frac{\sqrt{3}\,b\sqrt[6]{-\frac{1}{c^2}}\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3\sqrt[6]{-\frac{1}{c^2}}}-\frac{\sqrt{3}}{3}\right)}{8c}+\frac{\sqrt{3}\,b\sqrt[6]{-\frac{1}{c^2}}\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3\sqrt[6]{-\frac{1}{c^2}}}+\frac{\sqrt{3}}{3}\right)}{8c}+\frac{b\operatorname{atan}\left(cx^3\right)}{4c^2\sqrt[3]{-\frac{1}{c^2}}} & \text{for}\ c\neq 0\\ \frac{ax^4}{4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**4/4 + b*x**4*atan(c*x**3)/4 - 3*b*x/(4*c) - 3*b*(-1/c**2)**
(1/6)*log(4*x**2 - 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(16*c) + 3*b*
(-1/c**2)**(1/6)*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(1
6*c) + sqrt(3)*b*(-1/c**2)**(1/6)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - s
qrt(3)/3)/(8*c) + sqrt(3)*b*(-1/c**2)**(1/6)*atan(2*sqrt(3)*x/(3*(-1/c**2)*
*(1/6)) + sqrt(3)/3)/(8*c) + b*atan(c*x**3)/(4*c**2*(-1/c**2)**(1/3)), Ne(c
, 0)), (a*x**4/4, True))
```

Giac [A]
time = 0.48, size = 167, normalized size = 0.96

$$\frac{1}{16}\,bc^{7}\left(\frac{\sqrt{3}\,\log\left(x^{2}+\frac{\sqrt{3}\,x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{8}|c|^{\frac{1}{3}}}-\frac{\sqrt{3}\,\log\left(x^{2}-\frac{\sqrt{3}\,x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{8}|c|^{\frac{1}{3}}}+\frac{2\,\arctan\left(\left(2\,x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^{8}|c|^{\frac{1}{3}}}+\frac{2\,\arctan\left(\left(2\,x-\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^{8}|c|^{\frac{1}{3}}}+\frac{4\,\arctan\left(x|c|^{\frac{1}{3}}\right)}{c^{8}|c|^{\frac{1}{3}}}\right)+\frac{bcx^{4}\arctan\left(cx^{3}\right)+acx^{4}-3\,bx}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/16*b*c^7*(sqrt(3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/(c^8
*abs(c)^(1/3)) - sqrt(3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))
/(c^8*abs(c)^(1/3)) + 2*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/(
c^8*abs(c)^(1/3)) + 2*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/(c^
8*abs(c)^(1/3)) + 4*arctan(x*abs(c)^(1/3))/(c^8*abs(c)^(1/3))) + 1/4*(b*c*x
^4*arctan(c*x^3) + a*c*x^4 - 3*b*x)/c
```

Mupad [B]
time = 1.16, size = 114, normalized size = 0.66

$$\frac{a\,x^4}{4}-\frac{b\,\left(\mathrm{atan}\left({\left(-1\right)}^{2/3}\,c^{1/3}\,x\right)-\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)+2\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{2/3}\,c^{1/3}\,x\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\right)}{8\,c^{4/3}}+\frac{b\,x^4\,\mathrm{atan}\left(c\,x^3\right)}{4}-\frac{3\,b\,x}{4\,c}-\frac{\sqrt{3}\,b\,\left(\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)+\mathrm{atan}\left({\left(-1\right)}^{2/3}\,c^{1/3}\,x\right)\right)\,1{}\mathrm{i}}{8\,c^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c*x^3)),x)
```

```
[Out] (a*x^4)/4 - (b*(atan((-1)^(2/3)*c^(1/3)*x) - atan((c^(1/3)*x*(3^(1/2)*1i +
1))/2) + 2*atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i + 1))/2)))/(8*c^(4/3)) +
(b*x^4*atan(c*x^3))/4 - (3*b*x)/(4*c) - (3^(1/2)*b*(atan((c^(1/3)*x*(3^(1/2
)*1i + 1))/2) + atan((-1)^(2/3)*c^(1/3)*x))*1i)/(8*c^(4/3))
```

3.105 $\int (a+b\mathbf{ArcTan}(cx^3))\,dx$

Optimal. Leaf size=101

$$ax+bx\text{ArcTan}(cx^3)+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}}+\frac{b\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)}{4\sqrt[3]{c}}$$

```
[Out] a*x+b*x*arctan(c*x^3)+1/2*b*ln(1+c^(2/3)*x^2)/c^(1/3)-1/4*b*ln(1-c^(2/3)*x^
2+c^(4/3)*x^4)/c^(1/3)+1/2*b*arctan(1/3*(1-2*c^(2/3)*x^2)*3^(1/2))*3^(1/2)/
c^(1/3)
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {4930, 281, 298, 31, 648, 631, 210, 642}

$$ax+\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}}+bx\text{ArcTan}(cx^3)+\frac{b\log\left(c^{2/3}x^2+1\right)}{2\sqrt[3]{c}}-\frac{b\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{4\sqrt[3]{c}}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTan[c*x^3],x]
```

```
[Out] a*x + b*x*ArcTan[c*x^3] + (Sqrt[3]*b*ArcTan[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/(
2*c^(1/3)) + (b*Log[1 + c^(2/3)*x^2])/(2*c^(1/3)) - (b*Log[1 - c^(2/3)*x^2
+ c^(4/3)*x^4])/(4*c^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\int \left(a+b\tan^{-1}\left(cx^3\right)\right)\,dx = ax+b\int \tan^{-1}\left(cx^3\right)\,dx$$
$$= ax+bx\tan^{-1}\left(cx^3\right)-(3bc)\int \frac{x^3}{1+c^2x^6}\,dx$$
$$= ax+bx\tan^{-1}\left(cx^3\right)-\frac{1}{2}(3bc)\text{Subst}\left(\int \frac{x}{1+c^2x^3}\,dx,x,x^2\right)$$
$$= ax+bx\tan^{-1}\left(cx^3\right)+\frac{1}{2}\left(b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{1}{1+c^{2/3}x}\,dx,x,x^2\right)-\frac{1}{2}\left(b\sqrt[3]{c}\right)\text{Sub}$$
$$= ax+bx\tan^{-1}\left(cx^3\right)+\frac{b\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\text{Subst}\left(\int \frac{-c^{2/3}+2c^{4/3}x}{1-c^{2/3}x+c^{4/3}x^2}\,dx,x,x^2\right)}{4\sqrt[3]{c}}-$$
$$= ax+bx\tan^{-1}\left(cx^3\right)+\frac{b\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)}{4\sqrt[3]{c}}-\frac{(3b)\text{S}}{}$$
$$= ax+bx\tan^{-1}\left(cx^3\right)+\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}}+\frac{b\log\left(1+c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\log\left(1\right.}{}$$

Mathematica [A]
time = 0.03, size = 131, normalized size = 1.30

$$ax+bx\text{ArcTan}\left(cx^3\right)-\frac{b\left(-2\sqrt{3}\,\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)-2\sqrt{3}\,\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{c}\,x\right)-2\log\left(1+c^{2/3}x^2\right)+\log\left(1-\sqrt{3}\,\sqrt[3]{c}\,x+c^{2/3}x^2\right)+\log\left(1+\sqrt{3}\,\sqrt[3]{c}\,x+c^{2/3}x^2\right)\right)}{4\sqrt[3]{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTan[c*x^3],x]
```

```
[Out] a*x + b*x*ArcTan[c*x^3] - (b*(-2*Sqrt[3]*ArcTan[Sqrt[3] - 2*c^(1/3)*x] - 2*
Sqrt[3]*ArcTan[Sqrt[3] + 2*c^(1/3)*x] - 2*Log[1 + c^(2/3)*x^2] + Log[1 - Sq
rt[3]*c^(1/3)*x + c^(2/3)*x^2] + Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2]))
/(4*c^(1/3))
```

Maple [A]
time = 0.03, size = 98, normalized size = 0.97

method	result

default	$ax+bx\arctan\left(c\,x^{3}\right)+\frac{b\ln\left(x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x^{4}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(\frac{2x^{2}}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctan(c*x^3),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*x*arctan(c*x^3)+1/2*b/c/(1/c^2)^(1/3)*ln(x^2+(1/c^2)^(1/3))-1/4*b/c/(
1/c^2)^(1/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))-1/2*b*3^(1/2)/c/(1/c^2
)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1/3)*x^2-1))
```

Maxima [A]
time = 0.47, size = 92, normalized size = 0.91

$$-\frac{1}{4}\left(c\left(\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,c^{\frac{4}{3}}x^{2}-c^{\frac{2}{3}}\right)}{3\,c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}+\frac{\log\left(c^{\frac{4}{3}}x^{4}-c^{\frac{2}{3}}x^{2}+1\right)}{c^{\frac{4}{3}}}-\frac{2\,\log\left(\frac{c^{\frac{2}{3}}x^{2}+1}{c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}\right)-4\,x\arctan\left(cx^{3}\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x^3),x, algorithm="maxima")
```

```
[Out] -1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3))/c^(2/3))/c^
(4/3) + log(c^(4/3)*x^4 - c^(2/3)*x^2 + 1)/c^(4/3) - 2*log((c^(2/3)*x^2 + 1
)/c^(2/3))/c^(4/3)) - 4*x*arctan(c*x^3))*b + a*x
```

Fricas [A]
time = 1.33, size = 234, normalized size = 2.32

$$\left[\frac{4\,bcx\arctan\left(cx^{3}\right)+\sqrt{3}\,bc\sqrt{-\frac{1}{c^{\frac{2}{3}}}}\,\log\left(\frac{2\,c^{2}x^{6}-3\,c^{\frac{2}{3}}x^{2}-\sqrt{3}\,\left(2\,c^{\frac{5}{3}}x^{4}+cx^{2}-c^{\frac{1}{3}}\right)\sqrt{-\frac{1}{c^{\frac{2}{3}}}}-1}{c^{2}x^{6}+1}\right)+4\,acx-bc^{\frac{2}{3}}\log\left(c^{2}x^{4}-c^{\frac{4}{3}}x^{2}+c^{\frac{2}{3}}\right)+2\,bc^{\frac{2}{3}}\log\left(cx^{2}+c^{\frac{1}{3}}\right)}{4\,c},\ \frac{4\,bcx\arctan\left(cx^{3}\right)+2\,\sqrt{3}\,bc^{\frac{2}{3}}\arctan\left(-\frac{\sqrt{3}\,\left(2\,cx^{2}-c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right)+4\,acx-bc^{\frac{2}{3}}\log\left(c^{2}x^{4}-c^{\frac{4}{3}}x^{2}+c^{\frac{2}{3}}\right)+2\,bc^{\frac{2}{3}}\log\left(cx^{2}+c^{\frac{1}{3}}\right)}{4\,c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x^3),x, algorithm="fricas")
```

```
[Out] [1/4*(4*b*c*x*arctan(c*x^3) + sqrt(3)*b*c*sqrt(-1/c^(2/3))*log((2*c^2*x^6 -
 3*c^(2/3)*x^2 - sqrt(3)*(2*c^(5/3)*x^4 + c*x^2 - c^(1/3))*sqrt(-1/c^(2/3))
 - 1)/(c^2*x^6 + 1)) + 4*a*c*x - b*c^(2/3)*log(c^2*x^4 - c^(4/3)*x^2 + c^(2
/3)) + 2*b*c^(2/3)*log(c*x^2 + c^(1/3)))/c, 1/4*(4*b*c*x*arctan(c*x^3) + 2*
sqrt(3)*b*c^(2/3)*arctan(-1/3*sqrt(3)*(2*c*x^2 - c^(1/3))/c^(1/3)) + 4*a*c*
```

```
x - b*c^(2/3)*log(c^2*x^4 - c^(4/3)*x^2 + c^(2/3)) + 2*b*c^(2/3)*log(c*x^2
+ c^(1/3)))/c]
```

Sympy [A]
time = 12.94, size = 755, normalized size = 7.48

$$ax + b\left(\begin{cases} -\infty i x & \text{for}\: c = -\frac{i}{x^{3}} \\ \infty i x & \text{for}\: c = \frac{i}{x^{3}} \\ 0 & \text{for}\: c = 0 \\ -\frac{4c^{4}x^{6}\left(-\frac{1}{c^{2}}\right)^{\frac{5}{3}}\log\left(x-\sqrt[6]{-\frac{1}{c^{2}}}\right)}{4cx^{6}+\frac{4}{c}} + \frac{3c^{4}x^{6}\left(-\frac{1}{c^{2}}\right)^{\frac{5}{3}}\log\left(4x^{2}-4x\sqrt[6]{-\frac{1}{c^{2}}}+4\sqrt[3]{-\frac{1}{c^{2}}}\right)}{4cx^{6}+\frac{4}{c}} - \frac{c^{4}x^{6}\left(-\frac{1}{c^{2}}\right)^{\frac{5}{3}}\log\left(4x^{2}+4x\sqrt[6]{-\frac{1}{c^{2}}}+4\sqrt[3]{-\frac{1}{c^{2}}}\right)}{4cx^{6}+\frac{4}{c}} - \frac{2\sqrt{3}c^{4}x^{6}\left(-\frac{1}{c^{2}}\right)^{\frac{5}{3}}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3\sqrt[6]{-\frac{1}{c^{2}}}}-\frac{\sqrt{3}}{3}\right)}{4cx^{6}+\frac{4}{c}} + \frac{2\sqrt{3}c^{4}x^{6}\left(-\frac{1}{c^{2}}\right)^{\frac{5}{3}}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3\sqrt[6]{-\frac{1}{c^{2}}}}+\frac{\sqrt{3}}{3}\right)}{4cx^{6}+\frac{4}{c}} - \frac{4c^{4}x^{6}\left(-\frac{1}{c^{2}}\right)^{\frac{5}{3}}\log(2)}{4cx^{6}+\frac{4}{c}} - \frac{4c^{3}x^{6}\left(-\frac{1}{c^{2}}\right)^{\frac{7}{6}}\operatorname{atan}(cx^{3})}{4cx^{6}+\frac{4}{c}} - \frac{4c^{2}\left(-\frac{1}{c^{2}}\right)^{\frac{5}{3}}\log\left(x-\sqrt[6]{-\frac{1}{c^{2}}}\right)}{4cx^{6}+\frac{4}{c}} + \frac{3c^{2}\left(-\frac{1}{c^{2}}\right)^{\frac{5}{3}}\log\left(4x^{2}-4x\sqrt[6]{-\frac{1}{c^{2}}}+4\sqrt[3]{-\frac{1}{c^{2}}}\right)}{4cx^{6}+\frac{4}{c}} - \frac{c^{2}\left(-\frac{1}{c^{2}}\right)^{\frac{5}{3}}\log\left(4x^{2}+4x\sqrt[6]{-\frac{1}{c^{2}}}+4\sqrt[3]{-\frac{1}{c^{2}}}\right)}{4cx^{6}+\frac{4}{c}} - \frac{2\sqrt{3}c^{2}\left(-\frac{1}{c^{2}}\right)^{\frac{5}{3}}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3\sqrt[6]{-\frac{1}{c^{2}}}}-\frac{\sqrt{3}}{3}\right)}{4cx^{6}+\frac{4}{c}} + \frac{2\sqrt{3}c^{2}\left(-\frac{1}{c^{2}}\right)^{\frac{5}{3}}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3\sqrt[6]{-\frac{1}{c^{2}}}}+\frac{\sqrt{3}}{3}\right)}{4cx^{6}+\frac{4}{c}} - \frac{4c^{2}\left(-\frac{1}{c^{2}}\right)^{\frac{5}{3}}\log(2)}{4cx^{6}+\frac{4}{c}} + \frac{4cx^{7}\operatorname{atan}(cx^{3})}{4cx^{6}+\frac{4}{c}} - \frac{4c\left(-\frac{1}{c^{2}}\right)^{\frac{7}{6}}\operatorname{atan}(cx^{3})}{4cx^{6}+\frac{4}{c}} + \frac{4x\operatorname{atan}(cx^{3})}{4c^{2}x^{6}+4} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atan(c*x**3),x)
```

```
[Out] a*x + b*Piecewise((-oo*I*x, Eq(c, -I/x**3)), (oo*I*x, Eq(c, I/x**3)), (0, E
q(c, 0)), (-4*c**4*x**6*(-1/c**2)**(5/3)*log(x - (-1/c**2)**(1/6))/(4*c*x**
6 + 4/c) + 3*c**4*x**6*(-1/c**2)**(5/3)*log(4*x**2 - 4*x*(-1/c**2)**(1/6) +
 4*(-1/c**2)**(1/3))/(4*c*x**6 + 4/c) - c**4*x**6*(-1/c**2)**(5/3)*log(4*x*
*2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(4*c*x**6 + 4/c) - 2*sqrt(3
)*c**4*x**6*(-1/c**2)**(5/3)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3
)/3)/(4*c*x**6 + 4/c) + 2*sqrt(3)*c**4*x**6*(-1/c**2)**(5/3)*atan(2*sqrt(3)
*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/(4*c*x**6 + 4/c) - 4*c**4*x**6*(-1/c**
2)**(5/3)*log(2)/(4*c*x**6 + 4/c) - 4*c**3*x**6*(-1/c**2)**(7/6)*atan(c*x**
3)/(4*c*x**6 + 4/c) - 4*c**2*(-1/c**2)**(5/3)*log(x - (-1/c**2)**(1/6))/(4*
c*x**6 + 4/c) + 3*c**2*(-1/c**2)**(5/3)*log(4*x**2 - 4*x*(-1/c**2)**(1/6) +
 4*(-1/c**2)**(1/3))/(4*c*x**6 + 4/c) - c**2*(-1/c**2)**(5/3)*log(4*x**2 +
4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(4*c*x**6 + 4/c) - 2*sqrt(3)*c**
2*(-1/c**2)**(5/3)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3)/3)/(4*c*
x**6 + 4/c) + 2*sqrt(3)*c**2*(-1/c**2)**(5/3)*atan(2*sqrt(3)*x/(3*(-1/c**2)
**(1/6)) + sqrt(3)/3)/(4*c*x**6 + 4/c) - 4*c**2*(-1/c**2)**(5/3)*log(2)/(4*
c*x**6 + 4/c) + 4*c*x**7*atan(c*x**3)/(4*c*x**6 + 4/c) - 4*c*(-1/c**2)**(7/
6)*atan(c*x**3)/(4*c*x**6 + 4/c) + 4*x*atan(c*x**3)/(4*c**2*x**6 + 4), True
))
```

Giac [A]
time = 0.43, size = 95, normalized size = 0.94

$$-\frac{1}{4}\left(c\left(\frac{2\sqrt{3}|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{2}-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^{2}}+\frac{|c|^{\frac{2}{3}}\log\left(x^{4}-\frac{x^{2}}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^{2}}-\frac{2\log\left(x^{2}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{4}{3}}}\right)-4x\arctan\left(cx^{3}\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c*x^3),x, algorithm="giac")
```

```
[Out] -1/4*(c*(2*sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2/3))
*abs(c)^(2/3))/c^2 + abs(c)^(2/3)*log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)^(4/
3))/c^2 - 2*log(x^2 + 1/abs(c)^(2/3))/abs(c)^(4/3)) - 4*x*arctan(c*x^3))*b
+ a*x
```

Mupad [B]
time = 2.30, size = 91, normalized size = 0.90

$$a\,x+b\,x\,\mathrm{atan}\left(c\,x^3\right)+\frac{b\,\ln\left(c^{2/3}\,x^2+1\right)}{2\,c^{1/3}}-\frac{\ln\left(2-4\,c^{2/3}\,x^2+\sqrt{3}\,2\mathrm{i}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{4\,c^{1/3}}-\frac{\ln\left(4\,c^{2/3}\,x^2-2+\sqrt{3}\,2\mathrm{i}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{4\,c^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atan(c*x^3),x)
```

```
[Out] a*x + b*x*atan(c*x^3) + (b*log(c^(2/3)*x^2 + 1))/(2*c^(1/3)) - (log(3^(1/2)
*2i - 4*c^(2/3)*x^2 + 2)*(b - 3^(1/2)*b*1i))/(4*c^(1/3)) - (log(3^(1/2)*2i
+ 4*c^(2/3)*x^2 - 2)*(b + 3^(1/2)*b*1i))/(4*c^(1/3))
```

3.106 $\int \frac{a+b\text{ArcTan}(cx^3)}{x^3}\,dx$

Optimal. Leaf size=165

$$\frac{1}{2}bc^{2/3}\text{ArcTan}\left(\sqrt[3]{c}\,x\right)-\frac{a+b\text{ArcTan}(cx^3)}{2x^2}-\frac{1}{4}bc^{2/3}\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)+\frac{1}{4}bc^{2/3}\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{c}\,x\right)$$

```
[Out] 1/2*b*c^(2/3)*arctan(c^(1/3)*x)+1/2*(-a-b*arctan(c*x^3))/x^2+1/4*b*c^(2/3)*
arctan(2*c^(1/3)*x-3^(1/2))+1/4*b*c^(2/3)*arctan(2*c^(1/3)*x+3^(1/2))-1/8*b
*c^(2/3)*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)+1/8*b*c^(2/3)*ln(1+c^(
2/3)*x^2+c^(1/3)*x*3^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4946, 215, 648, 632, 210, 642, 209}

$$-\frac{a+b\text{ArcTan}(cx^3)}{2x^2}+\frac{1}{2}bc^{2/3}\text{ArcTan}\left(\sqrt[3]{c}\,x\right)-\frac{1}{4}bc^{2/3}\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)+\frac{1}{4}bc^{2/3}\text{ArcTan}\left(2\sqrt[3]{c}\,x+\sqrt{3}\right)-\frac{1}{8}\sqrt{3}\,bc^{2/3}\log\left(c^{2/3}x^2-\sqrt{3}\,\sqrt[3]{c}\,x+1\right)+\frac{1}{8}\sqrt{3}\,bc^{2/3}\log\left(c^{2/3}x^2+\sqrt{3}\,\sqrt[3]{c}\,x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x^3,x]
```

```
[Out] (b*c^(2/3)*ArcTan[c^(1/3)*x])/2 - (a + b*ArcTan[c*x^3])/(2*x^2) - (b*c^(2/3
)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/4 + (b*c^(2/3)*ArcTan[Sqrt[3] + 2*c^(1/3)*
x])/4 - (Sqrt[3]*b*c^(2/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/8 + (S
qrt[3]*b*c^(2/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/8
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 215

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r/(a*n)), Sum[u
```

```
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}\left(cx^3\right)}{x^3}\,dx = -\frac{a+b\tan^{-1}\left(cx^3\right)}{2x^2}+\frac{1}{2}(3bc)\int \frac{1}{1+c^2x^6}\,dx$$

$$= -\frac{a+b\tan^{-1}\left(cx^3\right)}{2x^2}+\frac{1}{2}(bc)\int \frac{1}{1+c^{2/3}x^2}\,dx+\frac{1}{2}(bc)\int \frac{1-\frac{1}{2}\sqrt{3}\,\sqrt[3]{c}\,x}{1-\sqrt{3}\,\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx$$

$$= \frac{1}{2}bc^{2/3}\tan^{-1}\left(\sqrt[3]{c}\,x\right)-\frac{a+b\tan^{-1}\left(cx^3\right)}{2x^2}-\frac{1}{8}\left(\sqrt{3}\,bc^{2/3}\right)\int \frac{-\sqrt{3}\,\sqrt[3]{c}\,+2c^{2/3}x}{1-\sqrt{3}\,\sqrt[3]{c}\,x+c^{2/3}x^2}$$

$$= \frac{1}{2}bc^{2/3}\tan^{-1}\left(\sqrt[3]{c}\,x\right)-\frac{a+b\tan^{-1}\left(cx^3\right)}{2x^2}-\frac{1}{8}\sqrt{3}\,bc^{2/3}\log\left(1-\sqrt{3}\,\sqrt[3]{c}\,x+c^{2/3}x^2\right)$$

$$= \frac{1}{2}bc^{2/3}\tan^{-1}\left(\sqrt[3]{c}\,x\right)-\frac{a+b\tan^{-1}\left(cx^3\right)}{2x^2}-\frac{1}{4}bc^{2/3}\tan^{-1}\left(\sqrt{3}\,-2\sqrt[3]{c}\,x\right)+\frac{1}{4}bc^{2/3}$$

Mathematica [A]
time = 0.03, size = 170, normalized size = 1.03

$$-\frac{a}{2x^2}+\frac{1}{2}bc^{2/3}\text{ArcTan}\left(\sqrt[3]{c}\,x\right)-\frac{b\text{ArcTan}(cx^3)}{2x^2}-\frac{1}{4}bc^{2/3}\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)+\frac{1}{4}bc^{2/3}\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{c}\,x\right)-\frac{1}{8}\sqrt{3}\,bc^{2/3}\log\left(1-\sqrt{3}\,\sqrt[3]{c}\,x+c^{2/3}x^2\right)+\frac{1}{8}\sqrt{3}\,bc^{2/3}\log\left(1+\sqrt{3}\,\sqrt[3]{c}\,x+c^{2/3}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^3,x]
```

```
[Out] -1/2*a/x^2 + (b*c^(2/3)*ArcTan[c^(1/3)*x])/2 - (b*ArcTan[c*x^3])/(2*x^2) -
(b*c^(2/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/4 + (b*c^(2/3)*ArcTan[Sqrt[3] + 2
*c^(1/3)*x])/4 - (Sqrt[3]*b*c^(2/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2
])/8 + (Sqrt[3]*b*c^(2/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/8
```

Maple [A]
time = 0.08, size = 148, normalized size = 0.90

method	result
default	$-\frac{a}{2x^2}-\frac{b\arctan(c\,x^3)}{2x^2}-\frac{bc\sqrt{3}\,\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\ln\left(x^2-\sqrt{3}\,\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{8}+\frac{bc\left(\frac{1}{c^2}\right)^{\frac{1}{6}}\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{4}+\frac{bc\sqrt{3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/x^2-1/2*b/x^2*arctan(c*x^3)-1/8*b*c*3^(1/2)*(1/c^2)^(1/6)*ln(x^2-3^(
1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/4*b*c*(1/c^2)^(1/6)*arctan(2*x/(1/c^2
)^(1/6)-3^(1/2))+1/8*b*c*3^(1/2)*(1/c^2)^(1/6)*ln(x^2+3^(1/2)*(1/c^2)^(1/6)
*x+(1/c^2)^(1/3))+1/4*b*c*(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))+1
/2*b*c*(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6))
```

Maxima [A]
time = 0.47, size = 137, normalized size = 0.83

$$\frac{1}{8}\left(\left(\frac{\sqrt{3}\,\log\left(c^{\frac{2}{3}}x^2+\sqrt{3}\,c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}-\frac{\sqrt{3}\,\log\left(c^{\frac{2}{3}}x^2-\sqrt{3}\,c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}+\frac{4\,\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{1}{3}}}+\frac{2\,\arctan\left(\frac{2\,c^{\frac{2}{3}}x+\sqrt{3}\,c^{\frac{1}{3}}}{c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}+\frac{2\,\arctan\left(\frac{2\,c^{\frac{2}{3}}x-\sqrt{3}\,c^{\frac{1}{3}}}{c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}\right)c-\frac{4\,\arctan\left(cx^3\right)}{x^2}\right)b-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*((sqrt(3)*log(c^(2/3)*x^2 + sqrt(3)*c^(1/3)*x + 1)/c^(1/3) - sqrt(3)*lo
g(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1)/c^(1/3) + 4*arctan(c^(1/3)*x)/c^(1/3
) + 2*arctan((2*c^(2/3)*x + sqrt(3)*c^(1/3))/c^(1/3))/c^(1/3) + 2*arctan((2
*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/3))/c^(1/3))*c - 4*arctan(c*x^3)/x^2)*b
- 1/2*a/x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(119) = 238$.
time = 1.53, size = 505, normalized size = 3.06

$$\frac{\sqrt{3}(b^6c^4)^{\frac16}x^2\log\left(4b^2c^2x^2+4\sqrt{3}(b^6c^4)^{\frac16}bcx+4(b^6c^4)^{\frac13}\right)-\sqrt{3}(b^6c^4)^{\frac16}x^2\log\left(4b^2c^2x^2-4\sqrt{3}(b^6c^4)^{\frac16}bcx+4(b^6c^4)^{\frac13}\right)+\sqrt{3}(b^6c^4)^{\frac16}x^2\log\left(b^2c^2x^2+\sqrt{3}(b^6c^4)^{\frac16}bcx+(b^6c^4)^{\frac13}\right)-\sqrt{3}(b^6c^4)^{\frac16}x^2\log\left(b^2c^2x^2-\sqrt{3}(b^6c^4)^{\frac16}bcx+(b^6c^4)^{\frac13}\right)-8(b^6c^4)^{\frac16}x^2\arctan\left(-\frac{\sqrt{3}b^6c^4+2(b^6c^4)^{\frac56}bcx-2(b^6c^4)^{\frac56}\sqrt{b^2c^2x^2+\sqrt{3}(b^6c^4)^{\frac16}bcx+(b^6c^4)^{\frac13}}}{b^6c^4}\right)-8(b^6c^4)^{\frac16}x^2\arctan\left(\frac{\sqrt{3}b^6c^4-2(b^6c^4)^{\frac56}bcx+2(b^6c^4)^{\frac56}\sqrt{b^2c^2x^2-\sqrt{3}(b^6c^4)^{\frac16}bcx+(b^6c^4)^{\frac13}}}{b^6c^4}\right)-16(b^6c^4)^{\frac16}x^2\arctan\left(-\frac{(b^6c^4)^{\frac56}bcx-(b^6c^4)^{\frac56}\sqrt{b^2c^2x^2+(b^6c^4)^{\frac13}}}{b^6c^4}\right)-8b\arctan(cx^3)-8a}{16x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(3)*(b^6*c^4)^(1/6)*x^2*log(4*b^2*c^2*x^2 + 4*sqrt(3)*(b^6*c^4)^(
1/6)*b*c*x + 4*(b^6*c^4)^(1/3)) - sqrt(3)*(b^6*c^4)^(1/6)*x^2*log(4*b^2*c^2
*x^2 - 4*sqrt(3)*(b^6*c^4)^(1/6)*b*c*x + 4*(b^6*c^4)^(1/3)) + sqrt(3)*(b^6*
c^4)^(1/6)*x^2*log(b^2*c^2*x^2 + sqrt(3)*(b^6*c^4)^(1/6)*b*c*x + (b^6*c^4)^
(1/3)) - sqrt(3)*(b^6*c^4)^(1/6)*x^2*log(b^2*c^2*x^2 - sqrt(3)*(b^6*c^4)^(1
/6)*b*c*x + (b^6*c^4)^(1/3)) - 8*(b^6*c^4)^(1/6)*x^2*arctan(-(sqrt(3)*b^6*c
^4 + 2*(b^6*c^4)^(5/6)*b*c*x - 2*(b^6*c^4)^(5/6)*sqrt(b^2*c^2*x^2 + sqrt(3)
*(b^6*c^4)^(1/6)*b*c*x + (b^6*c^4)^(1/3)))/(b^6*c^4)) - 8*(b^6*c^4)^(1/6)*x
^2*arctan((sqrt(3)*b^6*c^4 - 2*(b^6*c^4)^(5/6)*b*c*x + 2*(b^6*c^4)^(5/6)*sq
rt(b^2*c^2*x^2 - sqrt(3)*(b^6*c^4)^(1/6)*b*c*x + (b^6*c^4)^(1/3)))/(b^6*c^4
)) - 16*(b^6*c^4)^(1/6)*x^2*arctan(-((b^6*c^4)^(5/6)*b*c*x - (b^6*c^4)^(5/6
)*sqrt(b^2*c^2*x^2 + (b^6*c^4)^(1/3)))/(b^6*c^4)) - 8*b*arctan(c*x^3) - 8*a
)/x^2
```

Sympy [A]
time = 29.05, size = 245, normalized size = 1.48

$$\begin{cases} -\frac{a}{2x^2}+\frac{b\,\mathrm{atan}\,(cx^3)}{2\sqrt[3]{-\frac{1}{c^2}}}-\frac{b\,\mathrm{atan}\,(cx^3)}{2x^2}+\frac{3b\log\left(4x^2-4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{8c\left(-\frac{1}{c^2}\right)^{\frac56}}-\frac{3b\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{8c\left(-\frac{1}{c^2}\right)^{\frac56}}-\frac{\sqrt{3}\,b\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3\sqrt[6]{-\frac{1}{c^2}}}-\frac{\sqrt{3}}{3}\right)}{4c\left(-\frac{1}{c^2}\right)^{\frac56}}-\frac{\sqrt{3}\,b\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x}{3\sqrt[6]{-\frac{1}{c^2}}}+\frac{\sqrt{3}}{3}\right)}{4c\left(-\frac{1}{c^2}\right)^{\frac56}} & \text{for}\ c\neq 0 \\ -\frac{a}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) + b*atan(c*x**3)/(2*(-1/c**2)**(1/3)) - b*atan(c*x**
3)/(2*x**2) + 3*b*log(4*x**2 - 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(
8*c*(-1/c**2)**(5/6)) - 3*b*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)
**(1/3))/(8*c*(-1/c**2)**(5/6)) - sqrt(3)*b*atan(2*sqrt(3)*x/(3*(-1/c**2)**
(1/6)) - sqrt(3)/3)/(4*c*(-1/c**2)**(5/6)) - sqrt(3)*b*atan(2*sqrt(3)*x/(3*
(-1/c**2)**(1/6)) + sqrt(3)/3)/(4*c*(-1/c**2)**(5/6)), Ne(c, 0)), (-a/(2*x*
*2), True))
```

Giac [A]
time = 0.47, size = 137, normalized size = 0.83

$$\frac{1}{8}\left(\frac{\sqrt{3}\,\log\left(x^2+\frac{\sqrt{3}\,x}{|c|^{\frac13}}+\frac{1}{|c|^{\frac23}}\right)}{|c|^{\frac13}}-\frac{\sqrt{3}\,\log\left(x^2-\frac{\sqrt{3}\,x}{|c|^{\frac13}}+\frac{1}{|c|^{\frac23}}\right)}{|c|^{\frac13}}+\frac{2\,\arctan\left(\left(2\,x+\frac{\sqrt{3}}{|c|^{\frac13}}\right)|c|^{\frac13}\right)}{|c|^{\frac13}}+\frac{2\,\arctan\left(\left(2\,x-\frac{\sqrt{3}}{|c|^{\frac13}}\right)|c|^{\frac13}\right)}{|c|^{\frac13}}+\frac{4\,\arctan\left(x|c|^{\frac13}\right)}{|c|^{\frac13}}\right)bc-\frac{b\arctan\left(cx^3\right)+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3
) - sqrt(3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3)
 + 2*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/abs(c)^(1/3) + 2*arc
tan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/3))/abs(c)^(1/3) + 4*arctan(x*ab
s(c)^(1/3))/abs(c)^(1/3))*b*c - 1/2*(b*arctan(c*x^3) + a)/x^2
```

Mupad [B]
time = 0.91, size = 107, normalized size = 0.65

$$-\frac{a}{2\,x^2}-\frac{b\,c^{2/3}\,\left(\frac{\mathrm{atan}\left({\left(-1\right)}^{2/3}\,c^{1/3}\,x\right)}{2}-\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)}{2}+\mathrm{atan}\left(\frac{{\left(-1\right)}^{2/3}\,c^{1/3}\,x\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\right)}{2}-\frac{b\,\mathrm{atan}\left(c\,x^3\right)}{2\,x^2}-\frac{\sqrt{3}\,b\,c^{2/3}\,\left(\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)+\mathrm{atan}\left({\left(-1\right)}^{2/3}\,c^{1/3}\,x\right)\right)\,1{}\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))/x^3,x)
```

```
[Out] - a/(2*x^2) - (b*c^(2/3)*(atan((-1)^(2/3)*c^(1/3)*x)/2 - atan((c^(1/3)*x*(3
^(1/2)*1i + 1))/2)/2 + atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i + 1))/2)))/2
- (b*atan(c*x^3))/(2*x^2) - (3^(1/2)*b*c^(2/3)*(atan((c^(1/3)*x*(3^(1/2)*1i
 + 1))/2) + atan((-1)^(2/3)*c^(1/3)*x))*1i)/4
```

3.107 $\int \frac{a+b\mathbf{ArcTan}(cx^3)}{x^6}\,dx$

Optimal. Leaf size=115

$$-\frac{3bc}{10x^2}-\frac{a+b\text{ArcTan}(cx^3)}{5x^5}+\frac{1}{10}\sqrt{3}\,bc^{5/3}\text{ArcTan}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)+\frac{1}{10}bc^{5/3}\log\left(1+c^{2/3}x^2\right)-\frac{1}{20}bc^{5/3}\log\left(1-c\right.$$

```
[Out] -3/10*b*c/x^2+1/5*(-a-b*arctan(c*x^3))/x^5+1/10*b*c^(5/3)*ln(1+c^(2/3)*x^2)
-1/20*b*c^(5/3)*ln(1-c^(2/3)*x^2+c^(4/3)*x^4)+1/10*b*c^(5/3)*arctan(1/3*(1-
2*c^(2/3)*x^2)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {4946, 281, 331, 298, 31, 648, 631, 210, 642}

$$-\frac{a+b\text{ArcTan}(cx^3)}{5x^5}+\frac{1}{10}\sqrt{3}\,bc^{5/3}\text{ArcTan}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)+\frac{1}{10}bc^{5/3}\log\left(c^{2/3}x^2+1\right)-\frac{1}{20}bc^{5/3}\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)-\frac{3bc}{10x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x^6,x]
```

```
[Out] (-3*b*c)/(10*x^2) - (a + b*ArcTan[c*x^3])/(5*x^5) + (Sqrt[3]*b*c^(5/3)*ArcT
an[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/10 + (b*c^(5/3)*Log[1 + c^(2/3)*x^2])/10 -
 (b*c^(5/3)*Log[1 - c^(2/3)*x^2 + c^(4/3)*x^4])/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
```

```
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx^3)}{x^6}\,dx = -\frac{a+b\tan^{-1}(cx^3)}{5x^5}+\frac{1}{5}(3bc)\int \frac{1}{x^3(1+c^2x^6)}\,dx$$

$$= -\frac{a+b\tan^{-1}(cx^3)}{5x^5}+\frac{1}{10}(3bc)\text{Subst}\left(\int \frac{1}{x^2(1+c^2x^3)}\,dx,x,x^2\right)$$

$$= -\frac{3bc}{10x^2}-\frac{a+b\tan^{-1}(cx^3)}{5x^5}-\frac{1}{10}(3bc^3)\,\text{Subst}\left(\int \frac{x}{1+c^2x^3}\,dx,x,x^2\right)$$

$$= -\frac{3bc}{10x^2}-\frac{a+b\tan^{-1}(cx^3)}{5x^5}+\frac{1}{10}(bc^{7/3})\,\text{Subst}\left(\int \frac{1}{1+c^{2/3}x}\,dx,x,x^2\right)-\frac{1}{10}(bc^{7/3}$$

$$= -\frac{3bc}{10x^2}-\frac{a+b\tan^{-1}(cx^3)}{5x^5}+\frac{1}{10}bc^{5/3}\log\left(1+c^{2/3}x^2\right)-\frac{1}{20}(bc^{5/3})\,\text{Subst}\left(\int \frac{-c^2}{1-c}\right.$$

$$= -\frac{3bc}{10x^2}-\frac{a+b\tan^{-1}(cx^3)}{5x^5}+\frac{1}{10}bc^{5/3}\log\left(1+c^{2/3}x^2\right)-\frac{1}{20}bc^{5/3}\log\left(1-c^{2/3}x^2+c\right.$$

$$= -\frac{3bc}{10x^2}-\frac{a+b\tan^{-1}(cx^3)}{5x^5}+\frac{1}{10}\sqrt{3}\,bc^{5/3}\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)+\frac{1}{10}bc^{5/3}\log\left(1+\right.$$

Mathematica [A]

time = 0.03, size = 183, normalized size = 1.59

$$-\frac{a}{5x^5}-\frac{3bc}{10x^2}-\frac{b\text{ArcTan}(cx^3)}{5x^5}+\frac{1}{10}\sqrt{3}\,bc^{5/3}\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)+\frac{1}{10}\sqrt{3}\,bc^{5/3}\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{c}\,x\right)+\frac{1}{10}bc^{5/3}\log\left(1+c^{2/3}x^2\right)-\frac{1}{20}bc^{5/3}\log\left(1-\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2\right)-\frac{1}{20}bc^{5/3}\log\left(1+\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^6,x]
```

```
[Out] -1/5*a/x^5 - (3*b*c)/(10*x^2) - (b*ArcTan[c*x^3])/(5*x^5) + (Sqrt[3]*b*c^(5
/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/10 + (Sqrt[3]*b*c^(5/3)*ArcTan[Sqrt[3] +
 2*c^(1/3)*x])/10 + (b*c^(5/3)*Log[1 + c^(2/3)*x^2])/10 - (b*c^(5/3)*Log[1
- Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/20 - (b*c^(5/3)*Log[1 + Sqrt[3]*c^(1/3)
*x + c^(2/3)*x^2])/20
```

Maple [A]

time = 0.07, size = 105, normalized size = 0.91

method	result
default	$-\frac{a}{5x^5}-\frac{b\arctan(c\,x^3)}{5x^5}-\frac{3bc}{10x^2}+\frac{bc\ln\left(x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{10\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-\frac{bc\ln\left(x^4-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{20\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-\right.}{3}\right.}{10\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}$

risch	$\frac{ib\ln(icx^3+1)}{10x^5} - \frac{ib\ln(-icx^3+1)}{10x^5} - \frac{3bc}{10x^2} - \frac{bc\ln\left(x+\left(\frac{i}{c}\right)^{\frac{1}{3}}\right)}{10\left(\frac{i}{c}\right)^{\frac{2}{3}}} + \frac{bc\ln\left(x^2-\left(\frac{i}{c}\right)^{\frac{1}{3}}x+\left(\frac{i}{c}\right)^{\frac{2}{3}}\right)}{20\left(\frac{i}{c}\right)^{\frac{2}{3}}} - \frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{i}{c}\right)^{\frac{1}{3}}} \cdots\right.}{3} \cdots\right.}{10\left(\frac{i}{c}\right)^{\frac{2}{3}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*a/x^5-1/5*b/x^5*arctan(c*x^3)-3/10*b*c/x^2+1/10*b*c/(1/c^2)^(1/3)*ln(x
^2+(1/c^2)^(1/3))-1/20*b*c/(1/c^2)^(1/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2)^(
2/3))-1/10*b*c*3^(1/2)/(1/c^2)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1/3)*x^
2-1))
```

Maxima [A]
time = 0.47, size = 102, normalized size = 0.89

$$-\frac{1}{20}\left(\left(\left(2\sqrt{3}\,c^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{4}{3}}x^2-c^{\frac{2}{3}}\right)}{3\,c^{\frac{2}{3}}}\right)+c^{\frac{2}{3}}\log\left(c^{\frac{4}{3}}x^4-c^{\frac{2}{3}}x^2+1\right)-2\,c^{\frac{2}{3}}\log\left(\frac{c^{\frac{2}{3}}x^2+1}{c^{\frac{2}{3}}}\right)+\frac{6}{x^2}\right)c+\frac{4\arctan\left(cx^3\right)}{x^5}\right)b-\frac{a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^6,x, algorithm="maxima")
```

```
[Out] -1/20*((2*sqrt(3)*c^(2/3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3))/c^(2
/3)) + c^(2/3)*log(c^(4/3)*x^4 - c^(2/3)*x^2 + 1) - 2*c^(2/3)*log((c^(2/3)*
x^2 + 1)/c^(2/3)) + 6/x^2)*c + 4*arctan(c*x^3)/x^5)*b - 1/5*a/x^5
```

Fricas [A]
time = 1.16, size = 121, normalized size = 1.05

$$-\frac{2\sqrt{3}\,b(c^2)^{\frac{1}{3}}cx^5\arctan\left(\frac{2}{3}\sqrt{3}\,(c^2)^{\frac{1}{3}}x^2-\frac{1}{3}\sqrt{3}\right)+b(c^2)^{\frac{1}{3}}cx^5\log\left(c^2x^4-(c^2)^{\frac{2}{3}}x^2+(c^2)^{\frac{1}{3}}\right)-2\,b(c^2)^{\frac{1}{3}}cx^5\log\left(c^2x^2+(c^2)^{\frac{2}{3}}\right)+6\,bcx^3+4\,b\arctan\left(cx^3\right)+4\,a}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^6,x, algorithm="fricas")
```

```
[Out] -1/20*(2*sqrt(3)*b*(c^2)^(1/3)*c*x^5*arctan(2/3*sqrt(3)*(c^2)^(1/3)*x^2 - 1
/3*sqrt(3)) + b*(c^2)^(1/3)*c*x^5*log(c^2*x^4 - (c^2)^(2/3)*x^2 + (c^2)^(1/
3)) - 2*b*(c^2)^(1/3)*c*x^5*log(c^2*x^2 + (c^2)^(2/3)) + 6*b*c*x^3 + 4*b*ar
ctan(c*x^3) + 4*a)/x^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(112) = 224$.
time = 58.14, size = 286, normalized size = 2.49

$$\begin{cases} -\frac{a}{5x^5}+\frac{bc^2\sqrt[6]{-\frac{1}{c^2}}\operatorname{atan}\left(cx^3\right)}{5}-\frac{bc\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{5\sqrt[3]{-\frac{1}{c^2}}}+\frac{3bc\log\left(4x^2-4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{20\sqrt[3]{-\frac{1}{c^2}}}-\frac{bc\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{20\sqrt[3]{-\frac{1}{c^2}}}-\frac{\sqrt{3}\,bc\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3\sqrt[6]{-\frac{1}{c^2}}}-\frac{\sqrt{3}}{3}\right)}{10\sqrt[3]{-\frac{1}{c^2}}}+\frac{\sqrt{3}\,bc\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3\sqrt[6]{-\frac{1}{c^2}}}+\frac{\sqrt{3}}{3}\right)}{10\sqrt[3]{-\frac{1}{c^2}}}-\frac{3bc}{10x^2}-\frac{b\operatorname{atan}\left(cx^3\right)}{5x^5} & \text{for}\ c\neq 0 \\ -\frac{a}{5x^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**6,x)
```

```
[Out] Piecewise((-a/(5*x**5) + b*c**2*(-1/c**2)**(1/6)*atan(c*x**3)/5 - b*c*log(x
 - (-1/c**2)**(1/6))/(5*(-1/c**2)**(1/3)) + 3*b*c*log(4*x**2 - 4*x*(-1/c**2
)**(1/6) + 4*(-1/c**2)**(1/3))/(20*(-1/c**2)**(1/3)) - b*c*log(4*x**2 + 4*x
*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(20*(-1/c**2)**(1/3)) - sqrt(3)*b*c
*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3)/3)/(10*(-1/c**2)**(1/3)) +
 sqrt(3)*b*c*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/(10*(-1/c**
2)**(1/3)) - 3*b*c/(10*x**2) - b*atan(c*x**3)/(5*x**5), Ne(c, 0)), (-a/(5*x
**5), True))
```

Giac [A]
time = 0.49, size = 108, normalized size = 0.94

$$-\frac{1}{20}\,bc^3\left(\frac{2\,\sqrt{3}\,|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^2-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^2}+\frac{|c|^{\frac{2}{3}}\log\left(x^4-\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^2}-\frac{2\,\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{4}{3}}}\right)-\frac{3\,bcx^3+2\,b\arctan\left(cx^3\right)+2\,a}{10\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^6,x, algorithm="giac")
```

```
[Out] -1/20*b*c^3*(2*sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2
/3))*abs(c)^(2/3))/c^2 + abs(c)^(2/3)*log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)
^(4/3))/c^2 - 2*log(x^2 + 1/abs(c)^(2/3))/abs(c)^(4/3)) - 1/10*(3*b*c*x^3 +
 2*b*arctan(c*x^3) + 2*a)/x^5
```

Mupad [B]
time = 2.57, size = 118, normalized size = 1.03

$$\frac{b\,c^{5/3}\,\ln\left(c^{2/3}\,x^2+1\right)}{10}-\frac{\frac{3\,b\,c\,x^3}{2}+a}{5\,x^5}-\frac{b\,\mathrm{atan}\left(c\,x^3\right)}{5\,x^5}-\frac{b\,c^{5/3}\,\ln\left(\sqrt{3}\,c^{2/3}\,x^2-c^{2/3}\,x^2\,1\mathrm{i}+2\mathrm{i}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{20}+\frac{b\,c^{5/3}\,\ln\left(-c^{2/3}\,x^2\,1\mathrm{i}-\sqrt{3}\,c^{2/3}\,x^2+2\mathrm{i}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))/x^6,x)
```

```
[Out] (b*c^(5/3)*log(c^(2/3)*x^2 + 1))/10 - (a + (3*b*c*x^3)/2)/(5*x^5) - (b*atan
(c*x^3))/(5*x^5) - (b*c^(5/3)*log(3^(1/2)*c^(2/3)*x^2 - c^(2/3)*x^2*1i + 2i
)*(3^(1/2)*1i + 1))/20 + (b*c^(5/3)*log(2i - 3^(1/2)*c^(2/3)*x^2 - c^(2/3)*
x^2*1i)*(3^(1/2)*1i - 1))/20
```

3.108 $\int x^7(a+b\textbf{ArcTan}(cx^3))\,dx$

Optimal. Leaf size=176

$$-\frac{3bx^5}{40c}+\frac{b\text{ArcTan}\left(\sqrt[3]{c}\,x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\text{ArcTan}\left(cx^3\right)\right)-\frac{b\text{ArcTan}\left(\sqrt{3}\,-2\sqrt[3]{c}\,x\right)}{16c^{8/3}}+\frac{b\text{ArcTan}\left(\sqrt{3}\,+2\sqrt[3]{c}\,x\right)}{16c^{8/3}}$$

```
[Out] -3/40*b*x^5/c+1/8*b*arctan(c^(1/3)*x)/c^(8/3)+1/8*x^8*(a+b*arctan(c*x^3))+1
/16*b*arctan(2*c^(1/3)*x-3^(1/2))/c^(8/3)+1/16*b*arctan(2*c^(1/3)*x+3^(1/2)
)/c^(8/3)+1/32*b*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)/c^(8/3)-1/32*b
*ln(1+c^(2/3)*x^2+c^(1/3)*x*3^(1/2))*3^(1/2)/c^(8/3)
```

Rubi [A]
time = 0.31, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.571$, Rules used = {4946, 327, 301, 648, 632, 210, 642, 209}

$$\frac{1}{8}x^8\left(a+b\text{ArcTan}\left(cx^3\right)\right)+\frac{b\text{ArcTan}\left(\sqrt[3]{c}\,x\right)}{8c^{8/3}}-\frac{b\text{ArcTan}\left(\sqrt{3}\,-2\sqrt[3]{c}\,x\right)}{16c^{8/3}}+\frac{b\text{ArcTan}\left(2\sqrt[3]{c}\,x+\sqrt{3}\right)}{16c^{8/3}}+\frac{\sqrt{3}\,b\log\left(c^{2/3}x^2-\sqrt{3}\,\sqrt[3]{c}\,x+1\right)}{32c^{8/3}}-\frac{\sqrt{3}\,b\log\left(c^{2/3}x^2+\sqrt{3}\,\sqrt[3]{c}\,x+1\right)}{32c^{8/3}}-\frac{3bx^5}{40c}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x^5)/(40*c) + (b*ArcTan[c^(1/3)*x])/(8*c^(8/3)) + (x^8*(a + b*ArcTan[
c*x^3]))/8 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(16*c^(8/3)) + (b*ArcTan[Sqr
t[3] + 2*c^(1/3)*x])/(16*c^(8/3)) + (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x +
c^(2/3)*x^2])/(32*c^(8/3)) - (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)
*x^2])/(32*c^(8/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
```

```
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int x^7\left(a+b\tan^{-1}\left(cx^3\right)\right)\,dx=\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{8}(3bc)\int\frac{x^{10}}{1+c^2x^6}\,dx$$

$$=-\frac{3bx^5}{40c}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{(3b)\int\frac{x^4}{1+c^2x^6}\,dx}{8c}$$

$$=-\frac{3bx^5}{40c}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\int\frac{1}{1+c^{2/3}x^2}\,dx}{8c^{7/3}}+\frac{b\int\frac{-\frac{1}{2}+\frac{1}{2}\sqrt{3}\sqrt[3]{c}\,x}{1-\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2}\,d}{8c^{7/3}}$$

$$=-\frac{3bx^5}{40c}+\frac{b\tan^{-1}\left(\sqrt[3]{c}\,x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{\left(\sqrt{3}\,b\right)\int\frac{-\sqrt{3}\sqrt[3]{c}+}{1-\sqrt{3}\sqrt[3]{c}\,x}}{32c^{8/3}}$$

$$=-\frac{3bx^5}{40c}+\frac{b\tan^{-1}\left(\sqrt[3]{c}\,x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{\sqrt{3}\,b\log\left(1-\sqrt{3}\sqrt[3]{c}\right.}{32c^{8/3}}$$

$$=-\frac{3bx^5}{40c}+\frac{b\tan^{-1}\left(\sqrt[3]{c}\,x\right)}{8c^{8/3}}+\frac{1}{8}x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}\right.}{16c^{8/3}}$$

Mathematica [A]
time = 0.06, size = 181, normalized size = 1.03

$$-\frac{3bx^5}{40c}+\frac{ax^8}{8}+\frac{b\text{ArcTan}\left(\sqrt[3]{c}\,x\right)}{8c^{8/3}}+\frac{1}{8}bx^8\text{ArcTan}\left(cx^3\right)-\frac{b\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)}{16c^{8/3}}+\frac{b\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{c}\,x\right)}{16c^{8/3}}+\frac{\sqrt{3}\,b\log\left(1-\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2\right)}{32c^{8/3}}-\frac{\sqrt{3}\,b\log\left(1+\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2\right)}{32c^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x^5)/(40*c) + (a*x^8)/8 + (b*ArcTan[c^(1/3)*x])/(8*c^(8/3)) + (b*x^8*
ArcTan[c*x^3])/8 - (b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(16*c^(8/3)) + (b*ArcT
an[Sqrt[3] + 2*c^(1/3)*x])/(16*c^(8/3)) + (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3
)*x + c^(2/3)*x^2])/(32*c^(8/3)) - (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c
^(2/3)*x^2])/(32*c^(8/3))
```

Maple [A]
time = 0.10, size = 167, normalized size = 0.95

method	result
default	$\frac{x^8a}{8}+\frac{x^8b\arctan\left(cx^3\right)}{8}-\frac{3bx^5}{40c}+\frac{b\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2-\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{32c}+\frac{b\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{16c^3\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}-\frac{b\sqrt{3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x^8*a+1/8*x^8*b*arctan(c*x^3)-3/40*b*x^5/c+1/32*b/c*3^(1/2)*(1/c^2)^(5/
6)*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/16*b/c^3/(1/c^2)^(1/6)*a
rctan(2*x/(1/c^2)^(1/6)-3^(1/2))-1/32*b/c*3^(1/2)*(1/c^2)^(5/6)*ln(x^2+3^(1
/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))+1/16*b/c^3/(1/c^2)^(1/6)*arctan(2*x/(1/c
^2)^(1/6)+3^(1/2))+1/8*b/c^3/(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6))
```

Maxima [**A**]
time = 0.47, size = 152, normalized size = 0.86

$$\frac{1}{8}\,ax^{8}+\frac{1}{160}\left(20\,x^{8}\arctan\left(cx^{3}\right)-\left(\frac{12\,x^{5}}{c^{2}}+\frac{5\left(\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^{2}+\sqrt{3}\,c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^{2}-\sqrt{3}\,c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{4\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{5}{3}}}-\frac{2\arctan\left(\frac{2\,c^{\frac{2}{3}}x+\sqrt{3}\,c^{\frac{1}{3}}}{c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}-\frac{2\arctan\left(\frac{2\,c^{\frac{2}{3}}x-\sqrt{3}\,c^{\frac{1}{3}}}{c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}\right)}{c^{2}}\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8 + 1/160*(20*x^8*arctan(c*x^3) - (12*x^5/c^2 + 5*(sqrt(3)*log(c^(2
/3)*x^2 + sqrt(3)*c^(1/3)*x + 1)/c^(5/3) - sqrt(3)*log(c^(2/3)*x^2 - sqrt(3
)*c^(1/3)*x + 1)/c^(5/3) - 4*arctan(c^(1/3)*x)/c^(5/3) - 2*arctan((2*c^(2/3
)*x + sqrt(3)*c^(1/3))/c^(1/3))/c^(5/3) - 2*arctan((2*c^(2/3)*x - sqrt(3)*c
^(1/3))/c^(1/3))/c^(5/3))/c^2)*c)*b
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 439 vs. 2(128) = 256.
time = 1.74, size = 439, normalized size = 2.49

$$\frac{20\,bcx^{8}\arctan\left(cx^{3}\right)+20\,acx^{8}-12\,bx^{5}-5\,\sqrt{3}\,c\left(\frac{b^{6}}{c^{16}}\right)^{\frac{1}{6}}\log\left(\sqrt{3}\,b^{5}c^{13}x\left(\frac{b^{6}}{c^{16}}\right)^{\frac{5}{6}}+b^{6}c^{10}\left(\frac{b^{6}}{c^{16}}\right)^{\frac{2}{3}}+b^{10}x^{2}\right)+5\,\sqrt{3}\,c\left(\frac{b^{6}}{c^{16}}\right)^{\frac{1}{6}}\log\left(-\sqrt{3}\,b^{5}c^{13}x\left(\frac{b^{6}}{c^{16}}\right)^{\frac{5}{6}}+b^{6}c^{10}\left(\frac{b^{6}}{c^{16}}\right)^{\frac{2}{3}}+b^{10}x^{2}\right)-20\,c\left(\frac{b^{6}}{c^{16}}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\,b^{5}c^{3}x\left(\frac{b^{6}}{c^{16}}\right)^{\frac{1}{6}}+\sqrt{3}\,b^{6}-2\,\sqrt{\sqrt{3}\,b^{5}c^{13}x\left(\frac{b^{6}}{c^{16}}\right)^{\frac{5}{6}}+b^{6}c^{10}\left(\frac{b^{6}}{c^{16}}\right)^{\frac{2}{3}}+b^{10}x^{2}}\,c^{3}\left(\frac{b^{6}}{c^{16}}\right)^{\frac{1}{6}}}{b^{6}}\right)-20\,c\left(\frac{b^{6}}{c^{16}}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\,b^{5}c^{3}x\left(\frac{b^{6}}{c^{16}}\right)^{\frac{1}{6}}-\sqrt{3}\,b^{6}-2\,\sqrt{-\sqrt{3}\,b^{5}c^{13}x\left(\frac{b^{6}}{c^{16}}\right)^{\frac{5}{6}}+b^{6}c^{10}\left(\frac{b^{6}}{c^{16}}\right)^{\frac{2}{3}}+b^{10}x^{2}}\,c^{3}\left(\frac{b^{6}}{c^{16}}\right)^{\frac{1}{6}}}{b^{6}}\right)-40\,c\left(\frac{b^{6}}{c^{16}}\right)^{\frac{1}{6}}\arctan\left(-\frac{b^{5}c^{3}x\left(\frac{b^{6}}{c^{16}}\right)^{\frac{1}{6}}-\sqrt{b^{6}c^{10}\left(\frac{b^{6}}{c^{16}}\right)^{\frac{2}{3}}+b^{10}x^{2}}\,c^{3}\left(\frac{b^{6}}{c^{16}}\right)^{\frac{1}{6}}}{b^{6}}\right)}{160\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/160*(20*b*c*x^8*arctan(c*x^3) + 20*a*c*x^8 - 12*b*x^5 - 5*sqrt(3)*c*(b^6/
c^16)^(1/6)*log(sqrt(3)*b^5*c^13*x*(b^6/c^16)^(5/6) + b^6*c^10*(b^6/c^16)^(
2/3) + b^10*x^2) + 5*sqrt(3)*c*(b^6/c^16)^(1/6)*log(-sqrt(3)*b^5*c^13*x*(b^
6/c^16)^(5/6) + b^6*c^10*(b^6/c^16)^(2/3) + b^10*x^2) - 20*c*(b^6/c^16)^(1/
6)*arctan(-(2*b^5*c^3*x*(b^6/c^16)^(1/6) + sqrt(3)*b^6 - 2*sqrt(sqrt(3)*b^5
*c^13*x*(b^6/c^16)^(5/6) + b^6*c^10*(b^6/c^16)^(2/3) + b^10*x^2)*c^3*(b^6/c
^16)^(1/6))/b^6) - 20*c*(b^6/c^16)^(1/6)*arctan(-(2*b^5*c^3*x*(b^6/c^16)^(1
/6) - sqrt(3)*b^6 - 2*sqrt(-sqrt(3)*b^5*c^13*x*(b^6/c^16)^(5/6) + b^6*c^10*
(b^6/c^16)^(2/3) + b^10*x^2)*c^3*(b^6/c^16)^(1/6))/b^6) - 40*c*(b^6/c^16)^(
1/6)*arctan(-(b^5*c^3*x*(b^6/c^16)^(1/6) - sqrt(b^6*c^10*(b^6/c^16)^(2/3) +
 b^10*x^2)*c^3*(b^6/c^16)^(1/6))/b^6))/c
```

Sympy [A]
time = 61.13, size = 264, normalized size = 1.50

$$\begin{cases} \frac{ax^8}{8} + \frac{bx^8\operatorname{atan}(cx^3)}{8} - \frac{3bx^5}{40c} + \frac{3b\log\left(4x^2-4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{32c^3\sqrt[6]{-\frac{1}{c^2}}} - \frac{3b\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{32c^3\sqrt[6]{-\frac{1}{c^2}}} + \frac{\sqrt{3}b\operatorname{atan}\left(\frac{2\sqrt{3}x}{3\sqrt[6]{-\frac{1}{c^2}}}-\frac{\sqrt{3}}{3}\right)}{16c^3\sqrt[6]{-\frac{1}{c^2}}} + \frac{\sqrt{3}b\operatorname{atan}\left(\frac{2\sqrt{3}x}{3\sqrt[6]{-\frac{1}{c^2}}}+\frac{\sqrt{3}}{3}\right)}{16c^3\sqrt[6]{-\frac{1}{c^2}}} - \frac{b\operatorname{atan}(cx^3)}{8c^4\left(-\frac{1}{c^2}\right)^{\frac{2}{3}}} & \text{for } c \neq 0 \\ \frac{ax^8}{8} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**8/8 + b*x**8*atan(c*x**3)/8 - 3*b*x**5/(40*c) + 3*b*log(4*x
**2 - 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(32*c**3*(-1/c**2)**(1/6))
 - 3*b*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(32*c**3*(-1
/c**2)**(1/6)) + sqrt(3)*b*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3)/
3)/(16*c**3*(-1/c**2)**(1/6)) + sqrt(3)*b*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1
/6)) + sqrt(3)/3)/(16*c**3*(-1/c**2)**(1/6)) - b*atan(c*x**3)/(8*c**4*(-1/c
**2)**(2/3)), Ne(c, 0)), (a*x**8/8, True))
```

Giac [A]
time = 0.49, size = 171, normalized size = 0.97

$$-\frac{1}{32}bc^{15}\left(\frac{\sqrt{3}\log\left(x^2+\frac{\sqrt{3}x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{16}|c|^{\frac{5}{3}}} - \frac{\sqrt{3}\log\left(x^2-\frac{\sqrt{3}x}{|c|^{\frac13}}+\frac{1}{|c|^{\frac23}}\right)}{c^{16}|c|^{\frac53}} - \frac{2|c|^{\frac13}\arctan\left(\left(2x+\frac{\sqrt3}{|c|^{\frac13}}\right)|c|^{\frac13}\right)}{c^{18}} - \frac{2|c|^{\frac13}\arctan\left(\left(2x-\frac{\sqrt3}{|c|^{\frac13}}\right)|c|^{\frac13}\right)}{c^{18}} - \frac{4|c|^{\frac13}\arctan\left(x|c|^{\frac13}\right)}{c^{18}}\right) + \frac{5bcx^8\arctan(cx^3)+5acx^8-3bx^5}{40c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] -1/32*b*c^15*(sqrt(3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/(c
^16*abs(c)^(5/3)) - sqrt(3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/
3))/(c^16*abs(c)^(5/3)) - 2*abs(c)^(1/3)*arctan((2*x + sqrt(3)/abs(c)^(1/3)
)*abs(c)^(1/3))/c^18 - 2*abs(c)^(1/3)*arctan((2*x - sqrt(3)/abs(c)^(1/3))*a
bs(c)^(1/3))/c^18 - 4*abs(c)^(1/3)*arctan(x*abs(c)^(1/3))/c^18) + 1/40*(5*b
*c*x^8*arctan(c*x^3) + 5*a*c*x^8 - 3*b*x^5)/c
```

Mupad [B]
time = 1.00, size = 122, normalized size = 0.69

$$\frac{a\,x^8}{8} - \frac{3\,b\,x^5}{40\,c} - \frac{b\left(\operatorname{atan}\left((-1)^{2/3}c^{1/3}x\right)+\operatorname{atan}\left(\frac{(-1)^{2/3}c^{1/3}x\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)+2\operatorname{atan}\left(\frac{(-1)^{2/3}c^{1/3}x\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)}{16\,c^{8/3}} + \frac{b\,x^8\operatorname{atan}(c\,x^3)}{8} + \frac{\sqrt{3}\,b\left(\operatorname{atan}\left((-1)^{2/3}c^{1/3}x\right)-\operatorname{atan}\left(\frac{(-1)^{2/3}c^{1/3}x\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)1\mathrm{i}}{16\,c^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*atan(c*x^3)),x)
```

```
[Out] (a*x^8)/8 - (3*b*x^5)/(40*c) - (b*(atan((-1)^(2/3)*c^(1/3)*x) + atan(((-1)^
(2/3)*c^(1/3)*x*(3^(1/2)*1i - 1))/2) + 2*atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2
)*1i + 1))/2)))/(16*c^(8/3)) + (b*x^8*atan(c*x^3))/8 + (3^(1/2)*b*(atan((-1
)^(2/3)*c^(1/3)*x) - atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i - 1))/2))*1i)/(
16*c^(8/3))
```

3.109 $\int x^4(a+b\mathbf{ArcTan}(cx^3))\,dx$

Optimal. Leaf size=117

$$-\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\text{ArcTan}\left(cx^3\right)\right)-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{b\log\left(1+c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)}{20c^{5/3}}$$

```
[Out] -3/10*b*x^2/c+1/5*x^5*(a+b*arctan(c*x^3))+1/10*b*ln(1+c^(2/3)*x^2)/c^(5/3)-
1/20*b*ln(1-c^(2/3)*x^2+c^(4/3)*x^4)/c^(5/3)-1/10*b*arctan(1/3*(1-2*c^(2/3)
*x^2)*3^(1/2))*3^(1/2)/c^(5/3)
```

Rubi [A]
time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {4946, 281, 327, 206, 31, 648, 631, 210, 642}

$$\frac{1}{5}x^5\left(a+b\text{ArcTan}\left(cx^3\right)\right)-\frac{\sqrt{3}\,b\text{ArcTan}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{b\log\left(c^{2/3}x^2+1\right)}{10c^{5/3}}-\frac{b\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)}{20c^{5/3}}-\frac{3bx^2}{10c}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x^2)/(10*c) + (x^5*(a + b*ArcTan[c*x^3]))/5 - (Sqrt[3]*b*ArcTan[(1 -
2*c^(2/3)*x^2)/Sqrt[3]])/(10*c^(5/3)) + (b*Log[1 + c^(2/3)*x^2])/(10*c^(5/3
)) - (b*Log[1 - c^(2/3)*x^2 + c^(4/3)*x^4])/(20*c^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int x^4\left(a+b\tan^{-1}\left(cx^3\right)\right)\,dx = \frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{5}(3bc)\int\frac{x^7}{1+c^2x^6}\,dx$$

$$= \frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{10}(3bc)\text{Subst}\left(\int\frac{x^3}{1+c^2x^3}\,dx,x,x^2\right)$$

$$= -\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{(3b)\text{Subst}\left(\int\frac{1}{1+c^2x^3}\,dx,x,x^2\right)}{10c}$$

$$= -\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\text{Subst}\left(\int\frac{1}{1+c^{2/3}x}\,dx,x,x^2\right)}{10c}+\frac{b\text{Subst}\left(\int \right.}{}$$

$$= -\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1+c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\text{Subst}\left(\int\frac{-c^{2/3}+2c^{4/3}}{1-c^{2/3}x+c^{4/3}}\right.}{20c^{5/3}}$$

$$= -\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\log\left(1+c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(1-c^{2/3}x^2+c^{4}\right.}{20c^{5/3}}$$

$$= -\frac{3bx^2}{10c}+\frac{1}{5}x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\sqrt{3}\,b\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{b\log\left(1+c^{2/3}x\right.}{10c^{5/3}}$$

Mathematica [A]
time = 0.02, size = 185, normalized size = 1.58

$$-\frac{3bx^2}{10c}+\frac{ax^5}{5}+\frac{1}{5}bx^5\text{ArcTan}\left(cx^3\right)-\frac{\sqrt{3}\,b\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)}{10c^{5/3}}-\frac{\sqrt{3}\,b\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{c}\,x\right)}{10c^{5/3}}+\frac{b\log\left(1+c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(1-\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2\right)}{20c^{5/3}}-\frac{b\log\left(1+\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2\right)}{20c^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (-3*b*x^2)/(10*c) + (a*x^5)/5 + (b*x^5*ArcTan[c*x^3])/5 - (Sqrt[3]*b*ArcTan
[Sqrt[3] - 2*c^(1/3)*x])/(10*c^(5/3)) - (Sqrt[3]*b*ArcTan[Sqrt[3] + 2*c^(1/
3)*x])/(10*c^(5/3)) + (b*Log[1 + c^(2/3)*x^2])/(10*c^(5/3)) - (b*Log[1 - Sq
rt[3]*c^(1/3)*x + c^(2/3)*x^2])/(20*c^(5/3)) - (b*Log[1 + Sqrt[3]*c^(1/3)*x
 + c^(2/3)*x^2])/(20*c^(5/3))
```

Maple [A]
time = 0.05, size = 113, normalized size = 0.97

method	result

default	$\frac{a\,x^{5}}{5}+\frac{b\,x^{5}\arctan\left(c\,x^{3}\right)}{5}-\frac{3b\,x^{2}}{10c}+\frac{b\ln\left(x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{10c^{3}\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x^{4}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}\right)}{20c^{3}\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{2}}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-\right.}{3}\right.}{10c^{3}\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}$
risch	$-\frac{i x^{5} b\ln\left(i c\,x^{3}+1\right)}{10}+\frac{i b\,x^{5}\ln\left(-i c\,x^{3}+1\right)}{10}-\frac{3b\,x^{2}}{10c}-\frac{i b\ln\left(x+\left(\frac{i}{c}\right)^{\frac{1}{3}}\right)}{10c^{2}\left(\frac{i}{c}\right)^{\frac{1}{3}}}+\frac{i b\ln\left(x^{2}-\left(\frac{i}{c}\right)^{\frac{1}{3}}x+\left(\frac{i}{c}\right)^{\frac{2}{3}}\right)}{20c^{2}\left(\frac{i}{c}\right)^{\frac{1}{3}}}+\frac{i b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}}{}\right.}{10c^{2}\left(\frac{i}{c}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*a*x^5+1/5*b*x^5*arctan(c*x^3)-3/10*b*x^2/c+1/10*b/c^3/(1/c^2)^(2/3)*ln(
x^2+(1/c^2)^(1/3))-1/20*b/c^3/(1/c^2)^(2/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2
)^(2/3))+1/10*b/c^3/(1/c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1/
3)*x^2-1))
```

Maxima [A]
time = 0.48, size = 106, normalized size = 0.91

$$\frac{1}{5}\,ax^{5}+\frac{1}{20}\left(4\,x^{5}\arctan\left(cx^{3}\right)-c\left(\frac{6\,x^{2}}{c^{2}}-\frac{2\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\left(2\,c^{\frac{4}{3}}x^{2}-c^{\frac{2}{3}}\right)}{3\,c^{\frac{2}{3}}}\right)}{c^{\frac{8}{3}}}+\frac{\log\left(c^{\frac{4}{3}}x^{4}-c^{\frac{2}{3}}x^{2}+1\right)}{c^{\frac{8}{3}}}-\frac{2\,\log\left(\frac{c^{\frac{2}{3}}x^{2}+1}{c^{\frac{2}{3}}}\right)}{c^{\frac{8}{3}}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/20*(4*x^5*arctan(c*x^3) - c*(6*x^2/c^2 - 2*sqrt(3)*arctan(1/3
*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3))/c^(2/3))/c^(8/3) + log(c^(4/3)*x^4 - c^(
2/3)*x^2 + 1)/c^(8/3) - 2*log((c^(2/3)*x^2 + 1)/c^(2/3))/c^(8/3)))*b
```

Fricas [A]
time = 1.60, size = 137, normalized size = 1.17

$$\frac{4\,bc^{3}x^{5}\arctan\left(cx^{3}\right)+4\,ac^{3}x^{5}-6\,bc^{2}x^{2}+2\,\sqrt{3}\,b(c^{2})^{\frac{1}{6}}c\arctan\left(\frac{\sqrt{3}\,\left(2\,(c^{2})^{\frac{2}{3}}x^{2}-(c^{2})^{\frac{1}{3}}\right)(c^{2})^{\frac{1}{6}}}{3\,c}\right)-b(c^{2})^{\frac{2}{3}}\log\left(c^{2}x^{4}-(c^{2})^{\frac{2}{3}}x^{2}+(c^{2})^{\frac{1}{3}}\right)+2\,b(c^{2})^{\frac{2}{3}}\log\left(c^{2}x^{2}+(c^{2})^{\frac{2}{3}}\right)}{20\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*c^3*x^5*arctan(c*x^3) + 4*a*c^3*x^5 - 6*b*c^2*x^2 + 2*sqrt(3)*b*(
c^2)^(1/6)*c*arctan(1/3*sqrt(3)*(2*(c^2)^(2/3)*x^2 - (c^2)^(1/3))*(c^2)^(1/
6)/c) - b*(c^2)^(2/3)*log(c^2*x^4 - (c^2)^(2/3)*x^2 + (c^2)^(1/3)) + 2*b*(c
^2)^(2/3)*log(c^2*x^2 + (c^2)^(2/3)))/c^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(109) = 218$.

time = 29.54, size = 292, normalized size = 2.50

$$\begin{cases} \frac{ax^5}{5} - \frac{bc^3\left(-\frac{1}{c^2}\right)^{\frac{7}{3}}\log\left(x - \sqrt[6]{-\frac{1}{c^2}}\right)}{5} + \frac{3bc^3\left(-\frac{1}{c^2}\right)^{\frac{7}{3}}\log\left(4x^2 - 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{20} - \frac{bc^3\left(-\frac{1}{c^2}\right)^{\frac{7}{3}}\log\left(4x^2 + 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{20} + \frac{\sqrt{3}\,bc^3\left(-\frac{1}{c^2}\right)^{\frac{7}{3}}\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3\sqrt[6]{-\frac{1}{c^2}}} - \frac{\sqrt{3}}{3}\right)}{10} - \frac{\sqrt{3}\,bc^3\left(-\frac{1}{c^2}\right)^{\frac{7}{3}}\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3\sqrt[6]{-\frac{1}{c^2}}} + \frac{\sqrt{3}}{3}\right)}{10} - \frac{bc^2\left(-\frac{1}{c^2}\right)^{\frac{11}{6}}\operatorname{atan}\left(cx^3\right)}{5} + \frac{bx^5\operatorname{atan}\left(cx^3\right)}{5} - \frac{3bx^2}{10c} & \text{for } c \neq 0 \\ \frac{ax^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**5/5 - b*c**3*(-1/c**2)**(7/3)*log(x - (-1/c**2)**(1/6))/5 +
 3*b*c**3*(-1/c**2)**(7/3)*log(4*x**2 - 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)*
*(1/3))/20 - b*c**3*(-1/c**2)**(7/3)*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*
(-1/c**2)**(1/3))/20 + sqrt(3)*b*c**3*(-1/c**2)**(7/3)*atan(2*sqrt(3)*x/(3*
(-1/c**2)**(1/6)) - sqrt(3)/3)/10 - sqrt(3)*b*c**3*(-1/c**2)**(7/3)*atan(2*
sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/10 - b*c**2*(-1/c**2)**(11/6)*a
tan(c*x**3)/5 + b*x**5*atan(c*x**3)/5 - 3*b*x**2/(10*c), Ne(c, 0)), (a*x**5
/5, True))
```

Giac [A]

time = 0.43, size = 119, normalized size = 1.02

$$\frac{1}{20}\,bc^9\left(\frac{2\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^2 - \frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^{10}|c|^{\frac{2}{3}}} - \frac{\log\left(x^4 - \frac{x^2}{|c|^{\frac{2}{3}}} + \frac{1}{|c|^{\frac{4}{3}}}\right)}{c^{10}|c|^{\frac{2}{3}}} + \frac{2\,\log\left(x^2 + \frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{10}|c|^{\frac{2}{3}}}\right) + \frac{2\,bcx^5\arctan\left(cx^3\right) + 2\,acx^5 - 3\,bx^2}{10\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/20*b*c^9*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2/3))*abs(c)^(2
/3))/(c^10*abs(c)^(2/3)) - log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/(c^
10*abs(c)^(2/3)) + 2*log(x^2 + 1/abs(c)^(2/3))/(c^10*abs(c)^(2/3))) + 1/10*
(2*b*c*x^5*arctan(c*x^3) + 2*a*c*x^5 - 3*b*x^2)/c
```

Mupad [B]

time = 1.94, size = 106, normalized size = 0.91

$$\frac{a\,x^5}{5} + \frac{b\,\ln\left(c^{2/3}\,x^2 + 1\right)}{10\,c^{5/3}} - \frac{3\,b\,x^2}{10\,c} - \frac{\ln\left(1 - 2\,c^{2/3}\,x^2 + \sqrt{3}\,1\mathrm{i}\right)\,\left(b + \sqrt{3}\,b\,1\mathrm{i}\right)}{20\,c^{5/3}} - \frac{\ln\left(2\,c^{2/3}\,x^2 - 1 + \sqrt{3}\,1\mathrm{i}\right)\,\left(b - \sqrt{3}\,b\,1\mathrm{i}\right)}{20\,c^{5/3}} + \frac{b\,x^5\,\mathrm{atan}\left(c\,x^3\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*atan(c*x^3)),x)
```

```
[Out] (a*x^5)/5 + (b*log(c^(2/3)*x^2 + 1))/(10*c^(5/3)) - (3*b*x^2)/(10*c) - (log
(3^(1/2)*1i - 2*c^(2/3)*x^2 + 1)*(b + 3^(1/2)*b*1i))/(20*c^(5/3)) - (log(3^
(1/2)*1i + 2*c^(2/3)*x^2 - 1)*(b - 3^(1/2)*b*1i))/(20*c^(5/3)) + (b*x^5*ata
n(c*x^3))/5
```

3.110 $\int x(a+b\mathbf{ArcTan}(cx^3))\,dx$

Optimal. Leaf size=165

$$-\frac{b\text{ArcTan}\left(\sqrt[3]{c}\,x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\text{ArcTan}\left(cx^3\right)\right)+\frac{b\text{ArcTan}\left(\sqrt{3}\,-2\sqrt[3]{c}\,x\right)}{4c^{2/3}}-\frac{b\text{ArcTan}\left(\sqrt{3}\,+2\sqrt[3]{c}\,x\right)}{4c^{2/3}}-\frac{\sqrt{3}\,b\,\mathrm{lo}}{}$$

```
[Out] -1/2*b*arctan(c^(1/3)*x)/c^(2/3)+1/2*x^2*(a+b*arctan(c*x^3))-1/4*b*arctan(2
*c^(1/3)*x-3^(1/2))/c^(2/3)-1/4*b*arctan(2*c^(1/3)*x+3^(1/2))/c^(2/3)-1/8*b
*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)/c^(2/3)+1/8*b*ln(1+c^(2/3)*x^2
+c^(1/3)*x*3^(1/2))*3^(1/2)/c^(2/3)
```

Rubi [A]

time = 0.29, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {4946, 301, 648, 632, 210, 642, 209}

$$\frac{1}{2}x^2\left(a+b\text{ArcTan}\left(cx^3\right)\right)-\frac{b\text{ArcTan}\left(\sqrt[3]{c}\,x\right)}{2c^{2/3}}+\frac{b\text{ArcTan}\left(\sqrt{3}\,-2\sqrt[3]{c}\,x\right)}{4c^{2/3}}-\frac{b\text{ArcTan}\left(2\sqrt[3]{c}\,x+\sqrt{3}\right)}{4c^{2/3}}-\frac{\sqrt{3}\,b\log\left(c^{2/3}x^2-\sqrt{3}\,\sqrt[3]{c}\,x+1\right)}{8c^{2/3}}+\frac{\sqrt{3}\,b\log\left(c^{2/3}x^2+\sqrt{3}\,\sqrt[3]{c}\,x+1\right)}{8c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] -1/2*(b*ArcTan[c^(1/3)*x])/c^(2/3) + (x^2*(a + b*ArcTan[c*x^3]))/2 + (b*Arc
Tan[Sqrt[3] - 2*c^(1/3)*x])/(4*c^(2/3)) - (b*ArcTan[Sqrt[3] + 2*c^(1/3)*x])
/(4*c^(2/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2
/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
```

```
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int x\left(a+b\tan^{-1}\left(cx^3\right)\right)\,dx=\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{1}{2}(3bc)\int\frac{x^4}{1+c^2x^6}\,dx$$

$$=\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{b\int\frac{1}{1+c^{2/3}x^2}\,dx}{2\sqrt[3]{c}}-\frac{b\int\frac{-\frac{1}{2}+\frac{1}{2}\sqrt{3}\sqrt[3]{c}\,x}{1-\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx}{2\sqrt[3]{c}}-\frac{b\int\frac{-\frac{1}{2}\cdots}{1+\sqrt{\cdots}}}{\cdots}$$

$$=-\frac{b\tan^{-1}\left(\sqrt[3]{c}\,x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\left(\sqrt{3}\,b\right)\int\frac{-\sqrt{3}\sqrt[3]{c}+2c^{2/3}x}{1-\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2}\,dx}{8c^{2/3}}$$

$$=-\frac{b\tan^{-1}\left(\sqrt[3]{c}\,x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)-\frac{\sqrt{3}\,b\log\left(1-\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x\cdots\right.}{8c^{2/3}}$$

$$=-\frac{b\tan^{-1}\left(\sqrt[3]{c}\,x\right)}{2c^{2/3}}+\frac{1}{2}x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)+\frac{b\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)}{4c^{2/3}}-\frac{b\tan\cdots}{\cdots}$$

Mathematica [A]
time = 0.02, size = 170, normalized size = 1.03

$$\frac{ax^2}{2}-\frac{b\text{ArcTan}\left(\sqrt[3]{c}\,x\right)}{2c^{2/3}}+\frac{1}{2}bx^2\text{ArcTan}\left(cx^3\right)+\frac{b\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)}{4c^{2/3}}-\frac{b\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{c}\,x\right)}{4c^{2/3}}-\frac{\sqrt{3}\,b\log\left(1-\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2\right)}{8c^{2/3}}+\frac{\sqrt{3}\,b\log\left(1+\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2\right)}{8c^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] (a*x^2)/2 - (b*ArcTan[c^(1/3)*x])/(2*c^(2/3)) + (b*x^2*ArcTan[c*x^3])/2 + (
b*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/(4*c^(2/3)) - (b*ArcTan[Sqrt[3] + 2*c^(1/3
)*x])/(4*c^(2/3)) - (Sqrt[3]*b*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8
*c^(2/3)) + (Sqrt[3]*b*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3)
)
```

Maple [A]
time = 0.06, size = 154, normalized size = 0.93

method	result
default	$\frac{a\,x^2}{2}+\frac{b\,x^2\arctan\left(c\,x^3\right)}{2}-\frac{bc\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2-\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{8}-\frac{b\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{4c\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\frac{bc\sqrt{3}\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c*x^3)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*x^2+1/2*b*x^2*arctan(c*x^3)-1/8*b*c*3^(1/2)*(1/c^2)^(5/6)*ln(x^2-3^(1
/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))-1/4*b/c/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)
^(1/6)-3^(1/2))+1/8*b*c*3^(1/2)*(1/c^2)^(5/6)*ln(x^2+3^(1/2)*(1/c^2)^(1/6)*
x+(1/c^2)^(1/3))-1/4*b/c/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^(1/6)+3^(1/2))-1/
2*b/c/(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6))
```

Maxima [A]
time = 0.46, size = 137, normalized size = 0.83

$$\frac{1}{2}\,ax^2+\frac{1}{8}\left(4\,x^2\arctan\left(cx^3\right)+c\left(\frac{\sqrt{3}\,\log\left(c^{\frac{2}{3}}x^2+\sqrt{3}\,c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{\sqrt{3}\,\log\left(c^{\frac{2}{3}}x^2-\sqrt{3}\,c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{4\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{5}{3}}}-\frac{2\arctan\left(\frac{2c^{\frac{2}{3}}x+\sqrt{3}\,c^{\frac{1}{3}}}{c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}-\frac{2\arctan\left(\frac{2c^{\frac{2}{3}}x-\sqrt{3}\,c^{\frac{1}{3}}}{c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}\right)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^3)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/8*(4*x^2*arctan(c*x^3) + c*(sqrt(3)*log(c^(2/3)*x^2 + sqrt(3)
*c^(1/3)*x + 1)/c^(5/3) - sqrt(3)*log(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1)/
c^(5/3) - 4*arctan(c^(1/3)*x)/c^(5/3) - 2*arctan((2*c^(2/3)*x + sqrt(3)*c^(
1/3))/c^(1/3))/c^(5/3) - 2*arctan((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/3))/
c^(5/3)))*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 408 vs. 2(119) = 238.
time = 1.68, size = 408, normalized size = 2.47

$$\frac{1}{2}\,bx^2\arctan\left(cx^3\right)+\frac{1}{2}\,ax^2+\frac{1}{8}\,\sqrt{3}\left(\frac{b^6}{c^4}\right)^{\frac{1}{6}}\log\left(b^{10}x^2+\sqrt{3}\left(\frac{b^6}{c^4}\right)^{\frac{5}{6}}b^5c^3x+\left(\frac{b^6}{c^4}\right)^{\frac{2}{3}}b^6c^2\right)-\frac{1}{8}\,\sqrt{3}\left(\frac{b^6}{c^4}\right)^{\frac{1}{6}}\log\left(b^{10}x^2-\sqrt{3}\left(\frac{b^6}{c^4}\right)^{\frac{5}{6}}b^5c^3x+\left(\frac{b^6}{c^4}\right)^{\frac{2}{3}}b^6c^2\right)+\frac{1}{2}\left(\frac{b^6}{c^4}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\left(\frac{b^6}{c^4}\right)^{\frac{1}{6}}b^5cx+\sqrt{3}\,b^6-2\sqrt{b^{10}x^2+\sqrt{3}\left(\frac{b^6}{c^4}\right)^{\frac{5}{6}}b^5c^3x+\left(\frac{b^6}{c^4}\right)^{\frac{2}{3}}b^6c^2}\left(\frac{b^6}{c^4}\right)^{\frac{1}{6}}c}{b^6}\right)+\frac{1}{2}\left(\frac{b^6}{c^4}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\left(\frac{b^6}{c^4}\right)^{\frac{1}{6}}b^5cx-\sqrt{3}\,b^6-2\sqrt{b^{10}x^2-\sqrt{3}\left(\frac{b^6}{c^4}\right)^{\frac{5}{6}}b^5c^3x+\left(\frac{b^6}{c^4}\right)^{\frac{2}{3}}b^6c^2}\left(\frac{b^6}{c^4}\right)^{\frac{1}{6}}c}{b^6}\right)+\left(\frac{b^6}{c^4}\right)^{\frac{1}{6}}\arctan\left(-\frac{\left(\frac{b^6}{c^4}\right)^{\frac{1}{6}}b^5cx-\sqrt{b^{10}x^2+\left(\frac{b^6}{c^4}\right)^{\frac{2}{3}}b^6c^2}\left(\frac{b^6}{c^4}\right)^{\frac{1}{6}}c}{b^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2*arctan(c*x^3) + 1/2*a*x^2 + 1/8*sqrt(3)*(b^6/c^4)^(1/6)*log(b^10*
x^2 + sqrt(3)*(b^6/c^4)^(5/6)*b^5*c^3*x + (b^6/c^4)^(2/3)*b^6*c^2) - 1/8*sq
rt(3)*(b^6/c^4)^(1/6)*log(b^10*x^2 - sqrt(3)*(b^6/c^4)^(5/6)*b^5*c^3*x + (b
^6/c^4)^(2/3)*b^6*c^2) + 1/2*(b^6/c^4)^(1/6)*arctan(-(2*(b^6/c^4)^(1/6)*b^5
*c*x + sqrt(3)*b^6 - 2*sqrt(b^10*x^2 + sqrt(3)*(b^6/c^4)^(5/6)*b^5*c^3*x +
(b^6/c^4)^(2/3)*b^6*c^2)*(b^6/c^4)^(1/6)*c)/b^6) + 1/2*(b^6/c^4)^(1/6)*arct
an(-(2*(b^6/c^4)^(1/6)*b^5*c*x - sqrt(3)*b^6 - 2*sqrt(b^10*x^2 - sqrt(3)*(b
^6/c^4)^(5/6)*b^5*c^3*x + (b^6/c^4)^(2/3)*b^6*c^2)*(b^6/c^4)^(1/6)*c)/b^6)
+ (b^6/c^4)^(1/6)*arctan(-((b^6/c^4)^(1/6)*b^5*c*x - sqrt(b^10*x^2 + (b^6/c
^4)^(2/3)*b^6*c^2)*(b^6/c^4)^(1/6)*c)/b^6)
```

Sympy [A]
time = 17.09, size = 246, normalized size = 1.49

$$\begin{cases}\frac{ax^2}{2}+\frac{bx^2\operatorname{atan}\left(cx^3\right)}{2}-\frac{3b\log\left(4x^2-4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{8c\sqrt[6]{-\frac{1}{c^2}}}+\frac{3b\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{8c\sqrt[6]{-\frac{1}{c^2}}}-\frac{\sqrt{3}\,b\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3\sqrt[6]{-\frac{1}{c^2}}}-\frac{\sqrt{3}}{3}\right)}{4c\sqrt[6]{-\frac{1}{c^2}}}-\frac{\sqrt{3}\,b\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3\sqrt[6]{-\frac{1}{c^2}}}+\frac{\sqrt{3}}{3}\right)}{4c\sqrt[6]{-\frac{1}{c^2}}}+\frac{b\operatorname{atan}\left(cx^3\right)}{2c^2\left(-\frac{1}{c^2}\right)^{\frac{2}{3}}} & \text{for}\ c\neq 0\\ \frac{ax^2}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c*x**3)),x)
```

```
[Out] Piecewise((a*x**2/2 + b*x**2*atan(c*x**3)/2 - 3*b*log(4*x**2 - 4*x*(-1/c**2
)**(1/6) + 4*(-1/c**2)**(1/3))/(8*c*(-1/c**2)**(1/6)) + 3*b*log(4*x**2 + 4*
x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/(8*c*(-1/c**2)**(1/6)) - sqrt(3)*b
*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3)/3)/(4*c*(-1/c**2)**(1/6))
- sqrt(3)*b*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/(4*c*(-1/c**
2)**(1/6)) + b*atan(c*x**3)/(2*c**2*(-1/c**2)**(2/3)), Ne(c, 0)), (a*x**2/2
, True))
```

Giac [A]
time = 0.48, size = 157, normalized size = 0.95

$$\frac{1}{8}\,bc^{5}\left(\frac{\sqrt{3}\,\log\left(x^{2}+\frac{\sqrt{3}\,x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{4}|c|^{\frac{5}{3}}}-\frac{\sqrt{3}\,|c|^{\frac{1}{3}}\log\left(x^{2}-\frac{\sqrt{3}\,x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{c^{6}}-\frac{2\,|c|^{\frac{1}{3}}\arctan\left(\left(2\,x+\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^{6}}-\frac{2\,|c|^{\frac{1}{3}}\arctan\left(\left(2\,x-\frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^{6}}-\frac{4\,|c|^{\frac{1}{3}}\arctan\left(x|c|^{\frac{1}{3}}\right)}{c^{6}}\right)+\frac{1}{2}\,bx^{2}\arctan\left(cx^{3}\right)+\frac{1}{2}\,ax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] 1/8*b*c^5*(sqrt(3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c)^(2/3))/(c^4*
abs(c)^(5/3)) - sqrt(3)*abs(c)^(1/3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/a
bs(c)^(2/3))/c^6 - 2*abs(c)^(1/3)*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c
)^(1/3))/c^6 - 2*abs(c)^(1/3)*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1
/3))/c^6 - 4*abs(c)^(1/3)*arctan(x*abs(c)^(1/3))/c^6) + 1/2*b*x^2*arctan(c*
x^3) + 1/2*a*x^2
```

Mupad [B]
time = 0.69, size = 113, normalized size = 0.68

$$\frac{a\,x^{2}}{2}+\frac{b\,\left(\mathrm{atan}\left({\left(-1\right)}^{2/3}\,c^{1/3}\,x\right)+\mathrm{atan}\left(\frac{{\left(-1\right)}^{2/3}\,c^{1/3}\,x\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)+2\,\mathrm{atan}\left(\frac{{\left(-1\right)}^{2/3}\,c^{1/3}\,x\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)}{4\,c^{2/3}}+\frac{b\,x^{2}\,\mathrm{atan}\left(c\,x^{3}\right)}{2}-\frac{\sqrt{3}\,b\,\left(\mathrm{atan}\left({\left(-1\right)}^{2/3}\,c^{1/3}\,x\right)-\mathrm{atan}\left(\frac{{\left(-1\right)}^{2/3}\,c^{1/3}\,x\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)\,1\mathrm{i}}{4\,c^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c*x^3)),x)
```

```
[Out] (a*x^2)/2 + (b*(atan((-1)^(2/3)*c^(1/3)*x) + atan(((-1)^(2/3)*c^(1/3)*x*(3^
(1/2)*1i - 1))/2) + 2*atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i + 1))/2)))/(4*
c^(2/3)) + (b*x^2*atan(c*x^3))/2 - (3^(1/2)*b*(atan((-1)^(2/3)*c^(1/3)*x) -
 atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i - 1))/2))*1i)/(4*c^(2/3))
```

3.111 $\int \frac{a+b\mathbf{ArcTan}(cx^3)}{x^2}\,dx$

Optimal. Leaf size=104

$$-\frac{a+b\text{ArcTan}(cx^3)}{x}-\frac{1}{2}\sqrt{3}\,b\sqrt[3]{c}\,\text{ArcTan}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)+\frac{1}{2}b\sqrt[3]{c}\,\log\left(1+c^{2/3}x^2\right)-\frac{1}{4}b\sqrt[3]{c}\,\log\left(1-c^{2/3}x^2+c\right.$$

[Out] (-a-b*arctan(c*x^3))/x+1/2*b*c^(1/3)*ln(1+c^(2/3)*x^2)-1/4*b*c^(1/3)*ln(1-c^(2/3)*x^2+c^(4/3)*x^4)-1/2*b*c^(1/3)*arctan(1/3*(1-2*c^(2/3)*x^2)*3^(1/2))*3^(1/2)

Rubi [A]
time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4946, 281, 206, 31, 648, 631, 210, 642}

$$-\frac{a+b\text{ArcTan}(cx^3)}{x}-\frac{1}{2}\sqrt{3}\,b\sqrt[3]{c}\,\text{ArcTan}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)+\frac{1}{2}b\sqrt[3]{c}\,\log\left(c^{2/3}x^2+1\right)-\frac{1}{4}b\sqrt[3]{c}\,\log\left(c^{4/3}x^4-c^{2/3}x^2+1\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcTan[c*x^3])/x^2,x]

[Out] -((a + b*ArcTan[c*x^3])/x) - (Sqrt[3]*b*c^(1/3)*ArcTan[(1 - 2*c^(2/3)*x^2)/Sqrt[3]])/2 + (b*c^(1/3)*Log[1 + c^(2/3)*x^2])/2 - (b*c^(1/3)*Log[1 - c^(2/3)*x^2 + c^(4/3)*x^4])/4

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}(cx^3)}{x^2}\,dx &= -\frac{a+b\tan^{-1}(cx^3)}{x}+(3bc)\int \frac{x}{1+c^2x^6}\,dx\\ &= -\frac{a+b\tan^{-1}(cx^3)}{x}+\frac{1}{2}(3bc)\text{Subst}\left(\int \frac{1}{1+c^2x^3}\,dx,x,x^2\right)\\ &= -\frac{a+b\tan^{-1}(cx^3)}{x}+\frac{1}{2}(bc)\text{Subst}\left(\int \frac{1}{1+c^{2/3}x}\,dx,x,x^2\right)+\frac{1}{2}(bc)\text{Subst}\left(\int \frac{}{1-\right.\\ &= -\frac{a+b\tan^{-1}(cx^3)}{x}+\frac{1}{2}b\sqrt[3]{c}\,\log\left(1+c^{2/3}x^2\right)-\frac{1}{4}\left(b\sqrt[3]{c}\right)\text{Subst}\left(\int \frac{-c^{2/3}+2c^{4/}}{1-c^{2/3}x+c^{4}}\right.\\ &= -\frac{a+b\tan^{-1}(cx^3)}{x}+\frac{1}{2}b\sqrt[3]{c}\,\log\left(1+c^{2/3}x^2\right)-\frac{1}{4}b\sqrt[3]{c}\,\log\left(1-c^{2/3}x^2+c^{4/3}x^4\right)+\\ &= -\frac{a+b\tan^{-1}(cx^3)}{x}-\frac{1}{2}\sqrt{3}\,b\sqrt[3]{c}\,\tan^{-1}\left(\frac{1-2c^{2/3}x^2}{\sqrt{3}}\right)+\frac{1}{2}b\sqrt[3]{c}\,\log\left(1+c^{2/3}x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 170, normalized size = 1.63

$$-\frac{a}{x}-\frac{b\text{ArcTan}(cx^3)}{x}-\frac{1}{2}\sqrt{3}\,b\sqrt[3]{c}\,\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)-\frac{1}{2}\sqrt{3}\,b\sqrt[3]{c}\,\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{c}\,x\right)+\frac{1}{2}b\sqrt[3]{c}\,\log\left(1+c^{2/3}x^2\right)-\frac{1}{4}b\sqrt[3]{c}\,\log\left(1-\sqrt{3}\,\sqrt[3]{c}\,x+c^{2/3}x^2\right)-\frac{1}{4}b\sqrt[3]{c}\,\log\left(1+\sqrt{3}\,\sqrt[3]{c}\,x+c^{2/3}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^2,x]
```

```
[Out] -(a/x) - (b*ArcTan[c*x^3])/x - (Sqrt[3]*b*c^(1/3)*ArcTan[Sqrt[3] - 2*c^(1/3
)*x])/2 - (Sqrt[3]*b*c^(1/3)*ArcTan[Sqrt[3] + 2*c^(1/3)*x])/2 + (b*c^(1/3)*
Log[1 + c^(2/3)*x^2])/2 - (b*c^(1/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(2/3)*x^
2])/4 - (b*c^(1/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2])/4
```

Maple [A]
time = 0.05, size = 104, normalized size = 1.00

method	result
default	$-\frac{a}{x}-\frac{b\arctan(c\,x^3)}{x}+\frac{b\ln\left(x^2+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x^4-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{2c\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}$

risch	$\frac{ib\ln(icx^3+1)}{2x} - \frac{ib\ln(-icx^3+1)}{2x} - \frac{ib\ln\left(x+\left(\frac{i}{c}\right)^{\frac{1}{3}}\right)}{2\left(\frac{i}{c}\right)^{\frac{1}{3}}} + \frac{ib\ln\left(x^2-\left(\frac{i}{c}\right)^{\frac{1}{3}}x+\left(\frac{i}{c}\right)^{\frac{2}{3}}\right)}{4\left(\frac{i}{c}\right)^{\frac{1}{3}}} + \frac{ib\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{i}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{2\left(\frac{i}{c}\right)^{\frac{1}{3}}} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/x-b/x*arctan(c*x^3)+1/2*b/c/(1/c^2)^(2/3)*ln(x^2+(1/c^2)^(1/3))-1/4*b/c/
(1/c^2)^(2/3)*ln(x^4-(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))+1/2*b/c/(1/c^2)^(2/3)
*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1/3)*x^2-1))
```

Maxima [A]
time = 0.48, size = 98, normalized size = 0.94

$$\frac{1}{4}\left(c\left(\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2-c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)}{c^{\frac{2}{3}}} - \frac{\log\left(c^{\frac{4}{3}}x^4-c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{2}{3}}} + \frac{2\log\left(\frac{c^{\frac{2}{3}}x^2+1}{c^{\frac{2}{3}}}\right)}{c^{\frac{2}{3}}}\right) - \frac{4\arctan\left(cx^3\right)}{x}\right)b - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 - c^(2/3))/c^(2/3))/c^(
2/3) - log(c^(4/3)*x^4 - c^(2/3)*x^2 + 1)/c^(2/3) + 2*log((c^(2/3)*x^2 + 1)
/c^(2/3))/c^(2/3)) - 4*arctan(c*x^3)/x)*b - a/x
```

Fricas [A]
time = 1.29, size = 90, normalized size = 0.87

$$\frac{2\sqrt{3}\,bc^{\frac{1}{3}}x\arctan\left(\frac{2}{3}\sqrt{3}\,c^{\frac{2}{3}}x^2-\frac{1}{3}\sqrt{3}\right) - bc^{\frac{1}{3}}x\log\left(c^2x^4-c^{\frac{4}{3}}x^2+c^{\frac{2}{3}}\right) + 2\,bc^{\frac{1}{3}}x\log\left(cx^2+c^{\frac{1}{3}}\right) - 4\,b\arctan\left(cx^3\right) - 4\,a}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*sqrt(3)*b*c^(1/3)*x*arctan(2/3*sqrt(3)*c^(2/3)*x^2 - 1/3*sqrt(3)) -
b*c^(1/3)*x*log(c^2*x^4 - c^(4/3)*x^2 + c^(2/3)) + 2*b*c^(1/3)*x*log(c*x^2
+ c^(1/3)) - 4*b*arctan(c*x^3) - 4*a)/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(97) = 194$.
time = 24.35, size = 262, normalized size = 2.52

$$\begin{cases} -\frac{a}{x} + bc^2\left(-\frac{1}{c^2}\right)^{\frac{5}{6}}\operatorname{atan}\left(cx^3\right) - bc\sqrt[3]{-\frac{1}{c^2}}\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right) + \frac{3bc\sqrt[3]{-\frac{1}{c^2}}\log\left(4x^2-4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{4} - \frac{bc\sqrt[3]{-\frac{1}{c^2}}\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{4} + \frac{\sqrt{3}\,bc\sqrt[3]{-\frac{1}{c^2}}\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3\sqrt[6]{-\frac{1}{c^2}}}-\frac{\sqrt{3}}{3}\right)}{2} - \frac{\sqrt{3}\,bc\sqrt[3]{-\frac{1}{c^2}}\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3\sqrt[6]{-\frac{1}{c^2}}}+\frac{\sqrt{3}}{3}\right)}{2} - \frac{b\operatorname{atan}\left(cx^3\right)}{x} & \text{for}\: c \neq 0 \\ -\frac{a}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**2,x)
```

```
[Out] Piecewise((-a/x + b*c**2*(-1/c**2)**(5/6)*atan(c*x**3) - b*c*(-1/c**2)**(1/
3)*log(x - (-1/c**2)**(1/6)) + 3*b*c*(-1/c**2)**(1/3)*log(4*x**2 - 4*x*(-1/
c**2)**(1/6) + 4*(-1/c**2)**(1/3))/4 - b*c*(-1/c**2)**(1/3)*log(4*x**2 + 4*
x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/4 + sqrt(3)*b*c*(-1/c**2)**(1/3)*a
tan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3)/3)/2 - sqrt(3)*b*c*(-1/c**2)
**(1/3)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/2 - b*atan(c*x**
3)/x, Ne(c, 0)), (-a/x, True))
```

Giac [A]
time = 0.43, size = 91, normalized size = 0.88

$$\frac{1}{4}\,bc\left(\frac{2\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2-\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{|c|^{\frac{2}{3}}}-\frac{\log\left(x^4-\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{|c|^{\frac{2}{3}}}+\frac{2\,\log\left(x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)}{|c|^{\frac{2}{3}}}\right)-\frac{b\arctan\left(cx^3\right)+a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1/abs(c)^(2/3))*abs(c)^(2/3)
)/abs(c)^(2/3) - log(x^4 - x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/abs(c)^(2/3)
+ 2*log(x^2 + 1/abs(c)^(2/3))/abs(c)^(2/3)) - (b*arctan(c*x^3) + a)/x
```

Mupad [B]
time = 1.83, size = 99, normalized size = 0.95

$$\frac{b\,c^{1/3}\,\ln\left(c^{2/3}\,x^2+1\right)}{2}-\frac{a}{x}-\frac{b\,\mathrm{atan}(c\,x^3)}{x}-\frac{b\,c^{1/3}\,\ln\left(-\sqrt{3}-c^{2/3}\,x^2\,2\mathrm{i}+1\mathrm{i}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4}+\frac{b\,c^{1/3}\,\ln\left(-\sqrt{3}+c^{2/3}\,x^2\,2\mathrm{i}-\mathrm{i}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))/x^2,x)
```

```
[Out] (b*c^(1/3)*log(c^(2/3)*x^2 + 1))/2 - a/x - (b*atan(c*x^3))/x - (b*c^(1/3)*l
og(1i - c^(2/3)*x^2*2i - 3^(1/2))*(3^(1/2)*1i + 1))/4 + (b*c^(1/3)*log(c^(2
/3)*x^2*2i - 3^(1/2) - 1i)*(3^(1/2)*1i - 1))/4
```

3.112 $\int \frac{a+b\text{ArcTan}(cx^3)}{x^5}\,dx$

Optimal. Leaf size=174

$$-\frac{3bc}{4x}-\frac{1}{4}bc^{4/3}\text{ArcTan}\left(\sqrt[3]{c}\,x\right)-\frac{a+b\text{ArcTan}(cx^3)}{4x^4}+\frac{1}{8}bc^{4/3}\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)-\frac{1}{8}bc^{4/3}\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{c}\right.$$

```
[Out] -3/4*b*c/x-1/4*b*c^(4/3)*arctan(c^(1/3)*x)+1/4*(-a-b*arctan(c*x^3))/x^4-1/8
*b*c^(4/3)*arctan(2*c^(1/3)*x-3^(1/2))-1/8*b*c^(4/3)*arctan(2*c^(1/3)*x+3^(
1/2))-1/16*b*c^(4/3)*ln(1+c^(2/3)*x^2-c^(1/3)*x*3^(1/2))*3^(1/2)+1/16*b*c^(
4/3)*ln(1+c^(2/3)*x^2+c^(1/3)*x*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4946, 331, 301, 648, 632, 210, 642, 209}

$$-\frac{a+b\text{ArcTan}(cx^3)}{4x^4}-\frac{1}{4}bc^{4/3}\text{ArcTan}\left(\sqrt[3]{c}\,x\right)+\frac{1}{8}bc^{4/3}\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)-\frac{1}{8}bc^{4/3}\text{ArcTan}\left(2\sqrt[3]{c}\,x+\sqrt{3}\right)-\frac{1}{16}\sqrt{3}\,bc^{4/3}\log\left(c^{2/3}x^2-\sqrt{3}\sqrt[3]{c}\,x+1\right)+\frac{1}{16}\sqrt{3}\,bc^{4/3}\log\left(c^{2/3}x^2+\sqrt{3}\sqrt[3]{c}\,x+1\right)-\frac{3bc}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])/x^5,x]
```

```
[Out] (-3*b*c)/(4*x) - (b*c^(4/3)*ArcTan[c^(1/3)*x])/4 - (a + b*ArcTan[c*x^3])/(4
*x^4) + (b*c^(4/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/8 - (b*c^(4/3)*ArcTan[Sqr
t[3] + 2*c^(1/3)*x])/8 - (Sqrt[3]*b*c^(4/3)*Log[1 - Sqrt[3]*c^(1/3)*x + c^(
2/3)*x^2])/16 + (Sqrt[3]*b*c^(4/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2/3)*x^2]
)/16
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 301

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
```

```
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))*Int[1/(r^2 + s^2*x^2), x] + Dist[2*(r^
(m + 1)/(a*n*s^m)), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}(cx^3)}{x^5}\,dx = -\frac{a+b\tan^{-1}(cx^3)}{4x^4}+\frac{1}{4}(3bc)\int \frac{1}{x^2(1+c^2x^6)}\,dx$$

$$= -\frac{3bc}{4x}-\frac{a+b\tan^{-1}(cx^3)}{4x^4}-\frac{1}{4}(3bc^3)\int \frac{x^4}{1+c^2x^6}\,dx$$

$$= -\frac{3bc}{4x}-\frac{a+b\tan^{-1}(cx^3)}{4x^4}-\frac{1}{4}(bc^{5/3})\int \frac{1}{1+c^{2/3}x^2}\,dx-\frac{1}{4}(bc^{5/3})\int \frac{-\frac{1}{2}+\frac{1}{2}\sqrt{3}}{1-\sqrt{3}\sqrt[3]{c}\,x}$$

$$= -\frac{3bc}{4x}-\frac{1}{4}bc^{4/3}\tan^{-1}(\sqrt[3]{c}\,x)-\frac{a+b\tan^{-1}(cx^3)}{4x^4}-\frac{1}{16}\left(\sqrt{3}\,bc^{4/3}\right)\int \frac{-\sqrt{3}\sqrt[3]{c}}{1-\sqrt{3}\sqrt[3]{c}}$$

$$= -\frac{3bc}{4x}-\frac{1}{4}bc^{4/3}\tan^{-1}(\sqrt[3]{c}\,x)-\frac{a+b\tan^{-1}(cx^3)}{4x^4}-\frac{1}{16}\sqrt{3}\,bc^{4/3}\log\left(1-\sqrt{3}\sqrt[3]{c}\,x\right.$$

$$= -\frac{3bc}{4x}-\frac{1}{4}bc^{4/3}\tan^{-1}(\sqrt[3]{c}\,x)-\frac{a+b\tan^{-1}(cx^3)}{4x^4}+\frac{1}{8}bc^{4/3}\tan^{-1}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)$$

Mathematica [A]
time = 0.04, size = 179, normalized size = 1.03

$$-\frac{a}{4x^4}-\frac{3bc}{4x}-\frac{1}{4}bc^{4/3}\text{ArcTan}(\sqrt[3]{c}\,x)-\frac{b\text{ArcTan}(cx^3)}{4x^4}+\frac{1}{8}bc^{4/3}\text{ArcTan}\left(\sqrt{3}-2\sqrt[3]{c}\,x\right)-\frac{1}{8}bc^{4/3}\text{ArcTan}\left(\sqrt{3}+2\sqrt[3]{c}\,x\right)-\frac{1}{16}\sqrt{3}\,bc^{4/3}\log\left(1-\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2\right)+\frac{1}{16}\sqrt{3}\,bc^{4/3}\log\left(1+\sqrt{3}\sqrt[3]{c}\,x+c^{2/3}x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])/x^5,x]
```

```
[Out] -1/4*a/x^4 - (3*b*c)/(4*x) - (b*c^(4/3)*ArcTan[c^(1/3)*x])/4 - (b*ArcTan[c*
x^3])/(4*x^4) + (b*c^(4/3)*ArcTan[Sqrt[3] - 2*c^(1/3)*x])/8 - (b*c^(4/3)*Ar
cTan[Sqrt[3] + 2*c^(1/3)*x])/8 - (Sqrt[3]*b*c^(4/3)*Log[1 - Sqrt[3]*c^(1/3)
*x + c^(2/3)*x^2])/16 + (Sqrt[3]*b*c^(4/3)*Log[1 + Sqrt[3]*c^(1/3)*x + c^(2
/3)*x^2])/16
```

Maple [A]
time = 0.07, size = 159, normalized size = 0.91

method	result
default	$-\frac{a}{4x^4}-\frac{b\arctan(c\,x^3)}{4x^4}-\frac{3bc}{4x}-\frac{b\,c^3\sqrt{3}\,\left(\frac{1}{c^2}\right)^{\frac{5}{6}}\ln\left(x^2-\sqrt{3}\,\left(\frac{1}{c^2}\right)^{\frac{1}{6}}x+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{16}-\frac{bc\arctan\left(\frac{2x}{\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{8\left(\frac{1}{c^2}\right)^{\frac{1}{6}}}+\frac{b\,c^3\sqrt{3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*a/x^4-1/4*b/x^4*arctan(c*x^3)-3/4*b*c/x-1/16*b*c^3*3^(1/2)*(1/c^2)^(5/
6)*ln(x^2-3^(1/2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))-1/8*b*c/(1/c^2)^(1/6)*arct
an(2*x/(1/c^2)^(1/6)-3^(1/2))+1/16*b*c^3*3^(1/2)*(1/c^2)^(5/6)*ln(x^2+3^(1/
2)*(1/c^2)^(1/6)*x+(1/c^2)^(1/3))-1/8*b*c/(1/c^2)^(1/6)*arctan(2*x/(1/c^2)^
(1/6)+3^(1/2))-1/4*b*c/(1/c^2)^(1/6)*arctan(x/(1/c^2)^(1/6))
```

<u>Maxima</u> [**A**]
time = 0.48, size = 147, normalized size = 0.84

$$\frac{1}{16}\left(\left(\left(c^2\left(\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2+\sqrt{3}\,c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{\sqrt{3}\log\left(c^{\frac{2}{3}}x^2-\sqrt{3}\,c^{\frac{1}{3}}x+1\right)}{c^{\frac{5}{3}}}-\frac{4\arctan\left(c^{\frac{1}{3}}x\right)}{c^{\frac{5}{3}}}-\frac{2\arctan\left(\frac{2c^{\frac{2}{3}}x+\sqrt{3}\,c^{\frac{1}{3}}}{c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}-\frac{2\arctan\left(\frac{2c^{\frac{2}{3}}x-\sqrt{3}\,c^{\frac{1}{3}}}{c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}\right)-\frac{12}{x}\right)c-\frac{4\arctan\left(cx^3\right)}{x^4}\right)b-\frac{a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^5,x, algorithm="maxima")
```

```
[Out] 1/16*((c^2*(sqrt(3)*log(c^(2/3)*x^2 + sqrt(3)*c^(1/3)*x + 1)/c^(5/3) - sqrt
(3)*log(c^(2/3)*x^2 - sqrt(3)*c^(1/3)*x + 1)/c^(5/3) - 4*arctan(c^(1/3)*x)/
c^(5/3) - 2*arctan((2*c^(2/3)*x + sqrt(3)*c^(1/3))/c^(1/3))/c^(5/3) - 2*arc
tan((2*c^(2/3)*x - sqrt(3)*c^(1/3))/c^(1/3))/c^(5/3)) - 12/x)*c - 4*arctan(
c*x^3)/x^4)*b - 1/4*a/x^4
```

<u>Fricas</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(126) = 252.
time = 1.62, size = 595, normalized size = 3.42

$$\frac{\sqrt{3}\left(b^6c^8\right)^{\frac{1}{6}}x^4\log\left(4b^{10}c^{14}x^2+4\left(b^6c^8\right)^{\frac{2}{3}}b^6c^8+4\sqrt{3}\left(b^6c^8\right)^{\frac{5}{6}}b^5c^7x\right)-\sqrt{3}\left(b^6c^8\right)^{\frac{1}{6}}x^4\log\left(4b^{10}c^{14}x^2+4\left(b^6c^8\right)^{\frac{2}{3}}b^6c^8-4\sqrt{3}\left(b^6c^8\right)^{\frac{5}{6}}b^5c^7x\right)+\sqrt{3}\left(b^6c^8\right)^{\frac{1}{6}}x^4\log\left(b^{10}c^{14}x^2+\left(b^6c^8\right)^{\frac{2}{3}}b^6c^8+\sqrt{3}\left(b^6c^8\right)^{\frac{5}{6}}b^5c^7x\right)-\sqrt{3}\left(b^6c^8\right)^{\frac{1}{6}}x^4\log\left(b^{10}c^{14}x^2+\left(b^6c^8\right)^{\frac{2}{3}}b^6c^8-\sqrt{3}\left(b^6c^8\right)^{\frac{5}{6}}b^5c^7x\right)+8\left(b^6c^8\right)^{\frac{1}{6}}x^4\arctan\left(-\frac{\sqrt{3}b^6c^8+2\left(b^6c^8\right)^{\frac{1}{6}}b^5c^7x-2\sqrt{b^{10}c^{14}x^2+\left(b^6c^8\right)^{\frac{2}{3}}b^6c^8+\sqrt{3}\left(b^6c^8\right)^{\frac{5}{6}}b^5c^7x}\left(b^6c^8\right)^{\frac{1}{6}}}{b^6c^8}\right)+8\left(b^6c^8\right)^{\frac{1}{6}}x^4\arctan\left(\frac{\sqrt{3}b^6c^8-2\left(b^6c^8\right)^{\frac{1}{6}}b^5c^7x+2\sqrt{b^{10}c^{14}x^2+\left(b^6c^8\right)^{\frac{2}{3}}b^6c^8-\sqrt{3}\left(b^6c^8\right)^{\frac{5}{6}}b^5c^7x}\left(b^6c^8\right)^{\frac{1}{6}}}{b^6c^8}\right)+16\left(b^6c^8\right)^{\frac{1}{6}}x^4\arctan\left(-\frac{\left(b^6c^8\right)^{\frac{1}{6}}b^5c^7x-\sqrt{b^{10}c^{14}x^2+\left(b^6c^8\right)^{\frac{2}{3}}b^6c^8}\left(b^6c^8\right)^{\frac{1}{6}}}{b^6c^8}\right)-24bcx^3-8b\arctan\left(cx^3\right)-8a}{32x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^5,x, algorithm="fricas")
```

```
[Out] 1/32*(sqrt(3)*(b^6*c^8)^(1/6)*x^4*log(4*b^10*c^14*x^2 + 4*(b^6*c^8)^(2/3)*b
^6*c^8 + 4*sqrt(3)*(b^6*c^8)^(5/6)*b^5*c^7*x) - sqrt(3)*(b^6*c^8)^(1/6)*x^4
*log(4*b^10*c^14*x^2 + 4*(b^6*c^8)^(2/3)*b^6*c^8 - 4*sqrt(3)*(b^6*c^8)^(5/6
)*b^5*c^7*x) + sqrt(3)*(b^6*c^8)^(1/6)*x^4*log(b^10*c^14*x^2 + (b^6*c^8)^(2
/3)*b^6*c^8 + sqrt(3)*(b^6*c^8)^(5/6)*b^5*c^7*x) - sqrt(3)*(b^6*c^8)^(1/6)*
x^4*log(b^10*c^14*x^2 + (b^6*c^8)^(2/3)*b^6*c^8 - sqrt(3)*(b^6*c^8)^(5/6)*b
^5*c^7*x) + 8*(b^6*c^8)^(1/6)*x^4*arctan(-(sqrt(3)*b^6*c^8 + 2*(b^6*c^8)^(1
/6)*b^5*c^7*x - 2*sqrt(b^10*c^14*x^2 + (b^6*c^8)^(2/3)*b^6*c^8 + sqrt(3)*(b
^6*c^8)^(5/6)*b^5*c^7*x)*(b^6*c^8)^(1/6))/(b^6*c^8)) + 8*(b^6*c^8)^(1/6)*x^
4*arctan((sqrt(3)*b^6*c^8 - 2*(b^6*c^8)^(1/6)*b^5*c^7*x + 2*sqrt(b^10*c^14*
x^2 + (b^6*c^8)^(2/3)*b^6*c^8 - sqrt(3)*(b^6*c^8)^(5/6)*b^5*c^7*x)*(b^6*c^8
)^(1/6))/(b^6*c^8)) + 16*(b^6*c^8)^(1/6)*x^4*arctan(-((b^6*c^8)^(1/6)*b^5*c
^7*x - sqrt(b^10*c^14*x^2 + (b^6*c^8)^(2/3)*b^6*c^8)*(b^6*c^8)^(1/6))/(b^6*
c^8)) - 24*b*c*x^3 - 8*b*arctan(c*x^3) - 8*a)/x^4
```

Sympy [A]
time = 45.93, size = 264, normalized size = 1.52

$$\begin{cases} -\frac{a}{4x^4} + \frac{3bc^3\left(-\frac{1}{c^2}\right)^{\frac{5}{6}}\log\left(4x^2 - 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{16} - \frac{3bc^3\left(-\frac{1}{c^2}\right)^{\frac{5}{6}}\log\left(4x^2 + 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{16} + \frac{\sqrt{3}\,bc^3\left(-\frac{1}{c^2}\right)^{\frac{5}{6}}\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3\sqrt[6]{-\frac{1}{c^2}}} - \frac{\sqrt{3}}{3}\right)}{8} + \frac{\sqrt{3}\,bc^3\left(-\frac{1}{c^2}\right)^{\frac{5}{6}}\operatorname{atan}\left(\frac{2\sqrt{3}\,x}{3\sqrt[6]{-\frac{1}{c^2}}} + \frac{\sqrt{3}}{3}\right)}{8} - \frac{bc^2\sqrt[3]{-\frac{1}{c^2}}\operatorname{atan}\left(cx^3\right)}{4} - \frac{3bc}{4x} - \frac{b\operatorname{atan}\left(cx^3\right)}{4x^4} & \text{for}\ c \neq 0 \\ -\frac{a}{4x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))/x**5,x)
```

```
[Out] Piecewise((-a/(4*x**4) + 3*b*c**3*(-1/c**2)**(5/6)*log(4*x**2 - 4*x*(-1/c**
2)**(1/6) + 4*(-1/c**2)**(1/3))/16 - 3*b*c**3*(-1/c**2)**(5/6)*log(4*x**2 +
 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/16 + sqrt(3)*b*c**3*(-1/c**2)**
(5/6)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) - sqrt(3)/3)/8 + sqrt(3)*b*c**3
*(-1/c**2)**(5/6)*atan(2*sqrt(3)*x/(3*(-1/c**2)**(1/6)) + sqrt(3)/3)/8 - b*
c**2*(-1/c**2)**(1/3)*atan(c*x**3)/4 - 3*b*c/(4*x) - b*atan(c*x**3)/(4*x**4
), Ne(c, 0)), (-a/(4*x**4), True))
```

Giac [A]
time = 0.59, size = 161, normalized size = 0.93

$$\frac{1}{16}\,bc^3\left(\frac{\sqrt{3}\,|c|^{\frac{1}{3}}\log\left(x^2 + \frac{\sqrt{3}\,x}{|c|^{\frac{1}{3}}} + \frac{1}{|c|^{\frac{2}{3}}}\right)}{c^2} - \frac{\sqrt{3}\,|c|^{\frac{1}{3}}\log\left(x^2 - \frac{\sqrt{3}\,x}{|c|^{\frac{1}{3}}} + \frac{1}{|c|^{\frac{2}{3}}}\right)}{c^2} - \frac{2\,|c|^{\frac{1}{3}}\arctan\left(\left(2\,x + \frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^2} - \frac{2\,|c|^{\frac{1}{3}}\arctan\left(\left(2\,x - \frac{\sqrt{3}}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^2} - \frac{4\,|c|^{\frac{1}{3}}\arctan\left(x|c|^{\frac{1}{3}}\right)}{c^2}\right) - \frac{3\,bcx^3 + b\arctan\left(cx^3\right) + a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))/x^5,x, algorithm="giac")
```

```
[Out] 1/16*b*c^3*(sqrt(3)*abs(c)^(1/3)*log(x^2 + sqrt(3)*x/abs(c)^(1/3) + 1/abs(c
)^(2/3))/c^2 - sqrt(3)*abs(c)^(1/3)*log(x^2 - sqrt(3)*x/abs(c)^(1/3) + 1/ab
s(c)^(2/3))/c^2 - 2*abs(c)^(1/3)*arctan((2*x + sqrt(3)/abs(c)^(1/3))*abs(c)
^(1/3))/c^2 - 2*abs(c)^(1/3)*arctan((2*x - sqrt(3)/abs(c)^(1/3))*abs(c)^(1/
3))/c^2 - 4*abs(c)^(1/3)*arctan(x*abs(c)^(1/3))/c^2) - 1/4*(3*b*c*x^3 + b*a
rctan(c*x^3) + a)/x^4
```

Mupad [B]
time = 0.71, size = 120, normalized size = 0.69

$$-\frac{a}{4\,x^4} + \frac{b\,c^{4/3}\left(\operatorname{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right) + \operatorname{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right) + 2\operatorname{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)}{8} - \frac{b\operatorname{atan}\left(c\,x^3\right)}{4\,x^4} - \frac{3\,b\,c}{4\,x} - \frac{\sqrt{3}\,b\,c^{4/3}\left(\operatorname{atan}\left((-1)^{2/3}\,c^{1/3}\,x\right) - \operatorname{atan}\left(\frac{(-1)^{2/3}\,c^{1/3}\,x\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\right)\,1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))/x^5,x)
```

```
[Out] (b*c^(4/3)*(atan((-1)^(2/3)*c^(1/3)*x) + atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2
)*1i - 1))/2) + 2*atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i + 1))/2)))/8 - a/(
4*x^4) - (b*atan(c*x^3))/(4*x^4) - (3*b*c)/(4*x) - (3^(1/2)*b*c^(4/3)*(atan
((-1)^(2/3)*c^(1/3)*x) - atan(((-1)^(2/3)*c^(1/3)*x*(3^(1/2)*1i - 1))/2))*1
i)/8
```

3.113 $\int x^{11}(a+b\mathbf{ArcTan}(cx^3))^2\,dx$

Optimal. Leaf size=124

$$\frac{abx^3}{6c^3}+\frac{b^2x^6}{36c^2}+\frac{b^2x^3\text{ArcTan}(cx^3)}{6c^3}-\frac{bx^9(a+b\text{ArcTan}(cx^3))}{18c}-\frac{(a+b\text{ArcTan}(cx^3))^2}{12c^4}+\frac{1}{12}x^{12}(a+b\text{ArcTan}(cx^3)$$

[Out] 1/6*a*b*x^3/c^3+1/36*b^2*x^6/c^2+1/6*b^2*x^3*arctan(c*x^3)/c^3-1/18*b*x^9*(a+b*arctan(c*x^3))/c-1/12*(a+b*arctan(c*x^3))^2/c^4+1/12*x^12*(a+b*arctan(c*x^3))^2-1/9*b^2*ln(c^2*x^6+1)/c^4

Rubi [A]
time = 0.18, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4948, 4946, 5036, 272, 45, 4930, 266, 5004}

$$-\frac{(a+b\text{ArcTan}(cx^3))^2}{12c^4}+\frac{1}{12}x^{12}(a+b\text{ArcTan}(cx^3))^2-\frac{bx^9(a+b\text{ArcTan}(cx^3))}{18c}+\frac{abx^3}{6c^3}+\frac{b^2x^3\text{ArcTan}(cx^3)}{6c^3}+\frac{b^2x^6}{36c^2}-\frac{b^2\log(c^2x^6+1)}{9c^4}$$

Antiderivative was successfully verified.

[In] Int[x^11*(a + b*ArcTan[c*x^3])^2,x]

[Out] (a*b*x^3)/(6*c^3) + (b^2*x^6)/(36*c^2) + (b^2*x^3*ArcTan[c*x^3])/(6*c^3) - (b*x^9*(a + b*ArcTan[c*x^3]))/(18*c) - (a + b*ArcTan[c*x^3])^2/(12*c^4) + (x^12*(a + b*ArcTan[c*x^3])^2)/12 - (b^2*Log[1 + c^2*x^6])/(9*c^4)

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
```

```
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\int x^{11}\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\,dx = \int\left(\frac{1}{4}x^{11}\left(2a+ib\log\left(1-icx^3\right)\right)^2+\frac{1}{2}bx^{11}\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1\right.\right.$$

$$=\frac{1}{4}\int x^{11}\left(2a+ib\log\left(1-icx^3\right)\right)^2\,dx+\frac{1}{2}b\int x^{11}\left(-2ia+b\log\left(1-icx^3\right)\right)\mathrm{l}$$

$$=\frac{1}{12}\mathrm{Subst}\left(\int x^3(2a+ib\log(1-icx))^2\,dx,x,x^3\right)+\frac{1}{6}b\mathrm{Subst}\left(\int x^3(-2ia+\right.$$

$$=\frac{1}{48}x^{12}\left(2a+ib\log\left(1-icx^3\right)\right)^2-\frac{1}{24}bx^{12}\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+ic\right.$$

$$=\frac{1}{48}x^{12}\left(2a+ib\log\left(1-icx^3\right)\right)^2-\frac{1}{24}bx^{12}\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+ic\right.$$

$$=\frac{1}{48}x^{12}\left(2a+ib\log\left(1-icx^3\right)\right)^2+\frac{1}{288}ib\left(2a+ib\log\left(1-icx^3\right)\right)\left(\frac{48(1-ic}{c^4}\right.$$

$$=\frac{abx^3}{12c^3}-\frac{bx^6(2ia-b\log(1-icx^3))}{48c^2}+\frac{ibx^9(2ia-b\log(1-icx^3))}{72c}+\frac{1}{96}bx^{12}$$

$$=\frac{abx^3}{12c^3}-\frac{bx^6(2ia-b\log(1-icx^3))}{48c^2}+\frac{ibx^9(2ia-b\log(1-icx^3))}{72c}+\frac{1}{96}bx^{12}$$

$$=\frac{abx^3}{12c^3}-\frac{55ib^2x^3}{288c^3}-\frac{5b^2x^6}{576c^2}+\frac{ib^2x^9}{864c}+\frac{b^2x^{12}}{384}-\frac{b^2(1-icx^3)^2}{16c^4}+\frac{b^2(1-icx^3)^3}{54c^4}$$

$$=\frac{abx^3}{12c^3}-\frac{55ib^2x^3}{288c^3}-\frac{5b^2x^6}{576c^2}+\frac{ib^2x^9}{864c}+\frac{b^2x^{12}}{384}-\frac{b^2(1-icx^3)^2}{16c^4}+\frac{b^2(1-icx^3)^3}{54c^4}$$

Mathematica [A]
time = 0.05, size = 121, normalized size = 0.98

$$\frac{cx^3(6ab+b^2cx^3-2abc^2x^6+3a^2c^3x^9)-2b(bcx^3(-3+c^2x^6)+a(3-3c^4x^{12}))\,\mathrm{ArcTan}(cx^3)+3b^2(-1+c^4x^{12})\,\mathrm{ArcTan}(cx^3)^2-4b^2\log\left(1+c^2x^6\right)}{36c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] (c*x^3*(6*a*b + b^2*c*x^3 - 2*a*b*c^2*x^6 + 3*a^2*c^3*x^9) - 2*b*(b*c*x^3*(
-3 + c^2*x^6) + a*(3 - 3*c^4*x^12))*ArcTan[c*x^3] + 3*b^2*(-1 + c^4*x^12)*A
rcTan[c*x^3]^2 - 4*b^2*Log[1 + c^2*x^6])/(36*c^4)
```

Maple [A]
time = 0.10, size = 151, normalized size = 1.22

method	result
default	$\frac{x^{12}a^2}{12}+\frac{b^2x^{12}\arctan(c\,x^3)^2}{12}-\frac{b^2\arctan(c\,x^3)x^9}{18c}+\frac{b^2x^3\arctan(c\,x^3)}{6c^3}-\frac{b^2\arctan(c\,x^3)^2}{12c^4}+\frac{b^2x^6}{36c^2}-\frac{b^2\ln(c^2x^6+1)}{9c^4}+\frac{ab\,x^1}{}$
risch	$-\frac{b^2(x^{12}c^4-1)\ln(ic\,x^3+1)^2}{48c^4}-\frac{ib(6a\,c^4x^{12}+3ib\,c^4x^{12}\ln(-ic\,x^3+1)-2b\,c^3x^9+6bc\,x^3-3ib\ln(-ic\,x^3+1))\ln(ic\,x^3+1)}{72c^4}-\frac{b^2x^{12}\ln(-}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a+b*arctan(c*x^3))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*x^12*a^2+1/12*b^2*x^12*arctan(c*x^3)^2-1/18*b^2*arctan(c*x^3)/c*x^9+1/
6*b^2*x^3*arctan(c*x^3)/c^3-1/12*b^2/c^4*arctan(c*x^3)^2+1/36*b^2*x^6/c^2-1
/9*b^2*ln(c^2*x^6+1)/c^4+1/6*a*b*x^12*arctan(c*x^3)-1/18*a*b/c*x^9+1/6*a*b*
x^3/c^3-1/6*a*b/c^4*arctan(c*x^3)
```

Maxima [A]
time = 0.55, size = 169, normalized size = 1.36

$$\frac{1}{12}\,b^2x^{12}\arctan\left(cx^3\right)^2+\frac{1}{12}\,a^2x^{12}+\frac{1}{18}\left(3\,x^{12}\arctan\left(cx^3\right)-c\left(\frac{c^2x^9-3\,x^3}{c^4}+\frac{3\,\arctan\left(cx^3\right)}{c^5}\right)\right)ab-\frac{1}{36}\left(2\,c\left(\frac{c^2x^9-3\,x^3}{c^4}+\frac{3\,\arctan\left(cx^3\right)}{c^5}\right)\arctan\left(cx^3\right)-\frac{c^2x^6+3\,\arctan\left(cx^3\right)^2-3\,\log\left(18\,c^7x^6+18\,c^5\right)-\log\left(c^2x^6+1\right)}{c^4}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/12*b^2*x^12*arctan(c*x^3)^2 + 1/12*a^2*x^12 + 1/18*(3*x^12*arctan(c*x^3)
- c*((c^2*x^9 - 3*x^3)/c^4 + 3*arctan(c*x^3)/c^5))*a*b - 1/36*(2*c*((c^2*x^
9 - 3*x^3)/c^4 + 3*arctan(c*x^3)/c^5)*arctan(c*x^3) - (c^2*x^6 + 3*arctan(c
*x^3)^2 - 3*log(18*c^7*x^6 + 18*c^5) - log(c^2*x^6 + 1))/c^4)*b^2
```

Fricas [A]
time = 1.35, size = 129, normalized size = 1.04

$$\frac{3\,a^2c^4x^{12}-2\,abc^3x^9+b^2c^2x^6+6\,abcx^3+3\,(b^2c^4x^{12}-b^2)\arctan\left(cx^3\right)^2-4\,b^2\log\left(c^2x^6+1\right)+2\,(3\,abc^4x^{12}-b^2c^3x^9+3\,b^2cx^3-3\,ab)\arctan\left(cx^3\right)}{36\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] 1/36*(3*a^2*c^4*x^12 - 2*a*b*c^3*x^9 + b^2*c^2*x^6 + 6*a*b*c*x^3 + 3*(b^2*c
^4*x^12 - b^2)*arctan(c*x^3)^2 - 4*b^2*log(c^2*x^6 + 1) + 2*(3*a*b*c^4*x^12
 - b^2*c^3*x^9 + 3*b^2*c*x^3 - 3*a*b)*arctan(c*x^3))/c^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(112) = 224$.
time = 151.09, size = 243, normalized size = 1.96

$$\begin{cases}\frac{a^2x^{12}}{12}+\frac{abx^{12}\operatorname{atan}(cx^3)}{6}-\frac{abx^9}{18c}+\frac{abx^3}{6c^3}-\frac{ab\operatorname{atan}(cx^3)}{6c^4}+\frac{b^2x^{12}\operatorname{atan}^2(cx^3)}{12}-\frac{b^2x^9\operatorname{atan}(cx^3)}{18c}+\frac{b^2x^6}{36c^2}+\frac{b^2x^3\operatorname{atan}(cx^3)}{6c^3}+\frac{2b^2\sqrt{-\frac{1}{c^2}}\operatorname{atan}(cx^3)}{9c^3}-\frac{2b^2\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{9c^4}-\frac{2b^2\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{9c^4}-\frac{b^2\operatorname{atan}^2(cx^3)}{12c^4} & \text{for}\ c\neq 0\\ \frac{a^2x^{12}}{12} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(a+b*atan(c*x**3))**2,x)
```

```
[Out] Piecewise((a**2*x**12/12 + a*b*x**12*atan(c*x**3)/6 - a*b*x**9/(18*c) + a*b
*x**3/(6*c**3) - a*b*atan(c*x**3)/(6*c**4) + b**2*x**12*atan(c*x**3)**2/12
- b**2*x**9*atan(c*x**3)/(18*c) + b**2*x**6/(36*c**2) + b**2*x**3*atan(c*x*
*3)/(6*c**3) + 2*b**2*sqrt(-1/c**2)*atan(c*x**3)/(9*c**3) - 2*b**2*log(x -
(-1/c**2)**(1/6))/(9*c**4) - 2*b**2*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(
-1/c**2)**(1/3))/(9*c**4) - b**2*atan(c*x**3)**2/(12*c**4), Ne(c, 0)), (a**
2*x**12/12, True))
```

Giac [A]
time = 0.47, size = 145, normalized size = 1.17

$$\frac{3\,a^2cx^{12} + 2\left(3\,cx^{12}\arctan\left(cx^3\right) - \frac{3\,\arctan\left(cx^3\right)}{c^3} - \frac{c^9x^9 - 3\,c^7x^3}{c^9}\right)ab + \left(3\,cx^{12}\arctan\left(cx^3\right)^2 - \frac{2\,c^3x^9\arctan\left(cx^3\right) - c^2x^6 - 6\,cx^3\arctan\left(cx^3\right) + 3\,\arctan\left(cx^3\right)^2 + 4\,\log\left(c^2x^6+1\right)}{c^3}\right)b^2}{36\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

```
[Out] 1/36*(3*a^2*c*x^12 + 2*(3*c*x^12*arctan(c*x^3) - 3*arctan(c*x^3)/c^3 - (c^9
*x^9 - 3*c^7*x^3)/c^9)*a*b + (3*c*x^12*arctan(c*x^3)^2 - (2*c^3*x^9*arctan(
c*x^3) - c^2*x^6 - 6*c*x^3*arctan(c*x^3) + 3*arctan(c*x^3)^2 + 4*log(c^2*x^
6 + 1))/c^3)*b^2)/c
```

Mupad [B]
time = 1.14, size = 150, normalized size = 1.21

$$\frac{a^2\,x^{12}}{12}-\frac{b^2\,\mathrm{atan}\left(c\,x^3\right)^2}{12\,c^4}+\frac{b^2\,x^{12}\,\mathrm{atan}\left(c\,x^3\right)^2}{12}-\frac{b^2\,\ln\left(c^2\,x^6+1\right)}{9\,c^4}+\frac{b^2\,x^6}{36\,c^2}+\frac{b^2\,x^3\,\mathrm{atan}\left(c\,x^3\right)}{6\,c^3}-\frac{b^2\,x^9\,\mathrm{atan}\left(c\,x^3\right)}{18\,c}+\frac{a\,b\,x^3}{6\,c^3}-\frac{a\,b\,x^9}{18\,c}-\frac{a\,b\,\mathrm{atan}\left(c\,x^3\right)}{6\,c^4}+\frac{a\,b\,x^{12}\,\mathrm{atan}\left(c\,x^3\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*atan(c*x^3))^2,x)
```

```
[Out] (a^2*x^12)/12 - (b^2*atan(c*x^3)^2)/(12*c^4) + (b^2*x^12*atan(c*x^3)^2)/12
- (b^2*log(c^2*x^6 + 1))/(9*c^4) + (b^2*x^6)/(36*c^2) + (b^2*x^3*atan(c*x^3
))/(6*c^3) - (b^2*x^9*atan(c*x^3))/(18*c) + (a*b*x^3)/(6*c^3) - (a*b*x^9)/(
18*c) - (a*b*atan(c*x^3))/(6*c^4) + (a*b*x^12*atan(c*x^3))/6
```

3.114 $\int x^8(a+b\mathbf{ArcTan}(cx^3))^2\,dx$

Optimal. Leaf size=154

$$\frac{b^2x^3}{9c^2}-\frac{b^2\text{ArcTan}(cx^3)}{9c^3}-\frac{bx^6(a+b\text{ArcTan}(cx^3))}{9c}-\frac{i(a+b\text{ArcTan}(cx^3))^2}{9c^3}+\frac{1}{9}x^9\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2-\frac{2b(a+b}{}$$

```
[Out] 1/9*b^2*x^3/c^2-1/9*b^2*arctan(c*x^3)/c^3-1/9*b*x^6*(a+b*arctan(c*x^3))/c-1
/9*I*(a+b*arctan(c*x^3))^2/c^3+1/9*x^9*(a+b*arctan(c*x^3))^2-2/9*b*(a+b*arc
tan(c*x^3))*ln(2/(1+I*c*x^3))/c^3-1/9*I*b^2*polylog(2,1-2/(1+I*c*x^3))/c^3
```

Rubi [A]
time = 0.17, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {4948, 4946, 5036, 327, 209, 5040, 4964, 2449, 2352}

$$-\frac{i(a+b\text{ArcTan}(cx^3))^2}{9c^3}-\frac{2b\log\left(\frac{2}{1+icx^3}\right)(a+b\text{ArcTan}(cx^3))}{9c^3}+\frac{1}{9}x^9\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2-\frac{bx^6(a+b\text{ArcTan}(cx^3))}{9c}-\frac{b^2\text{ArcTan}(cx^3)}{9c^3}-\frac{ib^2\text{Li}_2\left(1-\frac{2}{icx^3+1}\right)}{9c^3}+\frac{b^2x^3}{9c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] (b^2*x^3)/(9*c^2) - (b^2*ArcTan[c*x^3])/(9*c^3) - (b*x^6*(a + b*ArcTan[c*x^
3]))/(9*c) - ((I/9)*(a + b*ArcTan[c*x^3])^2)/c^3 + (x^9*(a + b*ArcTan[c*x^3
])^2)/9 - (2*b*(a + b*ArcTan[c*x^3])*Log[2/(1 + I*c*x^3)])/(9*c^3) - ((I/9)
*b^2*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c^3
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x^8\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\,dx = \int\left(\frac{1}{4}x^8\left(2a+ib\log\left(1-icx^3\right)\right)^2+\frac{1}{2}bx^8\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+ic\right.\right.$$

$$=\frac{1}{4}\int x^8\left(2a+ib\log\left(1-icx^3\right)\right)^2\,dx+\frac{1}{2}b\int x^8\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(\right.$$

$$=\frac{1}{12}\text{Subst}\left(\int x^2(2a+ib\log(1-icx))^2\,dx,x,x^3\right)+\frac{1}{6}b\text{Subst}\left(\int x^2(-2ia+b\right.$$

$$=\frac{1}{36}x^9\left(2a+ib\log\left(1-icx^3\right)\right)^2-\frac{1}{18}bx^9\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)$$

$$=\frac{1}{36}x^9\left(2a+ib\log\left(1-icx^3\right)\right)^2-\frac{1}{18}bx^9\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)$$

$$=\frac{1}{36}x^9\left(2a+ib\log\left(1-icx^3\right)\right)^2+\frac{1}{108}ib\left(2a+ib\log\left(1-icx^3\right)\right)\left(\frac{18i(1-icx^3}{c^3}\right.$$

$$=-\frac{iabx^3}{9c^2}+\frac{ibx^6(2ia-b\log(1-icx^3))}{36c}+\frac{1}{54}bx^9\left(2ia-b\log\left(1-icx^3\right)\right)+\frac{1}{36}$$

$$=-\frac{iabx^3}{9c^2}+\frac{ibx^6(2ia-b\log(1-icx^3))}{36c}+\frac{1}{54}bx^9\left(2ia-b\log\left(1-icx^3\right)\right)+\frac{1}{36}$$

$$=-\frac{iabx^3}{9c^2}+\frac{13b^2x^3}{108c^2}+\frac{ib^2x^6}{216c}+\frac{b^2x^9}{162}-\frac{ib^2(1-icx^3)^2}{24c^3}+\frac{ib^2(1-icx^3)^3}{162c^3}+\frac{ib^2(1}{}$$

$$=-\frac{iabx^3}{9c^2}+\frac{13b^2x^3}{108c^2}+\frac{ib^2x^6}{216c}+\frac{b^2x^9}{162}-\frac{ib^2(1-icx^3)^2}{24c^3}+\frac{ib^2(1-icx^3)^3}{162c^3}+\frac{ib^2(1}{}$$

Mathematica [A]
time = 0.20, size = 141, normalized size = 0.92

$$\frac{b^2cx^3-abc^2x^6+a^2c^3x^9+b^2(i+c^3x^9)\,\text{ArcTan}(cx^3)^2-b\text{ArcTan}(cx^3)\left(b+bc^2x^6-2ac^3x^9+2b\log\left(1+e^{2i\text{ArcTan}(cx^3)}\right)\right)+ab\log\left(1+c^2x^6\right)+ib^2\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx^3)}\right)}{9c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] (b^2*c*x^3 - a*b*c^2*x^6 + a^2*c^3*x^9 + b^2*(I + c^3*x^9)*ArcTan[c*x^3]^2
- b*ArcTan[c*x^3]*(b + b*c^2*x^6 - 2*a*c^3*x^9 + 2*b*Log[1 + E^((2*I)*ArcTa
n[c*x^3])]) + a*b*Log[1 + c^2*x^6] + I*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x^
3])])/(9*c^3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{8}\left(a+b\arctan\left(c\,x^{3}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a+b*arctan(c*x^3))^2,x)
```

```
[Out] int(x^8*(a+b*arctan(c*x^3))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/9*a^2*x^9 + 1/9*(2*x^9*arctan(c*x^3) - (x^6/c^2 - log(c^2*x^6 + 1)/c^4)*c
)*a*b + 1/144*(4*x^9*arctan(c*x^3)^2 - x^9*log(c^2*x^6 + 1)^2 + 144*integra
te(1/48*(4*c^2*x^14*log(c^2*x^6 + 1) - 8*c*x^11*arctan(c*x^3) + 36*(c^2*x^1
4 + x^8)*arctan(c*x^3)^2 + 3*(c^2*x^14 + x^8)*log(c^2*x^6 + 1)^2)/(c^2*x^6
+ 1), x))*b^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*x^8*arctan(c*x^3)^2 + 2*a*b*x^8*arctan(c*x^3) + a^2*x^8, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{8}\left(a+b\operatorname{atan}\left(cx^{3}\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*atan(c*x**3))**2,x)
```

```
[Out] Integral(x**8*(a + b*atan(c*x**3))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)^2*x^8, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^8\,\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*atan(c*x^3))^2,x)
[Out] int(x^8*(a + b*atan(c*x^3))^2, x)
```

3.115 $\int x^5(a+b\text{ArcTan}(cx^3))^2\,dx$

Optimal. Leaf size=90

$$-\frac{abx^3}{3c}-\frac{b^2x^3\text{ArcTan}(cx^3)}{3c}+\frac{(a+b\text{ArcTan}(cx^3))^2}{6c^2}+\frac{1}{6}x^6\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2+\frac{b^2\log\left(1+c^2x^6\right)}{6c^2}$$

```
[Out] -1/3*a*b*x^3/c-1/3*b^2*x^3*arctan(c*x^3)/c+1/6*(a+b*arctan(c*x^3))^2/c^2+1/
6*x^6*(a+b*arctan(c*x^3))^2+1/6*b^2*ln(c^2*x^6+1)/c^2
```

Rubi [A]

time = 0.10, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4948, 4946, 5036, 4930, 266, 5004}

$$\frac{(a+b\text{ArcTan}(cx^3))^2}{6c^2}+\frac{1}{6}x^6\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2-\frac{abx^3}{3c}-\frac{b^2x^3\text{ArcTan}(cx^3)}{3c}+\frac{b^2\log\left(c^2x^6+1\right)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] -1/3*(a*b*x^3)/c - (b^2*x^3*ArcTan[c*x^3])/(3*c) + (a + b*ArcTan[c*x^3])^2/
(6*c^2) + (x^6*(a + b*ArcTan[c*x^3])^2)/6 + (b^2*Log[1 + c^2*x^6])/(6*c^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
```

```
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\int x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\,dx=\int\left(\frac{1}{4}x^5\left(2a+ib\log\left(1-icx^3\right)\right)^2+\frac{1}{2}bx^5\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+ic\right.\right.$$

$$=\frac{1}{4}\int x^5\left(2a+ib\log\left(1-icx^3\right)\right)^2\,dx+\frac{1}{2}b\int x^5\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(\right.$$

$$=\frac{1}{12}\text{Subst}\left(\int x(2a+ib\log(1-icx))^2\,dx,x,x^3\right)+\frac{1}{6}b\text{Subst}\left(\int x(-2ia+b\log\right.$$

$$=-\frac{1}{12}bx^6\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)+\frac{1}{12}\text{Subst}\left(\int\left(-\frac{i(2a+ib\log}{}\right.\right.$$

$$=-\frac{1}{12}bx^6\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)-\frac{i\text{Subst}\left(\int(2a+ib\log(1-ic\right.}{12c}$$

$$=-\frac{1}{12}bx^6\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)-\frac{1}{12}b\text{Subst}\left(\int x(-2ia+b\log\right.$$

$$=-\frac{abx^3}{6c}+\frac{1}{24}bx^6\left(2ia-b\log\left(1-icx^3\right)\right)+\frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12c^2}$$

$$=-\frac{abx^3}{2c}-\frac{ib^2x^3}{4c}+\frac{b^2\left(1-icx^3\right)^2}{48c^2}+\frac{b^2\left(1+icx^3\right)^2}{48c^2}+\frac{1}{24}bx^6\left(2ia-b\log\left(1-ic\right.\right.$$

$$=-\frac{abx^3}{2c}+\frac{b^2x^6}{24}+\frac{b^2\left(1-icx^3\right)^2}{48c^2}+\frac{b^2\left(1+icx^3\right)^2}{48c^2}-\frac{b^2\log\left(i-cx^3\right)}{24c^2}+\frac{b^2(1-ic}{}$$

Mathematica [**A**]

time = 0.04, size = 85, normalized size = 0.94

$$\frac{acx^3(-2b+acx^3)+2b(a-bcx^3+ac^2x^6)\operatorname{ArcTan}(cx^3)+b^2(1+c^2x^6)\operatorname{ArcTan}(cx^3)^2+b^2\log(1+c^2x^6)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] (a*c*x^3*(-2*b + a*c*x^3) + 2*b*(a - b*c*x^3 + a*c^2*x^6)*ArcTan[c*x^3] + b
^2*(1 + c^2*x^6)*ArcTan[c*x^3]^2 + b^2*Log[1 + c^2*x^6])/(6*c^2)
```

Maple [A]
time = 0.14, size = 113, normalized size = 1.26

method	result
default	$\frac{x^6a^2}{6}+\frac{b^2x^6\arctan(cx^3)^2}{6}-\frac{b^2x^3\arctan(cx^3)}{3c}+\frac{b^2\arctan(cx^3)^2}{6c^2}+\frac{b^2\ln(c^2x^6+1)}{6c^2}+\frac{ab\,x^6\arctan(cx^3)}{3}-\frac{ab\,x^3}{3c}+\frac{ab\,\mathrm{a}}{}$
risch	$-\frac{b^2(c^2x^6+1)\ln(icx^3+1)^2}{24c^2}-\frac{ib(4a^2c^2x^6+2ix^6b\ln(-icx^3+1)ac^2-4abcx^3+b^2+2ib\ln(-icx^3+1)a)\ln(icx^3+1)}{24c^2a}+\frac{ib^3\ln(c^2x^6}{48c^2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x^3))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x^6*a^2+1/6*b^2*x^6*arctan(c*x^3)^2-1/3*b^2*x^3*arctan(c*x^3)/c+1/6*b^2
/c^2*arctan(c*x^3)^2+1/6*b^2*ln(c^2*x^6+1)/c^2+1/3*a*b*x^6*arctan(c*x^3)-1/
3*a*b*x^3/c+1/3*a*b/c^2*arctan(c*x^3)
```

Maxima [A]
time = 0.54, size = 126, normalized size = 1.40

$$\frac{1}{6}b^2x^6\arctan(cx^3)^2+\frac{1}{6}a^2x^6+\frac{1}{3}\left(x^6\arctan(cx^3)-c\left(\frac{x^3}{c^2}-\frac{\arctan(cx^3)}{c^3}\right)\right)ab-\frac{1}{6}\left(2c\left(\frac{x^3}{c^2}-\frac{\arctan(cx^3)}{c^3}\right)\arctan(cx^3)+\frac{\arctan(cx^3)^2-\log(6c^5x^6+6c^3)}{c^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6*arctan(c*x^3)^2 + 1/6*a^2*x^6 + 1/3*(x^6*arctan(c*x^3) - c*(x^3
/c^2 - arctan(c*x^3)/c^3))*a*b - 1/6*(2*c*(x^3/c^2 - arctan(c*x^3)/c^3)*arc
tan(c*x^3) + (arctan(c*x^3)^2 - log(6*c^5*x^6 + 6*c^3))/c^2)*b^2
```

Fricas [A]
time = 1.03, size = 91, normalized size = 1.01

$$\frac{a^2c^2x^6-2\,abcx^3+(b^2c^2x^6+b^2)\arctan(cx^3)^2+b^2\log(c^2x^6+1)+2\,(abc^2x^6-b^2cx^3+ab)\arctan(cx^3)}{6\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(a^2*c^2*x^6 - 2*a*b*c*x^3 + (b^2*c^2*x^6 + b^2)*arctan(c*x^3)^2 + b^2*
log(c^2*x^6 + 1) + 2*(a*b*c^2*x^6 - b^2*c*x^3 + a*b)*arctan(c*x^3))/c^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(78) = 156$.
time = 50.50, size = 194, normalized size = 2.16

$$\begin{cases} \frac{a^2x^6}{6} + \frac{abx^6 \operatorname{atan}(cx^3)}{3} - \frac{abx^3}{3c} + \frac{ab \operatorname{atan}(cx^3)}{3c^2} + \frac{b^2x^6 \operatorname{atan}^2(cx^3)}{6} - \frac{b^2x^3 \operatorname{atan}(cx^3)}{3c} - \frac{b^2\sqrt{-\frac{1}{c^2}} \operatorname{atan}(cx^3)}{3c} + \frac{b^2 \log\left(x - \sqrt[6]{-\frac{1}{c^2}}\right)}{3c^2} + \frac{b^2 \log\left(4x^2 + 4x\sqrt[6]{-\frac{1}{c^2}} + 4\sqrt[3]{-\frac{1}{c^2}}\right)}{3c^2} + \frac{b^2 \operatorname{atan}^2(cx^3)}{6c^2} & \text{for } c \neq 0 \\ \frac{a^2x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x**3))**2,x)
```

```
[Out] Piecewise((a**2*x**6/6 + a*b*x**6*atan(c*x**3)/3 - a*b*x**3/(3*c) + a*b*ata
n(c*x**3)/(3*c**2) + b**2*x**6*atan(c*x**3)**2/6 - b**2*x**3*atan(c*x**3)/(
3*c) - b**2*sqrt(-1/c**2)*atan(c*x**3)/(3*c) + b**2*log(x - (-1/c**2)**(1/6
))/(3*c**2) + b**2*log(4*x**2 + 4*x*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/
(3*c**2) + b**2*atan(c*x**3)**2/(6*c**2), Ne(c, 0)), (a**2*x**6/6, True))
```

Giac [A]
time = 0.42, size = 100, normalized size = 1.11

$$\frac{a^2cx^6 + \frac{2\left(c^2x^6 \arctan\left(cx^3\right) - cx^3 + \arctan\left(cx^3\right)\right)ab}{c} + \frac{\left(c^2x^6 \arctan\left(cx^3\right)^2 - 2\,cx^3 \arctan\left(cx^3\right) + \arctan\left(cx^3\right)^2 + \log\left(c^2x^6 + 1\right)\right)b^2}{c}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

```
[Out] 1/6*(a^2*c*x^6 + 2*(c^2*x^6*arctan(c*x^3) - c*x^3 + arctan(c*x^3))*a*b/c +
(c^2*x^6*arctan(c*x^3)^2 - 2*c*x^3*arctan(c*x^3) + arctan(c*x^3)^2 + log(c^
2*x^6 + 1))*b^2/c)/c
```

Mupad [B]
time = 0.72, size = 112, normalized size = 1.24

$$\frac{a^2\,x^6}{6} + \frac{b^2\,\mathrm{atan}\left(c\,x^3\right)^2}{6\,c^2} + \frac{b^2\,x^6\,\mathrm{atan}\left(c\,x^3\right)^2}{6} + \frac{b^2\,\ln\left(c^2\,x^6 + 1\right)}{6\,c^2} - \frac{b^2\,x^3\,\mathrm{atan}\left(c\,x^3\right)}{3\,c} - \frac{a\,b\,x^3}{3\,c} + \frac{a\,b\,\mathrm{atan}\left(c\,x^3\right)}{3\,c^2} + \frac{a\,b\,x^6\,\mathrm{atan}\left(c\,x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atan(c*x^3))^2,x)
```

```
[Out] (a^2*x^6)/6 + (b^2*atan(c*x^3)^2)/(6*c^2) + (b^2*x^6*atan(c*x^3)^2)/6 + (b^
2*log(c^2*x^6 + 1))/(6*c^2) - (b^2*x^3*atan(c*x^3))/(3*c) - (a*b*x^3)/(3*c)
 + (a*b*atan(c*x^3))/(3*c^2) + (a*b*x^6*atan(c*x^3))/3
```

3.116 $\int x^2(a+b\mathbf{ArcTan}(cx^3))^2\,dx$

Optimal. Leaf size=104

$$\frac{i(a+b\text{ArcTan}(cx^3))^2}{3c}+\frac{1}{3}x^3(a+b\text{ArcTan}(cx^3))^2+\frac{2b(a+b\text{ArcTan}(cx^3))\log\left(\frac{2}{1+icx^3}\right)}{3c}+\frac{ib^2\text{PolyLog}(2,1-\frac{2}{1+icx^3})}{3c}$$

```
[Out] 1/3*I*(a+b*arctan(c*x^3))^2/c+1/3*x^3*(a+b*arctan(c*x^3))^2+2/3*b*(a+b*arct
an(c*x^3))*ln(2/(1+I*c*x^3))/c+1/3*I*b^2*polylog(2,1-2/(1+I*c*x^3))/c
```

Rubi [A]
time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4948, 4930, 5040, 4964, 2449, 2352}

$$\frac{1}{3}x^3(a+b\text{ArcTan}(cx^3))^2+\frac{i(a+b\text{ArcTan}(cx^3))^2}{3c}+\frac{2b\log\left(\frac{2}{1+icx^3}\right)(a+b\text{ArcTan}(cx^3))}{3c}+\frac{ib^2\text{Li}_2\left(1-\frac{2}{icx^3+1}\right)}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] ((I/3)*(a + b*ArcTan[c*x^3])^2)/c + (x^3*(a + b*ArcTan[c*x^3])^2)/3 + (2*b*
(a + b*ArcTan[c*x^3])*Log[2/(1 + I*c*x^3)])/(3*c) + ((I/3)*b^2*PolyLog[2, 1
 - 2/(1 + I*c*x^3)])/c
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
```

```
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\,dx &= \int \left(\frac{1}{4}x^2\left(2a+ib\log\left(1-icx^3\right)\right)^2+\frac{1}{2}bx^2\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(1+ic\right.\right.\\
&= \frac{1}{4}\int x^2\left(2a+ib\log\left(1-icx^3\right)\right)^2\,dx+\frac{1}{2}b\int x^2\left(-2ia+b\log\left(1-icx^3\right)\right)\log\left(\right.\\
&= \frac{1}{12}\text{Subst}\left(\int (2a+ib\log(1-icx))^2\,dx,x,x^3\right)+\frac{1}{6}b\text{Subst}\left(\int\left(-2ia+b\log(1\right.\right.\\
&= -\frac{1}{6}bx^3\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+icx^3\right)+\frac{i\text{Subst}\left(\int(2a+ib\log(x))^2\,dx\right.}{12c}\\
&= \frac{i(1-icx^3)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12c}-\frac{1}{6}bx^3\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(1+i\right.\\
&= -\frac{1}{3}iabx^3-\frac{b^2x^3}{6}+\frac{i(1-icx^3)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12c}-\frac{ib^2(1+icx^3)\log\left(1\right.}{6c}\\
&= -\frac{1}{3}b^2x^3+\frac{ib^2(1-icx^3)\log\left(1-icx^3\right)}{6c}+\frac{i(1-icx^3)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12c}\\
&= -\frac{1}{6}b^2x^3+\frac{ib^2(1-icx^3)\log\left(1-icx^3\right)}{6c}+\frac{i(1-icx^3)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12c}\\
&= \frac{i(1-icx^3)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{12c}+\frac{ib\left(2ia-b\log\left(1-icx^3\right)\right)\log\left(\frac{1}{2}(1+ic\right.}{6c}
\end{aligned}$$

Mathematica [A]

time = 0.04, size = 107, normalized size = 1.03

$$\frac{b^2(-i+cx^3)\operatorname{ArcTan}(cx^3)^2+2b\operatorname{ArcTan}(cx^3)\left(acx^3+b\log\left(1+e^{2i\operatorname{ArcTan}(cx^3)}\right)\right)+a(acx^3-b\log(1+c^2x^6))-ib^2\operatorname{PolyLog}\left(2,-e^{2i\operatorname{ArcTan}(cx^3)}\right)}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x^3])^2,x]
```

```
[Out] (b^2*(-I + c*x^3)*ArcTan[c*x^3]^2 + 2*b*ArcTan[c*x^3]*(a*c*x^3 + b*Log[1 +
E^((2*I)*ArcTan[c*x^3])]) + a*(a*c*x^3 - b*Log[1 + c^2*x^6]) - I*b^2*PolyLo
g[2, -E^((2*I)*ArcTan[c*x^3])])/(3*c)
```

Maple [A]
time = 0.17, size = 142, normalized size = 1.37

method	result
derivativedivides	$\frac{c x^3 a^2-i \arctan(c x^3)^2 b^2+\arctan(c x^3)^2 b^2 c x^3-i \operatorname{polylog}\left(2,-\frac{(i c x^3+1)^2}{c^2 x^6+1}\right) b^2+2 \arctan(c x^3) \ln\left(1+\frac{(i c x^3+1)^2}{c^2 x^6+1}\right) b^2+2}{3c}$
default	$\frac{c x^3 a^2-i \arctan(c x^3)^2 b^2+\arctan(c x^3)^2 b^2 c x^3-i \operatorname{polylog}\left(2,-\frac{(i c x^3+1)^2}{c^2 x^6+1}\right) b^2+2 \arctan(c x^3) \ln\left(1+\frac{(i c x^3+1)^2}{c^2 x^6+1}\right) b^2+2}{3c}$
risch	$\frac{x^3 a^2}{3}+\frac{i b^2 \ln(i c x^3+1) \ln(-i c x^3+1)}{6c}+\frac{i b^2}{3c}+\frac{i b^2 \ln\left(\frac{1}{2}+\frac{i c x^3}{2}\right) \ln\left(\frac{1}{2}-\frac{i c x^3}{2}\right)}{3c}+\frac{i a^2}{3c}-\frac{i b^2 \ln\left(\frac{1}{2}+\frac{i c x^3}{2}\right) \ln(-i c x}{3c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x^3))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/c*(c*x^3*a^2-I*arctan(c*x^3)^2*b^2+arctan(c*x^3)^2*b^2*c*x^3-I*polylog(
2,-(1+I*c*x^3)^2/(c^2*x^6+1))*b^2+2*arctan(c*x^3)*ln(1+(1+I*c*x^3)^2/(c^2*x
^6+1))*b^2+2*a*b*c*x^3*arctan(c*x^3)-a*b*ln(c^2*x^6+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/48*(4*x^3*arctan(c*x^3)^2 - x^3*log(c^2*x^6 + 1)^2 + 576*c^
2*integrate(1/16*x^8*arctan(c*x^3)^2/(c^2*x^6 + 1), x) + 48*c^2*integrate(1
/16*x^8*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 192*c^2*integrate(1/16*x^8*l
og(c^2*x^6 + 1)/(c^2*x^6 + 1), x) + 4*arctan(c*x^3)^3/c - 384*c*integrate(1
/16*x^5*arctan(c*x^3)/(c^2*x^6 + 1), x) + 48*integrate(1/16*x^2*log(c^2*x^6
 + 1)^2/(c^2*x^6 + 1), x))*b^2 + 1/3*(2*c*x^3*arctan(c*x^3) - log(c^2*x^6 +
 1))*a*b/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
[Out] integral(b^2*x^2*arctan(c*x^3)^2 + 2*a*b*x^2*arctan(c*x^3) + a^2*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \operatorname{atan}\left(cx^3\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x**3))**2,x)
[Out] Integral(x**2*(a + b*atan(c*x**3))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)^2*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(a + b \operatorname{atan}\left(c x^3\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x^3))^2,x)
[Out] int(x^2*(a + b*atan(c*x^3))^2, x)
```

3.117 $\int \frac{\left(a+b\textbf{ArcTan}\left(cx^3\right)\right)^2}{x}\,dx$

Optimal. Leaf size=154

$$\frac{2}{3}\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)-\frac{1}{3}ib\left(a+b\text{ArcTan}\left(cx^3\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)+\frac{1}{3}ib($$

```
[Out] -2/3*(a+b*arctan(c*x^3))^2*arctanh(-1+2/(1+I*c*x^3))-1/3*I*b*(a+b*arctan(c*
x^3))*polylog(2,1-2/(1+I*c*x^3))+1/3*I*b*(a+b*arctan(c*x^3))*polylog(2,-1+2
/(1+I*c*x^3))-1/6*b^2*polylog(3,1-2/(1+I*c*x^3))+1/6*b^2*polylog(3,-1+2/(1+
I*c*x^3))
```

Rubi [A]
time = 0.21, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4944, 4942, 5108, 5004, 5114, 6745}

$$-\frac{1}{3}ib\text{Li}_2\left(1-\frac{2}{icx^3+1}\right)\left(a+b\text{ArcTan}\left(cx^3\right)\right)+\frac{1}{3}ib\text{Li}_2\left(\frac{2}{icx^3+1}-1\right)\left(a+b\text{ArcTan}\left(cx^3\right)\right)+\frac{2}{3}\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2-\frac{1}{6}b^2\text{Li}_3\left(1-\frac{2}{icx^3+1}\right)+\frac{1}{6}b^2\text{Li}_3\left(\frac{2}{icx^3+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])^2/x,x]
```

```
[Out] (2*(a + b*ArcTan[c*x^3])^2*ArcTanh[1 - 2/(1 + I*c*x^3)])/3 - (I/3)*b*(a + b
*ArcTan[c*x^3])*PolyLog[2, 1 - 2/(1 + I*c*x^3)] + (I/3)*b*(a + b*ArcTan[c*x
^3])*PolyLog[2, -1 + 2/(1 + I*c*x^3)] - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x^3)
])/6 + (b^2*PolyLog[3, -1 + 2/(1 + I*c*x^3)])/6
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}{x}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx,x,x^3\right)$$

$$= \frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)-\frac{1}{3}(4bc)\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(\right.}{}\right.$$

$$= \frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)+\frac{1}{3}(2bc)\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(\right.}{1+}\right.$$

$$= \frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)-\frac{1}{3}ib\left(a+b\tan^{-1}\left(cx^3\right)\right)\text{Li}_2\left(1-\right.$$

$$= \frac{2}{3}\left(a+b\tan^{-1}\left(cx^3\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)-\frac{1}{3}ib\left(a+b\tan^{-1}\left(cx^3\right)\right)\text{Li}_2\left(1-\right.$$

Mathematica [A]
time = 0.08, size = 167, normalized size = 1.08

$$\frac{1}{6}\left(4\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2\tanh^{-1}\left(1+\frac{2i}{-i+cx^3}\right)+b\left(2i\left(a+b\text{ArcTan}\left(cx^3\right)\right)\text{PolyLog}\left(2,\frac{i+cx^3}{i-cx^3}\right)-2i\left(a+b\text{ArcTan}\left(cx^3\right)\right)\text{PolyLog}\left(2,\frac{i+cx^3}{-i+cx^3}\right)+b\left(\text{PolyLog}\left(3,\frac{i+cx^3}{i-cx^3}\right)-\text{PolyLog}\left(3,\frac{i+cx^3}{-i+cx^3}\right)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^2/x,x]
```

```
[Out] (4*(a + b*ArcTan[c*x^3])^2*ArcTanh[1 + (2*I)/(-I + c*x^3)] + b*((2*I)*(a +
b*ArcTan[c*x^3])*PolyLog[2, (I + c*x^3)/(I - c*x^3)] - (2*I)*(a + b*ArcTan[
c*x^3])*PolyLog[2, (I + c*x^3)/(-I + c*x^3)] + b*(PolyLog[3, (I + c*x^3)/(I
 - c*x^3)] - PolyLog[3, (I + c*x^3)/(-I + c*x^3)])))/6
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\arctan\left(c\,x^{3}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^2/x,x)
```

```
[Out] int((a+b*arctan(c*x^3))^2/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + 1/16*integrate((12*b^2*arctan(c*x^3)^2 + b^2*log(c^2*x^6 + 1)^
2 + 32*a*b*arctan(c*x^3))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\operatorname{atan}\left(cx^{3}\right)\right)^{2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**2/x,x)
```

```
[Out] Integral((a + b*atan(c*x**3))**2/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^2/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))^2/x,x)
```

```
[Out] int((a + b*atan(c*x^3))^2/x, x)
```

3.118 $\int \frac{\left(a+b\text{ArcTan}(cx^3)\right)^2}{x^4}\,dx$

Optimal. Leaf size=100

$$-\frac{1}{3}ic\left(a+b\text{ArcTan}(cx^3)\right)^2-\frac{\left(a+b\text{ArcTan}(cx^3)\right)^2}{3x^3}+\frac{2}{3}bc\left(a+b\text{ArcTan}(cx^3)\right)\log\left(2-\frac{2}{1-icx^3}\right)-\frac{1}{3}ib^2c\text{Pol}$$

```
[Out] -1/3*I*c*(a+b*arctan(c*x^3))^2-1/3*(a+b*arctan(c*x^3))^2/x^3+2/3*b*c*(a+b*a
rctan(c*x^3))*ln(2-2/(1-I*c*x^3))-1/3*I*b^2*c*polylog(2,-1+2/(1-I*c*x^3))
```

Rubi [A]
time = 0.13, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$,
Rules used = {4948, 4946, 5044, 4988, 2497}

$$-\frac{1}{3}ic\left(a+b\text{ArcTan}(cx^3)\right)^2-\frac{\left(a+b\text{ArcTan}(cx^3)\right)^2}{3x^3}+\frac{2}{3}bc\log\left(2-\frac{2}{1-icx^3}\right)\left(a+b\text{ArcTan}(cx^3)\right)-\frac{1}{3}ib^2c\text{Li}_2\left(\frac{2}{1-icx^3}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])^2/x^4,x]
```

```
[Out] (-1/3*I)*c*(a + b*ArcTan[c*x^3])^2 - (a + b*ArcTan[c*x^3])^2/(3*x^3) + (2*b
*c*(a + b*ArcTan[c*x^3])*Log[2 - 2/(1 - I*c*x^3)])/3 - (I/3)*b^2*c*PolyLog[
2, -1 + 2/(1 - I*c*x^3)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
```

```
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx^3))^2}{x^4}\,dx = \int \left(\frac{(2a+ib\log(1-icx^3))^2}{4x^4} + \frac{b(-2ia+b\log(1-icx^3))\log(1+icx^3)}{2x^4} - \frac{b^2\log\ldots}{\ldots}\right.$$

$$= \frac{1}{4}\int \frac{(2a+ib\log(1-icx^3))^2}{x^4}\,dx + \frac{1}{2}b\int \frac{(-2ia+b\log(1-icx^3))\log(1+icx^3)}{x^4}\ldots$$

$$= \frac{1}{12}\text{Subst}\left(\int \frac{(2a+ib\log(1-icx))^2}{x^2}\,dx, x, x^3\right) + \frac{1}{6}b\,\text{Subst}\left(\int \frac{(-2ia+b\log(1-\ldots}{\ldots}\right.$$

$$= -\frac{(1-icx^3)(2a+ib\log(1-icx^3))^2}{12x^3} + \frac{b(2ia-b\log(1-icx^3))\log(1+icx^3)}{6x^3} + \ldots$$

$$= abc\log(x) - \frac{(1-icx^3)(2a+ib\log(1-icx^3))^2}{12x^3} + \frac{b(2ia-b\log(1-icx^3))\log(\ldots}{6x^3}$$

$$= abc\log(x) - \frac{(1-icx^3)(2a+ib\log(1-icx^3))^2}{12x^3} + \frac{b(2ia-b\log(1-icx^3))\log(\ldots}{6x^3}$$

$$= 2abc\log(x) - \frac{(1-icx^3)(2a+ib\log(1-icx^3))^2}{12x^3} + \frac{1}{6}ibc(2ia-b\log(1-icx^3))\ldots$$

$$= 2abc\log(x) - \frac{(1-icx^3)(2a+ib\log(1-icx^3))^2}{12x^3} + \frac{1}{6}ibc(2ia-b\log(1-icx^3))\ldots$$

$$= 2abc\log(x) - \frac{(1-icx^3)(2a+ib\log(1-icx^3))^2}{12x^3} + \frac{1}{6}ibc(2ia-b\log(1-icx^3))\ldots$$

Mathematica [A]

time = 0.10, size = 125, normalized size = 1.25

$$\frac{b^2(-1-icx^3)\operatorname{ArcTan}(cx^3)^2+2b\operatorname{ArcTan}(cx^3)\left(-a+bcx^3\log\left(1-e^{2i\operatorname{ArcTan}(cx^3)}\right)\right)-a(a-2bcx^3\log(cx^3)+bcx^3\log(1+c^2x^6))-ib^2cx^3\operatorname{PolyLog}\left(2,e^{2i\operatorname{ArcTan}(cx^3)}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^2/x^4,x]
```

```
[Out] (b^2*(-1 - I*c*x^3)*ArcTan[c*x^3]^2 + 2*b*ArcTan[c*x^3]*(-a + b*c*x^3*Log[1
 - E^((2*I)*ArcTan[c*x^3])]) - a*(a - 2*b*c*x^3*Log[c*x^3] + b*c*x^3*Log[1
+ c^2*x^6]) - I*b^2*c*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x^3])])/(3*x^3)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\arctan\left(c\,x^{3}\right)\right)^{2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^2/x^4,x)
```

```
[Out] int((a+b*arctan(c*x^3))^2/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*(c*(log(c^2*x^6 + 1) - log(x^6)) + 2*arctan(c*x^3)/x^3)*a*b + 1/48*(48
*x^3*integrate(-1/16*(4*c^2*x^6*log(c^2*x^6 + 1) - 8*c*x^3*arctan(c*x^3) -
12*(c^2*x^6 + 1)*arctan(c*x^3)^2 - (c^2*x^6 + 1)*log(c^2*x^6 + 1)^2)/(c^2*x
^10 + x^4), x) - 4*arctan(c*x^3)^2 + log(c^2*x^6 + 1)^2)*b^2/x^3 - 1/3*a^2/
x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atan}\,(cx^3))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**2/x**4,x)
```

```
[Out] Integral((a + b*atan(c*x**3))**2/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^2/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atan}(c\,x^3))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))^2/x^4,x)
```

```
[Out] int((a + b*atan(c*x^3))^2/x^4, x)
```

3.119 $\int \frac{\left(a+b\textbf{ArcTan}\left(cx^3\right)\right)^2}{x^7}\,dx$

Optimal. Leaf size=87

$$-\frac{bc(a+b\text{ArcTan}(cx^3))}{3x^3}-\frac{1}{6}c^2\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2-\frac{\left(a+b\text{ArcTan}(cx^3)\right)^2}{6x^6}+b^2c^2\log(x)-\frac{1}{6}b^2c^2\log\left(1+c^2x^6\right)$$

```
[Out] -1/3*b*c*(a+b*arctan(c*x^3))/x^3-1/6*c^2*(a+b*arctan(c*x^3))^2-1/6*(a+b*arc
tan(c*x^3))^2/x^6+b^2*c^2*ln(x)-1/6*b^2*c^2*ln(c^2*x^6+1)
```

Rubi [A]
time = 0.11, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4948, 4946, 5038, 272, 36, 29, 31, 5004}

$$-\frac{1}{6}c^2\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2-\frac{bc(a+b\text{ArcTan}(cx^3))}{3x^3}-\frac{\left(a+b\text{ArcTan}(cx^3)\right)^2}{6x^6}-\frac{1}{6}b^2c^2\log\left(c^2x^6+1\right)+b^2c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])^2/x^7,x]
```

```
[Out] -1/3*(b*c*(a + b*ArcTan[c*x^3]))/x^3 - (c^2*(a + b*ArcTan[c*x^3])^2)/6 - (a
 + b*ArcTan[c*x^3])^2/(6*x^6) + b^2*c^2*Log[x] - (b^2*c^2*Log[1 + c^2*x^6])
/6
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx^3))^2}{x^7}\,dx = \int \left(\frac{(2a+ib\log(1-icx^3))^2}{4x^7} + \frac{b(-2ia+b\log(1-icx^3))\log(1+icx^3)}{2x^7} - \frac{b^2 l\cdots}{}\right.$$

$$= \frac{1}{4}\int \frac{(2a+ib\log(1-icx^3))^2}{x^7}\,dx + \frac{1}{2}b\int \frac{(-2ia+b\log(1-icx^3))\log(1+icx^3\cdots}{x^7}$$

$$= \frac{1}{12}\text{Subst}\left(\int \frac{(2a+ib\log(1-icx))^2}{x^3}\,dx, x, x^3\right) + \frac{1}{6}b\text{Subst}\left(\int \frac{(-2ia+b\log(1\cdots}{}\right.$$

$$= -\frac{(2a+ib\log(1-icx^3))^2}{24x^6} + \frac{b(2ia-b\log(1-icx^3))\log(1+icx^3)}{12x^6} + \frac{b^2\log^2(\cdots}{24\cdots}$$

$$= -\frac{(2a+ib\log(1-icx^3))^2}{24x^6} + \frac{b(2ia-b\log(1-icx^3))\log(1+icx^3)}{12x^6} + \frac{b^2\log^2(\cdots}{24\cdots}$$

$$= -\frac{(2a+ib\log(1-icx^3))^2}{24x^6} + \frac{b(2ia-b\log(1-icx^3))\log(1+icx^3)}{12x^6} + \frac{b^2\log^2(\cdots}{24\cdots}$$

$$= -\frac{1}{2}iabc^2\log(x) + \frac{ibc(2ia-b\log(1-icx^3))}{12x^3} - \frac{bc(1-icx^3)(2a+ib\log(1-i\cdots}{12x^3}$$

$$= \frac{1}{4}b^2c^2\log(x) + \frac{ibc(2ia-b\log(1-icx^3))}{12x^3} - \frac{bc(1-icx^3)(2a+ib\log(1-icx^3\cdots}{12x^3}$$

$$= \frac{1}{2}b^2c^2\log(x) + \frac{ibc(2ia-b\log(1-icx^3))}{12x^3} - \frac{bc(1-icx^3)(2a+ib\log(1-icx^3\cdots}{12x^3}$$

Mathematica [A]
time = 0.04, size = 98, normalized size = 1.13

$$-\frac{a^2+2abcx^3+2b(a+bcx^3+ac^2x^6)\,\text{ArcTan}(cx^3)+b^2(1+c^2x^6)\,\text{ArcTan}(cx^3)^2-6b^2c^2x^6\log(x)+b^2c^2x^6\log(1+c^2x^6)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^2/x^7,x]
```

```
[Out] -1/6*(a^2 + 2*a*b*c*x^3 + 2*b*(a + b*c*x^3 + a*c^2*x^6)*ArcTan[c*x^3] + b^2
*(1 + c^2*x^6)*ArcTan[c*x^3]^2 - 6*b^2*c^2*x^6*Log[x] + b^2*c^2*x^6*Log[1 +
 c^2*x^6])/x^6
```

Maple [A]
time = 0.13, size = 118, normalized size = 1.36

method	result
default	$-\frac{a^2}{6x^6} - \frac{b^2\arctan(cx^3)^2}{6x^6} - \frac{b^2c\arctan(cx^3)}{3x^3} - \frac{b^2\arctan(cx^3)^2c^2}{6} + b^2c^2\ln(x) - \frac{b^2c^2\ln(c^2x^6+1)}{6} - \frac{ab\arctan(cx^3)}{3x^6}$

risch	$\frac{b^2(c^2x^6+1)\ln(icx^3+1)^2}{24x^6}+\frac{ib(ibc^2x^6\ln(-icx^3+1)+2bcx^3+2a+ib\ln(-icx^3+1))\ln(icx^3+1)}{12x^6}-\frac{4i\ln((-7ibc+ac)x^3+7b+ia)abc$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^2/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*a^2/x^6-1/6*b^2/x^6*arctan(c*x^3)^2-1/3*b^2*c*arctan(c*x^3)/x^3-1/6*b^
2*arctan(c*x^3)^2*c^2+b^2*c^2*ln(x)-1/6*b^2*c^2*ln(c^2*x^6+1)-1/3*a*b/x^6*a
rctan(c*x^3)-1/3*a*b*c/x^3-1/3*a*b*arctan(c*x^3)*c^2
```

Maxima [A]
time = 0.54, size = 110, normalized size = 1.26

$$-\frac{1}{3}\left(\left(c\arctan\left(cx^3\right)+\frac{1}{x^3}\right)c+\frac{\arctan\left(cx^3\right)}{x^6}\right)ab+\frac{1}{6}\left(\left(\arctan\left(cx^3\right)^2-\log\left(c^2x^6+1\right)+6\log\left(x\right)\right)c^2-2\left(c\arctan\left(cx^3\right)+\frac{1}{x^3}\right)c\arctan\left(cx^3\right)\right)b^2-\frac{b^2\arctan\left(cx^3\right)^2}{6x^6}-\frac{a^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^7,x, algorithm="maxima")
```

```
[Out] -1/3*((c*arctan(c*x^3) + 1/x^3)*c + arctan(c*x^3)/x^6)*a*b + 1/6*((arctan(c
*x^3)^2 - log(c^2*x^6 + 1) + 6*log(x))*c^2 - 2*(c*arctan(c*x^3) + 1/x^3)*c*
arctan(c*x^3))*b^2 - 1/6*b^2*arctan(c*x^3)^2/x^6 - 1/6*a^2/x^6
```

Fricas [A]
time = 1.24, size = 102, normalized size = 1.17

$$-\frac{b^2c^2x^6\log\left(c^2x^6+1\right)-6\,b^2c^2x^6\log\left(x\right)+2\,abcx^3+\left(b^2c^2x^6+b^2\right)\arctan\left(cx^3\right)^2+a^2+2\left(abc^2x^6+b^2cx^3+ab\right)\arctan\left(cx^3\right)}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(b^2*c^2*x^6*log(c^2*x^6 + 1) - 6*b^2*c^2*x^6*log(x) + 2*a*b*c*x^3 + (
b^2*c^2*x^6 + b^2)*arctan(c*x^3)^2 + a^2 + 2*(a*b*c^2*x^6 + b^2*c*x^3 + a*b
)*arctan(c*x^3))/x^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(80) = 160.
time = 72.19, size = 207, normalized size = 2.38

$$\begin{cases} -\frac{a^2}{6x^6}-\frac{abc^2\operatorname{atan}\left(cx^3\right)}{3}-\frac{abc}{3x^3}-\frac{ab\operatorname{atan}\left(cx^3\right)}{3x^6}+\frac{b^2c^3\sqrt{-\frac{1}{c^2}}\operatorname{atan}\left(cx^3\right)}{3}+b^2c^2\log\left(x\right)-\frac{b^2c^2\log\left(x-\sqrt[6]{-\frac{1}{c^2}}\right)}{3}-\frac{b^2c^2\log\left(4x^2+4x\sqrt[6]{-\frac{1}{c^2}}+4\sqrt[3]{-\frac{1}{c^2}}\right)}{3}-\frac{b^2c^2\operatorname{atan}^2\left(cx^3\right)}{6}-\frac{b^2c\operatorname{atan}\left(cx^3\right)}{3x^3}-\frac{b^2\operatorname{atan}^2\left(cx^3\right)}{6x^6} & \text{for}\: c\neq 0 \\ -\frac{a^2}{6x^6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**2/x**7,x)
```

```
[Out] Piecewise((-a**2/(6*x**6) - a*b*c**2*atan(c*x**3)/3 - a*b*c/(3*x**3) - a*b*
atan(c*x**3)/(3*x**6) + b**2*c**3*sqrt(-1/c**2)*atan(c*x**3)/3 + b**2*c**2*
```

```
log(x) - b**2*c**2*log(x - (-1/c**2)**(1/6))/3 - b**2*c**2*log(4*x**2 + 4*x
*(-1/c**2)**(1/6) + 4*(-1/c**2)**(1/3))/3 - b**2*c**2*atan(c*x**3)**2/6 - b
**2*c*atan(c*x**3)/(3*x**3) - b**2*atan(c*x**3)**2/(6*x**6), Ne(c, 0)), (-a
**2/(6*x**6), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^2/x^7, x)
```

Mupad [B]
time = 0.69, size = 152, normalized size = 1.75

$$b^2\,c^2\,\ln\left(x\right)-\frac{b^2\,c^2\,\mathrm{atan}\left(c\,x^3\right)^2}{6}-\frac{b^2\,\mathrm{atan}\left(c\,x^3\right)^2}{6\,x^6}-\frac{b^2\,c^2\,\ln\left(c^2\,x^6+1\right)}{6}-\frac{a^2}{6\,x^6}-\frac{b^2\,c\,\mathrm{atan}\left(c\,x^3\right)}{3\,x^3}-\frac{a\,b\,c}{3\,x^3}-\frac{a\,b\,c^2\,\mathrm{atan}\left(\frac{a^2\,c\,x^3}{a^2+49\,b^2}+\frac{49\,b^2\,c\,x^3}{a^2+49\,b^2}\right)}{3}-\frac{a\,b\,\mathrm{atan}\left(c\,x^3\right)}{3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))^2/x^7,x)
```

```
[Out] b^2*c^2*log(x) - (b^2*c^2*atan(c*x^3)^2)/6 - (b^2*atan(c*x^3)^2)/(6*x^6) -
(b^2*c^2*log(c^2*x^6 + 1))/6 - a^2/(6*x^6) - (b^2*c*atan(c*x^3))/(3*x^3) -
(a*b*c)/(3*x^3) - (a*b*c^2*atan((a^2*c*x^3)/(a^2 + 49*b^2) + (49*b^2*c*x^3)
/(a^2 + 49*b^2)))/3 - (a*b*atan(c*x^3))/(3*x^6)
```

3.120 $\int \frac{\left(a+b\mathbf{ArcTan}(cx^3)\right)^2}{x^{10}}\,dx$

Optimal. Leaf size=154

$$-\frac{b^2c^2}{9x^3}-\frac{1}{9}b^2c^3\text{ArcTan}(cx^3)-\frac{bc(a+b\text{ArcTan}(cx^3))}{9x^6}+\frac{1}{9}ic^3\left(a+b\text{ArcTan}(cx^3)\right)^2-\frac{\left(a+b\text{ArcTan}(cx^3)\right)^2}{9x^9}-\frac{2}{9}bc^3$$

```
[Out] -1/9*b^2*c^2/x^3-1/9*b^2*c^3*arctan(c*x^3)-1/9*b*c*(a+b*arctan(c*x^3))/x^6+
1/9*I*c^3*(a+b*arctan(c*x^3))^2-1/9*(a+b*arctan(c*x^3))^2/x^9-2/9*b*c^3*(a+
b*arctan(c*x^3))*ln(2-2/(1-I*c*x^3))+1/9*I*b^2*c^3*polylog(2,-1+2/(1-I*c*x^
3))
```

Rubi [A]
time = 0.19, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4948, 4946, 5038, 331, 209, 5044, 4988, 2497}

$$\frac{1}{9}ic^3\left(a+b\text{ArcTan}(cx^3)\right)^2-\frac{2}{9}bc^3\log\left(2-\frac{2}{1-icx^3}\right)\left(a+b\text{ArcTan}(cx^3)\right)-\frac{\left(a+b\text{ArcTan}(cx^3)\right)^2}{9x^9}-\frac{bc(a+b\text{ArcTan}(cx^3))}{9x^6}-\frac{1}{9}b^2c^3\text{ArcTan}(cx^3)+\frac{1}{9}ib^2c^3\text{Li}_2\left(\frac{2}{1-icx^3}-1\right)-\frac{b^2c^2}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])^2/x^10,x]
```

```
[Out] -1/9*(b^2*c^2)/x^3 - (b^2*c^3*ArcTan[c*x^3])/9 - (b*c*(a + b*ArcTan[c*x^3])
)/(9*x^6) + (I/9)*c^3*(a + b*ArcTan[c*x^3])^2 - (a + b*ArcTan[c*x^3])^2/(9*
x^9) - (2*b*c^3*(a + b*ArcTan[c*x^3])*Log[2 - 2/(1 - I*c*x^3)])/9 + (I/9)*b
^2*c^3*PolyLog[2, -1 + 2/(1 - I*c*x^3)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx^3))^2}{x^{10}}\,dx = \int \left(\frac{(2a+ib\log(1-icx^3))^2}{4x^{10}} + \frac{b(-2ia+b\log(1-icx^3))\log(1+icx^3)}{2x^{10}} - \frac{b^2\log\right.$$

$$= \frac{1}{4}\int \frac{(2a+ib\log(1-icx^3))^2}{x^{10}}\,dx + \frac{1}{2}b\int \frac{(-2ia+b\log(1-icx^3))\log(1+icx^3)}{x^{10}}$$

$$= \frac{1}{12}\text{Subst}\left(\int \frac{(2a+ib\log(1-icx))^2}{x^4}\,dx, x, x^3\right) + \frac{1}{6}b\text{Subst}\left(\int \frac{(-2ia+b\log(1-\right.\right.$$

$$= -\frac{(2a+ib\log(1-icx^3))^2}{36x^9} + \frac{b(2ia-b\log(1-icx^3))\log(1+icx^3)}{18x^9} + \frac{b^2\log^2(1}{36x}$$

$$= -\frac{(2a+ib\log(1-icx^3))^2}{36x^9} + \frac{b(2ia-b\log(1-icx^3))\log(1+icx^3)}{18x^9} + \frac{b^2\log^2(1}{36x}$$

$$= -\frac{(2a+ib\log(1-icx^3))^2}{36x^9} + \frac{b(2ia-b\log(1-icx^3))\log(1+icx^3)}{18x^9} + \frac{b^2\log^2(1}{36x}$$

$$= -\frac{1}{3}abc^3\log(x) + \frac{ibc(2ia-b\log(1-icx^3))}{36x^6} + \frac{bc^2(2ia-b\log(1-icx^3))}{18x^3} - \frac{bc(2}{}$$

$$= -\frac{1}{3}abc^3\log(x) + \frac{ibc(2ia-b\log(1-icx^3))}{36x^6} + \frac{bc^2(2ia-b\log(1-icx^3))}{18x^3} - \frac{bc(2}{}$$

$$= -\frac{b^2c^2}{18x^3} - \frac{2}{3}abc^3\log(x) + \frac{1}{6}ib^2c^3\log(x) + \frac{ibc(2ia-b\log(1-icx^3))}{36x^6} + \frac{bc^2(2ia-}{}$$

$$= -\frac{b^2c^2}{18x^3} - \frac{2}{3}abc^3\log(x) + \frac{1}{6}ib^2c^3\log(x) + \frac{ibc(2ia-b\log(1-icx^3))}{36x^6} + \frac{bc^2(2ia-}{}$$

Mathematica [A]
time = 0.26, size = 167, normalized size = 1.08

$$-\frac{a^2+abcx^3+b^2c^2x^6+b^2(1-ic^3x^9)\text{ArcTan}(cx^3)^2+b\text{ArcTan}(cx^3)\left(2a+bcx^3+bc^3x^9+2bc^3x^9\log\left(1-e^{2i\text{ArcTan}(cx^3)}\right)\right)+2abc^3x^9\log(cx^3)-abc^3x^9\log(1+c^2x^6)-ib^2c^3x^9\text{PolyLog}\left(2,e^{2i\text{ArcTan}(cx^3)}\right)}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^2/x^10,x]
```

```
[Out] -1/9*(a^2 + a*b*c*x^3 + b^2*c^2*x^6 + b^2*(1 - I*c^3*x^9)*ArcTan[c*x^3]^2 +
 b*ArcTan[c*x^3]*(2*a + b*c*x^3 + b*c^3*x^9 + 2*b*c^3*x^9*Log[1 - E^((2*I)*
ArcTan[c*x^3])]) + 2*a*b*c^3*x^9*Log[c*x^3] - a*b*c^3*x^9*Log[1 + c^2*x^6]
- I*b^2*c^3*x^9*PolyLog[2, E^((2*I)*ArcTan[c*x^3])])/x^9
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\arctan(c\,x^3))^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^2/x^10,x)
```

```
[Out] int((a+b*arctan(c*x^3))^2/x^10,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^10,x, algorithm="maxima")
```

```
[Out] 1/9*((c^2*log(c^2*x^6 + 1) - c^2*log(x^6) - 1/x^6)*c - 2*arctan(c*x^3)/x^9)
*a*b + 1/144*(144*x^9*integrate(-1/48*(4*c^2*x^6*log(c^2*x^6 + 1) - 8*c*x^3
*arctan(c*x^3) - 36*(c^2*x^6 + 1)*arctan(c*x^3)^2 - 3*(c^2*x^6 + 1)*log(c^2
*x^6 + 1)^2)/(c^2*x^16 + x^10), x) - 4*arctan(c*x^3)^2 + log(c^2*x^6 + 1)^2
)*b^2/x^9 - 1/9*a^2/x^9
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^10,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)/x^10, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}{\left(c x^{3} \right)}\right)^{2}}{x^{10}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**2/x**10,x)
```

```
[Out] Integral((a + b*atan(c*x**3))**2/x**10, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^2/x^10,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^2/x^10, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^2}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))^2/x^10,x)
```

```
[Out] int((a + b*atan(c*x^3))^2/x^10, x)
```

3.121 $\int x^8(a+b\text{ArcTan}(cx^3))^3\,dx$

Optimal. Leaf size=240

$$\frac{ab^2x^3}{3c^2}+\frac{b^3x^3\text{ArcTan}(cx^3)}{3c^2}-\frac{b(a+b\text{ArcTan}(cx^3))^2}{6c^3}-\frac{bx^6(a+b\text{ArcTan}(cx^3))^2}{6c}-\frac{i(a+b\text{ArcTan}(cx^3))^3}{9c^3}+\frac{1}{9}x^9($$

```
[Out] 1/3*a*b^2*x^3/c^2+1/3*b^3*x^3*arctan(c*x^3)/c^2-1/6*b*(a+b*arctan(c*x^3))^2
/c^3-1/6*b*x^6*(a+b*arctan(c*x^3))^2/c-1/9*I*(a+b*arctan(c*x^3))^3/c^3+1/9*
x^9*(a+b*arctan(c*x^3))^3-1/3*b*(a+b*arctan(c*x^3))^2*ln(2/(1+I*c*x^3))/c^3
-1/6*b^3*ln(c^2*x^6+1)/c^3-1/3*I*b^2*(a+b*arctan(c*x^3))*polylog(2,1-2/(1+I
*c*x^3))/c^3-1/6*b^3*polylog(3,1-2/(1+I*c*x^3))/c^3
```

Rubi [A]
time = 0.33, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {4948, 4946, 5036, 4930, 266, 5004, 5040, 4964, 5114, 6745}

$$-\frac{ib^2\text{Li}_2\left(1-\frac{2}{icx^3+1}\right)(a+b\text{ArcTan}(cx^3))}{3c^3}-\frac{i(a+b\text{ArcTan}(cx^3))^3}{9c^3}-\frac{b(a+b\text{ArcTan}(cx^3))^2}{6c^3}-\frac{b\log\left(\frac{2}{1+icx^3}\right)(a+b\text{ArcTan}(cx^3))^2}{3c^3}+\frac{1}{9}x^9(a+b\text{ArcTan}(cx^3))^3-\frac{bx^6(a+b\text{ArcTan}(cx^3))^2}{6c}+\frac{ab^2x^3}{3c^2}+\frac{b^3x^3\text{ArcTan}(cx^3)}{3c^2}-\frac{b^3\text{Li}_3\left(1-\frac{2}{icx^3+1}\right)}{6c^3}-\frac{b^3\log(c^2x^6+1)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] (a*b^2*x^3)/(3*c^2) + (b^3*x^3*ArcTan[c*x^3])/(3*c^2) - (b*(a + b*ArcTan[c*
x^3])^2)/(6*c^3) - (b*x^6*(a + b*ArcTan[c*x^3])^2)/(6*c) - ((I/9)*(a + b*Ar
cTan[c*x^3])^3)/c^3 + (x^9*(a + b*ArcTan[c*x^3])^3)/9 - (b*(a + b*ArcTan[c*
x^3])^2*Log[2/(1 + I*c*x^3)])/(3*c^3) - (b^3*Log[1 + c^2*x^6])/(6*c^3) - ((
I/3)*b^2*(a + b*ArcTan[c*x^3])*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c^3 - (b^3*
PolyLog[3, 1 - 2/(1 + I*c*x^3)])/(6*c^3)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int x^8 \left(a+b\tan^{-1}\left(cx^3\right)\right)^3\,dx = \int \left(\frac{1}{8}x^8\left(2a+ib\log\left(1-icx^3\right)\right)^3+\frac{3}{8}ibx^8\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(1+\right.\right.$$

$$= \frac{1}{8}\int x^8\left(2a+ib\log\left(1-icx^3\right)\right)^3\,dx+\frac{1}{8}(3ib)\int x^8\left(-2ia+b\log\left(1-icx^3\right)\right)^2$$

$$= \frac{1}{24}\text{Subst}\left(\int x^2(2a+ib\log(1-icx))^3\,dx,x,x^3\right)+\frac{1}{8}(ib)\text{Subst}\left(\int x^2(-2ia+\right.$$

$$= \frac{1}{24}ibx^9\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)+\frac{1}{24}ib^2x^9\left(2ia-b\log\left(1-icx\right.\right.$$

$$= \frac{1}{24}ibx^9\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)+\frac{1}{24}ib^2x^9\left(2ia-b\log\left(1-icx\right.\right.$$

$$= \frac{1}{24}ibx^9\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)+\frac{1}{24}ib^2x^9\left(2ia-b\log\left(1-icx\right.\right.$$

$$= -\frac{1}{72}ibx^9\left(2ia-b\log\left(1-icx^3\right)\right)^2-\frac{i\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24c^3}+\frac{i(\cdots}{}$$

$$= -\frac{1}{72}ibx^9\left(2ia-b\log\left(1-icx^3\right)\right)^2-\frac{b\left(1-icx^3\right)\left(2ia-b\log\left(1-icx^3\right)\right)^2}{24c^3}-\frac{b(\cdots}{}$$

$$= \frac{ab^2x^3}{3c^2}+\frac{ib^3x^3}{6c^2}-\frac{b^3\left(1-icx^3\right)^2}{32c^3}-\frac{b^3\left(1+icx^3\right)^2}{32c^3}+\frac{b^3\left(1+icx^3\right)^3}{324c^3}+\frac{ib^3\left(i+cx^3\cdots\right.}{324c^3}$$

$$= \frac{ab^2x^3}{3c^2}-\frac{b^3\left(1-icx^3\right)^2}{32c^3}-\frac{b^3\left(1+icx^3\right)^2}{32c^3}+\frac{b^3\left(1+icx^3\right)^3}{324c^3}+\frac{ib^3\left(i+cx^3\right)^3}{324c^3}-\frac{b^3(\cdots}{}$$

$$= \frac{ab^2x^3}{2c^2}+\frac{ib^3x^3}{18c^2}-\frac{b^3\left(1-icx^3\right)^2}{24c^3}-\frac{b^3\left(1+icx^3\right)^2}{24c^3}+\frac{b^3\left(1+icx^3\right)^3}{324c^3}+\frac{ib^3\left(i+cx^3\cdots\right.}{324c^3}$$

$$= \frac{ab^2x^3}{2c^2}+\frac{7ib^3x^3}{216c^2}-\frac{5b^3x^6}{432c}+\frac{1}{324}ib^3x^9-\frac{b^3\left(1-icx^3\right)^2}{48c^3}-\frac{b^3\left(1-icx^3\right)^3}{324c^3}-\frac{b^3(1\cdots}{}$$

$$= \frac{ab^2x^3}{2c^2}+\frac{7ib^3x^3}{216c^2}-\frac{5b^3x^6}{432c}+\frac{1}{324}ib^3x^9-\frac{b^3\left(1-icx^3\right)^2}{48c^3}-\frac{b^3\left(1-icx^3\right)^3}{324c^3}-\frac{b^3(1\cdots}{}$$

<u>Mathematica</u> [A]
time = 0.35, size = 346, normalized size = 1.44

$$\frac{6ab^2cx^3-3a^2bc^2x^6+2a^3c^3x^9-6ab^2\text{ArcTan}(cx^3)+6b^3cx^3\text{ArcTan}(cx^3)-6ab^2c^2x^6\text{ArcTan}(cx^3)+6a^2bc^3x^9\text{ArcTan}(cx^3)+6iab^2\text{ArcTan}(cx^3)^2-3b^3\text{ArcTan}(cx^3)^2-3b^3c^2x^6\text{ArcTan}(cx^3)^2+6ab^2c^3x^9\text{ArcTan}(cx^3)^2+2ib^3\text{ArcTan}(cx^3)^3+2b^3c^3x^9\text{ArcTan}(cx^3)^3-12ab^2\text{ArcTan}(cx^3)\log\left(1+e^{2i\text{ArcTan}(cx^3)}\right)-6b^3\text{ArcTan}(cx^3)^2\log\left(1+e^{2i\text{ArcTan}(cx^3)}\right)+3a^2b\log\left(1+c^2x^6\right)-3b^3\log\left(1+c^2x^6\right)+6ib^2(a+b\text{ArcTan}(cx^3))\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx^3)}\right)-3b^3\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(cx^3)}\right)}{18c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] (6*a*b^2*c*x^3 - 3*a^2*b*c^2*x^6 + 2*a^3*c^3*x^9 - 6*a*b^2*ArcTan[c*x^3] +
6*b^3*c*x^3*ArcTan[c*x^3] - 6*a*b^2*c^2*x^6*ArcTan[c*x^3] + 6*a^2*b*c^3*x^9
*ArcTan[c*x^3] + (6*I)*a*b^2*ArcTan[c*x^3]^2 - 3*b^3*ArcTan[c*x^3]^2 - 3*b^
3*c^2*x^6*ArcTan[c*x^3]^2 + 6*a*b^2*c^3*x^9*ArcTan[c*x^3]^2 + (2*I)*b^3*Arc
Tan[c*x^3]^3 + 2*b^3*c^3*x^9*ArcTan[c*x^3]^3 - 12*a*b^2*ArcTan[c*x^3]*Log[1
 + E^((2*I)*ArcTan[c*x^3])] - 6*b^3*ArcTan[c*x^3]^2*Log[1 + E^((2*I)*ArcTan
[c*x^3])] + 3*a^2*b*Log[1 + c^2*x^6] - 3*b^3*Log[1 + c^2*x^6] + (6*I)*b^2*(
a + b*ArcTan[c*x^3])*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])] - 3*b^3*PolyLog[3
, -E^((2*I)*ArcTan[c*x^3])])/(18*c^3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^{8}\left(a+b\arctan\left(c\,x^{3}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a+b*arctan(c*x^3))^3,x)
```

```
[Out] int(x^8*(a+b*arctan(c*x^3))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] 1/72*b^3*x^9*arctan(c*x^3)^3 - 1/96*b^3*x^9*arctan(c*x^3)*log(c^2*x^6 + 1)^
2 + 1/9*a^3*x^9 + 1/6*(2*x^9*arctan(c*x^3) - (x^6/c^2 - log(c^2*x^6 + 1)/c^
4)*c)*a^2*b + integrate(1/32*(4*b^3*c^2*x^14*arctan(c*x^3)*log(c^2*x^6 + 1)
 + 28*(b^3*c^2*x^14 + b^3*x^8)*arctan(c*x^3)^3 + 4*(24*a*b^2*c^2*x^14 - b^3
*c*x^11 + 24*a*b^2*x^8)*arctan(c*x^3)^2 + (b^3*c*x^11 + 3*(b^3*c^2*x^14 + b
^3*x^8)*arctan(c*x^3))*log(c^2*x^6 + 1)^2)/(c^2*x^6 + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^8*arctan(c*x^3)^3 + 3*a*b^2*x^8*arctan(c*x^3)^2 + 3*a^2*b*x^
8*arctan(c*x^3) + a^3*x^8, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^8\left(a+b\,\mathrm{atan}\left(cx^3\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(a+b*atan(c*x**3))**3,x)
```

```
[Out] Integral(x**8*(a + b*atan(c*x**3))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^3*x^8, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^8\,\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*atan(c*x^3))^3,x)
```

```
[Out] int(x^8*(a + b*atan(c*x^3))^3, x)
```

3.122 $\int x^5(a+b\mathbf{ArcTan}(cx^3))^3\,dx$

Optimal. Leaf size=147

$$-\frac{ib(a+b\text{ArcTan}(cx^3))^2}{2c^2}-\frac{bx^3(a+b\text{ArcTan}(cx^3))^2}{2c}+\frac{(a+b\text{ArcTan}(cx^3))^3}{6c^2}+\frac{1}{6}x^6(a+b\text{ArcTan}(cx^3))^3-\frac{b^2(a}{}$$

```
[Out] -1/2*I*b*(a+b*arctan(c*x^3))^2/c^2-1/2*b*x^3*(a+b*arctan(c*x^3))^2/c+1/6*(a
+b*arctan(c*x^3))^3/c^2+1/6*x^6*(a+b*arctan(c*x^3))^3-b^2*(a+b*arctan(c*x^3
))*ln(2/(1+I*c*x^3))/c^2-1/2*I*b^3*polylog(2,1-2/(1+I*c*x^3))/c^2
```

Rubi [A]
time = 0.20, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {4948, 4946, 5036, 4930, 5040, 4964, 2449, 2352, 5004}

$$-\frac{b^2\log\left(\frac{2}{1+icx^3}\right)(a+b\text{ArcTan}(cx^3))}{c^2}+\frac{(a+b\text{ArcTan}(cx^3))^3}{6c^2}-\frac{ib(a+b\text{ArcTan}(cx^3))^2}{2c^2}-\frac{bx^3(a+b\text{ArcTan}(cx^3))^2}{2c}+\frac{1}{6}x^6(a+b\text{ArcTan}(cx^3))^3-\frac{ib^3\text{Li}_2\left(1-\frac{2}{icx^3+1}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] ((-1/2*I)*b*(a + b*ArcTan[c*x^3])^2)/c^2 - (b*x^3*(a + b*ArcTan[c*x^3])^2)/
(2*c) + (a + b*ArcTan[c*x^3])^3/(6*c^2) + (x^6*(a + b*ArcTan[c*x^3])^3)/6 -
 (b^2*(a + b*ArcTan[c*x^3])*Log[2/(1 + I*c*x^3)])/c^2 - ((I/2)*b^3*PolyLog[
2, 1 - 2/(1 + I*c*x^3)])/c^2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int x^5\left(a+b\tan^{-1}\left(cx^3\right)\right)^3\,dx = \int \left(\frac{1}{8}x^5\left(2a+ib\log\left(1-icx^3\right)\right)^3+\frac{3}{8}ibx^5\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(1\right.\right.$$

$$= \frac{1}{8}\int x^5\left(2a+ib\log\left(1-icx^3\right)\right)^3\,dx+\frac{1}{8}(3ib)\int x^5\left(-2ia+b\log\left(1-icx^3\right)\right)$$

$$= \frac{1}{24}\text{Subst}\left(\int x(2a+ib\log(1-icx))^3\,dx,x,x^3\right)+\frac{1}{8}(ib)\text{Subst}\left(\int x(-2ia+\right.$$

$$= \frac{1}{16}ibx^6\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)+\frac{1}{16}ib^2x^6\left(2ia-b\log\left(1-ic\right.\right.$$

$$= \frac{1}{16}ibx^6\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)+\frac{1}{16}ib^2x^6\left(2ia-b\log\left(1-ic\right.\right.$$

$$= \frac{1}{16}ibx^6\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)+\frac{1}{16}ib^2x^6\left(2ia-b\log\left(1-ic\right.\right.$$

$$= \frac{\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24c^2}-\frac{\left(1-icx^3\right)^2\left(2a+ib\log\left(1-icx^3\right)\right)^3}{48c^2}+$$

$$= \frac{ib(1-icx^3)\left(2ia-b\log\left(1-icx^3\right)\right)^2}{16c^2}-\frac{ib(1-icx^3)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{8c^2}$$

$$= \frac{3iab^2x^3}{4c}+\frac{3b^3x^3}{8c}-\frac{ib^3(1-icx^3)^2}{64c^2}+\frac{ib^3(1+icx^3)^2}{64c^2}+\frac{ib(1-icx^3)\left(2ia-b\right.}{16c^2}$$

$$= \frac{3iab^2x^3}{4c}+\frac{3b^3x^3}{4c}-\frac{ib^3(1-icx^3)^2}{64c^2}+\frac{ib^3(1+icx^3)^2}{64c^2}-\frac{3ib^3(1-icx^3)\log\left(1\right.}{8c^2}$$

$$= \frac{iab^2x^3}{2c}+\frac{5b^3x^3}{8c}-\frac{3ib^3(1-icx^3)\log\left(1-icx^3\right)}{8c^2}+\frac{ib^2\left(1-icx^3\right)^2\left(2ia-b\log\right.}{32c^2}$$

$$= \frac{iab^2x^3}{2c}+\frac{b^3x^3}{2c}-\frac{ib^3(1-icx^3)\log\left(1-icx^3\right)}{4c^2}+\frac{ib^2\left(1-icx^3\right)^2\left(2ia-b\log\left(1\right.\right.}{32c^2}$$

Mathematica [A]
time = 0.09, size = 170, normalized size = 1.16

$$\frac{3b^2(a+ac^2x^6+b(i-cx^3))\,\text{ArcTan}(cx^3)^2+b^3(1+c^2x^6)\,\text{ArcTan}(cx^3)^3+3b\text{ArcTan}(cx^3)\left(a(a-2bcx^3+ac^2x^6)-2b^2\log\left(1+e^{2i\text{ArcTan}(cx^3)}\right)\right)+a(acx^3(-3b+acx^3)+3b^2\log(1+c^2x^6))+3ib^3\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx^3)}\right)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] (3*b^2*(a + a*c^2*x^6 + b*(I - c*x^3))*ArcTan[c*x^3]^2 + b^3*(1 + c^2*x^6)*
ArcTan[c*x^3]^3 + 3*b*ArcTan[c*x^3]*(a*(a - 2*b*c*x^3 + a*c^2*x^6) - 2*b^2*
Log[1 + E^((2*I)*ArcTan[c*x^3])]) + a*(a*c*x^3*(-3*b + a*c*x^3) + 3*b^2*Log
[1 + c^2*x^6]) + (3*I)*b^3*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])])/(6*c^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.65, size = 935, normalized size = 6.36

method	result	size
risch	Expression too large to display	935

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arctan(c*x^3))^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/16*I*b^3*(c^2*x^6+1)/c^2*ln(1-I*c*x^3)^2+1/16*b^2*(2*a*c*x^3-b)^2/a/c^2*
ln(1-I*c*x^3)-1/16*b*(4*I*a^3*c^2*x^6-8*I*a^2*b*c*x^3+4*I*ln(1-I*c*x^3)*a*b
^2-4*ln(1-I*c*x^3)*a^2*b+ln(1-I*c*x^3)*b^3+4*I*a*b^2)/a/c^2)*ln(1+I*c*x^3)-
1/48*I*b^3/c^2*ln(1-I*c*x^3)^3+1/6*x^6*a^3-1/16*b^2*(I*b*c^2*x^6*ln(1-I*c*x
^3)+2*a*c^2*x^6-2*b*c*x^3+I*b*ln(1-I*c*x^3)+2*I*b+2*a)/c^2*ln(1+I*c*x^3)^2-
1/2*b*a^2/c*x^3+1/48*I*b^3*(c^2*x^6+1)/c^2*ln(1+I*c*x^3)^3-1/4*b^3/c^2*arct
an(c*x^3)+1/8*I*b^3/c^2*ln(1-I*c*x^3)^2+1/2/c^2*b*a^2*arctan(c*x^3)+1/4*I*b
*a^2*x^6*ln(1-I*c*x^3)-1/48*I*b^3*x^6*ln(1-I*c*x^3)^3+1/8*I*b^3/c^2*ln(c^2*
x^6+1)+1/8*b^3/c*x^3*ln(1-I*c*x^3)^2-1/2*I/c*b^2*a*x^3*ln(1-I*c*x^3)+3/4*I/
c*b^2*Sum(2/3*(ln(x-_alpha)*ln(1-I*c*x^3)+3*c*(-1/3*ln(x-_alpha)*(ln((RootO
f(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=1)-x+_alpha)/RootOf(_Z^
2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=1))+ln((RootOf(_Z^2+_Z*RootO
f(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=2)-x+_alpha)/RootOf(_Z^2+_Z*RootOf(c*_
Z^3-I)+RootOf(c*_Z^3-I)^2,index=2))+ln(1/2*(2*(I/c)^(1/3)+x-_alpha)/(I/c)^(
1/3)))/c-1/3*(dilog((RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,ind
ex=1)-x+_alpha)/RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=1)
)+dilog((RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=2)-x+_alp
ha)/RootOf(_Z^2+_Z*RootOf(c*_Z^3-I)+RootOf(c*_Z^3-I)^2,index=2))+dilog(1/2*
(2*(I/c)^(1/3)+x-_alpha)/(I/c)^(1/3)))/c))*b/c,_alpha=RootOf(c*_Z^3-RootOf(
_Z^2+1,index=1)))-1/8*b^2*a*x^6*ln(1-I*c*x^3)^2+1/2/c^2*b^2*a*ln(c^2*x^6+1)
-1/8/c^2*b^2*a*ln(1-I*c*x^3)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*b^2*x^6*arctan(c*x^3)^2 + 1/6*a^3*x^6 + 1/2*(x^6*arctan(c*x^3) - c*(x
^3/c^2 - arctan(c*x^3)/c^3))*a^2*b - 1/2*(2*c*(x^3/c^2 - arctan(c*x^3)/c^3)
```

```
*arctan(c*x^3) + (arctan(c*x^3)^2 - log(6*c^5*x^6 + 6*c^3))/c^2)*a*b^2 + 1/
192*(4*x^6*arctan(c*x^3)^3 - 3*x^6*arctan(c*x^3)*log(c^2*x^6 + 1)^2 + 192*i
ntegrate(1/64*(12*c^2*x^11*arctan(c*x^3)*log(c^2*x^6 + 1) - 12*c*x^8*arctan
(c*x^3)^2 + 56*(c^2*x^11 + x^5)*arctan(c*x^3)^3 + 3*(c*x^8 + 2*(c^2*x^11 +
x^5)*arctan(c*x^3))*log(c^2*x^6 + 1)^2)/(c^2*x^6 + 1), x))*b^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
[Out] integral(b^3*x^5*arctan(c*x^3)^3 + 3*a*b^2*x^5*arctan(c*x^3)^2 + 3*a^2*b*x^
5*arctan(c*x^3) + a^3*x^5, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \left(a + b \operatorname{atan}\left(cx^3\right)\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*atan(c*x**3))**3,x)
[Out] Integral(x**5*(a + b*atan(c*x**3))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)^3*x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \left(a + b \operatorname{atan}\left(c\,x^3\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*atan(c*x^3))^3,x)
[Out] int(x^5*(a + b*atan(c*x^3))^3, x)
```

3.123 $\int x^2(a + b\textbf{ArcTan}(cx^3))^3 \, dx$

Optimal. Leaf size=139

$$\frac{i(a + b\text{ArcTan}(cx^3))^3}{3c} + \frac{1}{3}x^3(a + b\text{ArcTan}(cx^3))^3 + \frac{b(a + b\text{ArcTan}(cx^3))^2 \log\left(\frac{2}{1+icx^3}\right)}{c} + \frac{ib^2(a + b\text{ArcTan}(cx^3)}{}$$

```
[Out] 1/3*I*(a+b*arctan(c*x^3))^3/c+1/3*x^3*(a+b*arctan(c*x^3))^3+b*(a+b*arctan(c
*x^3))^2*ln(2/(1+I*c*x^3))/c+I*b^2*(a+b*arctan(c*x^3))*polylog(2,1-2/(1+I*c
*x^3))/c+1/2*b^3*polylog(3,1-2/(1+I*c*x^3))/c
```

Rubi [A]
time = 0.18, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {4948, 4930, 5040, 4964, 5004, 5114, 6745}

$$\frac{ib^2\text{Li}_2\left(1-\frac{2}{icx^3+1}\right)(a + b\text{ArcTan}(cx^3))}{c} + \frac{1}{3}x^3(a + b\text{ArcTan}(cx^3))^3 + \frac{i(a + b\text{ArcTan}(cx^3))^3}{3c} + \frac{b\log\left(\frac{2}{1+icx^3}\right)(a + b\text{ArcTan}(cx^3))^2}{c} + \frac{b^3\text{Li}_3\left(1-\frac{2}{icx^3+1}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] ((I/3)*(a + b*ArcTan[c*x^3])^3)/c + (x^3*(a + b*ArcTan[c*x^3])^3)/3 + (b*(a
 + b*ArcTan[c*x^3])^2*Log[2/(1 + I*c*x^3)])/c + (I*b^2*(a + b*ArcTan[c*x^3]
)*PolyLog[2, 1 - 2/(1 + I*c*x^3)])/c + (b^3*PolyLog[3, 1 - 2/(1 + I*c*x^3)]
)/(2*c)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2\left(a+b\tan^{-1}\left(cx^3\right)\right)^3\,dx &= \int \left(\frac{1}{8}x^2\left(2a+ib\log\left(1-icx^3\right)\right)^3+\frac{3}{8}ibx^2\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(1+\right.\right.\\
&= \frac{1}{8}\int x^2\left(2a+ib\log\left(1-icx^3\right)\right)^3\,dx+\frac{1}{8}(3ib)\int x^2\left(-2ia+b\log\left(1-icx^3\right)\right)^2\\
&= \frac{1}{24}\text{Subst}\left(\int (2a+ib\log(1-icx))^3\,dx,x,x^3\right)+\frac{1}{8}(ib)\text{Subst}\left(\int (-2ia+b\log\right.\\
&= \frac{1}{8}ibx^3\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)+\frac{1}{8}ib^2x^3\left(2ia-b\log\left(1-icx^3\right)\right.\\
&= \frac{i(1-icx^3)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24c}+\frac{1}{8}ibx^3\left(2ia-b\log\left(1-icx^3\right)\right)^2\log\left(1+\right.\\
&= \frac{b(1-icx^3)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{8c}+\frac{i(1-icx^3)\left(2a+ib\log\left(1-icx^3\right)\right)^3}{24c}+\\
&= -\frac{1}{2}ab^2x^3-\frac{1}{4}ib^3x^3+\frac{b(1-icx^3)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{8c}+\frac{i(1-icx^3)\left(2a+\right.}{2}\\
&= -\frac{1}{2}ab^2x^3+\frac{b^3(1-icx^3)\log\left(1-icx^3\right)}{4c}+\frac{b(1-icx^3)\left(2ia-b\log\left(1-icx^3\right)\right)^2}{8c}\\
&= \frac{1}{4}ib^3x^3+\frac{b^3(1-icx^3)\log\left(1-icx^3\right)}{4c}+\frac{b(1-icx^3)\left(2ia-b\log\left(1-icx^3\right)\right)^2}{8c}+\\
&= \frac{b(1-icx^3)\left(2ia-b\log\left(1-icx^3\right)\right)^2}{8c}+\frac{b(1-icx^3)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{8c}+\\
&= \frac{b(1-icx^3)\left(2ia-b\log\left(1-icx^3\right)\right)^2}{8c}+\frac{b(1-icx^3)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{8c}+
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 224, normalized size = 1.61

$$
\frac{2a^3cx^3+6a^2bcx^3\text{ArcTan}(cx^3)-6iab^2\text{ArcTan}(cx^3)^2+6ab^2cx^3\text{ArcTan}(cx^3)^2-2ib^3\text{ArcTan}(cx^3)^3+2b^3cx^3\text{ArcTan}(cx^3)^3+12ab^2\text{ArcTan}(cx^3)\log\left(1+e^{2i\text{ArcTan}(cx^3)}\right)+6b^3\text{ArcTan}(cx^3)^2\log\left(1+e^{2i\text{ArcTan}(cx^3)}\right)-3a^2b\log\left(1+c^2x^6\right)-6ib^2(a+b\text{ArcTan}(cx^3))\text{PolyLog}\left(2,-e^{2i\text{ArcTan}(cx^3)}\right)+3b^3\text{PolyLog}\left(3,-e^{2i\text{ArcTan}(cx^3)}\right)}{6c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTan[c*x^3])^3,x]
```

```
[Out] (2*a^3*c*x^3 + 6*a^2*b*c*x^3*ArcTan[c*x^3] - (6*I)*a*b^2*ArcTan[c*x^3]^2 +
6*a*b^2*c*x^3*ArcTan[c*x^3]^2 - (2*I)*b^3*ArcTan[c*x^3]^3 + 2*b^3*c*x^3*Arc
Tan[c*x^3]^3 + 12*a*b^2*ArcTan[c*x^3]*Log[1 + E^((2*I)*ArcTan[c*x^3])] + 6*
b^3*ArcTan[c*x^3]^2*Log[1 + E^((2*I)*ArcTan[c*x^3])] - 3*a^2*b*Log[1 + c^2*
```

```
x^6] - (6*I)*b^2*(a + b*ArcTan[c*x^3])*PolyLog[2, -E^((2*I)*ArcTan[c*x^3])]
 + 3*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x^3])])/(6*c)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(128) = 256$.
time = 0.16, size = 289, normalized size = 2.08

method	result
derivativedivides	$\frac{c x^3 a^3 - i b^3 \arctan(c x^3)^3 + b^3 \arctan(c x^3)^3 c x^3 + 3 b^3 \arctan(c x^3)^2 \ln\left(1 + \frac{(i c x^3+1)^2}{c^2 x^6+1}\right) - 3 i b^3 \arctan(c x^3) \operatorname{polylog}\left(2, -\right.}{}$
default	$\frac{c x^3 a^3 - i b^3 \arctan(c x^3)^3 + b^3 \arctan(c x^3)^3 c x^3 + 3 b^3 \arctan(c x^3)^2 \ln\left(1 + \frac{(i c x^3+1)^2}{c^2 x^6+1}\right) - 3 i b^3 \arctan(c x^3) \operatorname{polylog}\left(2, -\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c*x^3))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/c*(c*x^3*a^3-I*b^3*arctan(c*x^3)^3+b^3*arctan(c*x^3)^3*c*x^3+3*b^3*arct
an(c*x^3)^2*ln(1+(1+I*c*x^3)^2/(c^2*x^6+1))-3*I*b^3*arctan(c*x^3)*polylog(2
,-(1+I*c*x^3)^2/(c^2*x^6+1))+3/2*b^3*polylog(3,-(1+I*c*x^3)^2/(c^2*x^6+1))+
3*a^2*b*c*x^3*arctan(c*x^3)-3/2*a^2*b*ln(c^2*x^6+1)-3*I*arctan(c*x^3)^2*a*b
^2+3*arctan(c*x^3)^2*a*b^2*c*x^3-3*I*polylog(2,-(1+I*c*x^3)^2/(c^2*x^6+1))*
a*b^2+6*arctan(c*x^3)*ln(1+(1+I*c*x^3)^2/(c^2*x^6+1))*a*b^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*x^3*arctan(c*x^3)^3 - 1/32*b^3*x^3*arctan(c*x^3)*log(c^2*x^6 + 1)^
2 + 1/3*a^3*x^3 + 7/96*b^3*arctan(c*x^3)^4/c + 28*b^3*c^2*integrate(1/32*x^
8*arctan(c*x^3)^3/(c^2*x^6 + 1), x) + 3*b^3*c^2*integrate(1/32*x^8*arctan(c
*x^3)*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 96*a*b^2*c^2*integrate(1/32*x^
8*arctan(c*x^3)^2/(c^2*x^6 + 1), x) + 12*b^3*c^2*integrate(1/32*x^8*arctan(
c*x^3)*log(c^2*x^6 + 1)/(c^2*x^6 + 1), x) + 1/3*a*b^2*arctan(c*x^3)^3/c - 1
2*b^3*c*integrate(1/32*x^5*arctan(c*x^3)^2/(c^2*x^6 + 1), x) + 3*b^3*c*inte
grate(1/32*x^5*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 3*b^3*integrate(1/32*
x^2*arctan(c*x^3)*log(c^2*x^6 + 1)^2/(c^2*x^6 + 1), x) + 1/2*(2*c*x^3*arcta
n(c*x^3) - log(c^2*x^6 + 1))*a^2*b/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
[Out] integral(b^3*x^2*arctan(c*x^3)^3 + 3*a*b^2*x^2*arctan(c*x^3)^2 + 3*a^2*b*x^
2*arctan(c*x^3) + a^3*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(a+b\operatorname{atan}\left(cx^{3}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c*x**3))**3,x)
[Out] Integral(x**2*(a + b*atan(c*x**3))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
[Out] integrate((b*arctan(c*x^3) + a)^3*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c*x^3))^3,x)
[Out] int(x^2*(a + b*atan(c*x^3))^3, x)
```

3.124 $\int \frac{\left(a+b\mathbf{ArcTan}\left(cx^3\right)\right)^3}{x}\,dx$

Optimal. Leaf size=232

$$\frac{2}{3}\left(a+b\text{ArcTan}\left(cx^3\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)-\frac{1}{2}ib\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2\text{PolyLog}\left(2,1-\frac{2}{1+icx^3}\right)+\frac{1}{2}ib$$

```
[Out] -2/3*(a+b*arctan(c*x^3))^3*arctanh(-1+2/(1+I*c*x^3))-1/2*I*b*(a+b*arctan(c*
x^3))^2*polylog(2,1-2/(1+I*c*x^3))+1/2*I*b*(a+b*arctan(c*x^3))^2*polylog(2,
-1+2/(1+I*c*x^3))-1/2*b^2*(a+b*arctan(c*x^3))*polylog(3,1-2/(1+I*c*x^3))+1/
2*b^2*(a+b*arctan(c*x^3))*polylog(3,-1+2/(1+I*c*x^3))+1/4*I*b^3*polylog(4,1
-2/(1+I*c*x^3))-1/4*I*b^3*polylog(4,-1+2/(1+I*c*x^3))
```

Rubi [A]
time = 0.33, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4944, 4942, 5108, 5004, 5114, 5118, 6745}

$$-\frac{1}{2}b^2\text{Li}_3\left(1-\frac{2}{icx^3+1}\right)\left(a+b\text{ArcTan}\left(cx^3\right)\right)+\frac{1}{2}b^2\text{Li}_3\left(\frac{2}{icx^3+1}-1\right)\left(a+b\text{ArcTan}\left(cx^3\right)\right)-\frac{1}{2}ib\text{Li}_2\left(1-\frac{2}{icx^3+1}\right)\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2+\frac{1}{2}ib\text{Li}_2\left(\frac{2}{icx^3+1}-1\right)\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2+\frac{2}{3}\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right)\left(a+b\text{ArcTan}\left(cx^3\right)\right)^3+\frac{1}{4}ib^3\text{Li}_4\left(1-\frac{2}{icx^3+1}\right)-\frac{1}{4}ib^3\text{Li}_4\left(\frac{2}{icx^3+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])^3/x,x]
```

```
[Out] (2*(a + b*ArcTan[c*x^3])^3*ArcTanh[1 - 2/(1 + I*c*x^3)])/3 - (I/2)*b*(a + b
*ArcTan[c*x^3])^2*PolyLog[2, 1 - 2/(1 + I*c*x^3)] + (I/2)*b*(a + b*ArcTan[c
*x^3])^2*PolyLog[2, -1 + 2/(1 + I*c*x^3)] - (b^2*(a + b*ArcTan[c*x^3])*Poly
Log[3, 1 - 2/(1 + I*c*x^3)])/2 + (b^2*(a + b*ArcTan[c*x^3])*PolyLog[3, -1 +
 2/(1 + I*c*x^3)])/2 + (I/4)*b^3*PolyLog[4, 1 - 2/(1 + I*c*x^3)] - (I/4)*b^
3*PolyLog[4, -1 + 2/(1 + I*c*x^3)]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx^3))^3}{x}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{(a+b\tan^{-1}(cx))^3}{x}\,dx, x, x^3\right)$$

$$= \frac{2}{3}(a+b\tan^{-1}(cx^3))^3\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right) - (2bc)\text{Subst}\left(\int \frac{(a+b\tan^{-1}(c$$

$$= \frac{2}{3}(a+b\tan^{-1}(cx^3))^3\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right) + (bc)\text{Subst}\left(\int \frac{(a+b\tan^{-1}(c}{1+}$$

$$= \frac{2}{3}(a+b\tan^{-1}(cx^3))^3\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right) - \frac{1}{2}ib(a+b\tan^{-1}(cx^3))^2\text{Li}_2\Big($$

$$= \frac{2}{3}(a+b\tan^{-1}(cx^3))^3\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right) - \frac{1}{2}ib(a+b\tan^{-1}(cx^3))^2\text{Li}_2\Big($$

$$= \frac{2}{3}(a+b\tan^{-1}(cx^3))^3\tanh^{-1}\left(1-\frac{2}{1+icx^3}\right) - \frac{1}{2}ib(a+b\tan^{-1}(cx^3))^2\text{Li}_2\Big($$

Mathematica [A]
time = 0.13, size = 248, normalized size = 1.07

$$\frac{2}{3}(a+b\text{ArcTan}(cx^3))^3\tanh^{-1}\left(1+\frac{2i}{-i+cx^3}\right)+\frac{1}{4}ib\left(2(a+b\text{ArcTan}(cx^3))^2\text{PolyLog}\left(2,\frac{i+cx^3}{i-cx^3}\right)-2(a+b\text{ArcTan}(cx^3))^2\text{PolyLog}\left(2,\frac{i+cx^3}{-i+cx^3}\right)+b\left(-2i(a+b\text{ArcTan}(cx^3))\text{PolyLog}\left(3,\frac{i+cx^3}{i-cx^3}\right)+2i(a+b\text{ArcTan}(cx^3))\text{PolyLog}\left(3,\frac{i+cx^3}{-i+cx^3}\right)+b\left(-\text{PolyLog}\left(4,\frac{i+cx^3}{i-cx^3}\right)+\text{PolyLog}\left(4,\frac{i+cx^3}{-i+cx^3}\right)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^3/x,x]
```

```
[Out] (2*(a + b*ArcTan[c*x^3])^3*ArcTanh[1 + (2*I)/(-I + c*x^3)])/3 + (I/4)*b*(2*
(a + b*ArcTan[c*x^3])^2*PolyLog[2, (I + c*x^3)/(I - c*x^3)] - 2*(a + b*ArcT
an[c*x^3])^2*PolyLog[2, (I + c*x^3)/(-I + c*x^3)] + b*((-2*I)*(a + b*ArcTan
[c*x^3])*PolyLog[3, (I + c*x^3)/(I - c*x^3)] + (2*I)*(a + b*ArcTan[c*x^3])*
PolyLog[3, (I + c*x^3)/(-I + c*x^3)] + b*(-PolyLog[4, (I + c*x^3)/(I - c*x^
3)] + PolyLog[4, (I + c*x^3)/(-I + c*x^3)])))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\arctan(c\,x^3))^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^3/x,x)
```

```
[Out] int((a+b*arctan(c*x^3))^3/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + 1/32*integrate((28*b^3*arctan(c*x^3)^3 + 3*b^3*arctan(c*x^3)*l
og(c^2*x^6 + 1)^2 + 96*a*b^2*arctan(c*x^3)^2 + 96*a^2*b*arctan(c*x^3))/x, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c*
x^3) + a^3)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{atan}\,(cx^3))^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**3/x,x)
```

```
[Out] Integral((a + b*atan(c*x**3))**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)}^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))^3/x,x)
```

```
[Out] int((a + b*atan(c*x^3))^3/x, x)
```

3.125 $\int \frac{\left(a+b\mathbf{ArcTan}\left(cx^3\right)\right)^3}{x^4}\,dx$

Optimal. Leaf size=133

$$-\frac{1}{3}ic\left(a+b\text{ArcTan}\left(cx^3\right)\right)^3-\frac{\left(a+b\text{ArcTan}\left(cx^3\right)\right)^3}{3x^3}+bc\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2\log\left(2-\frac{2}{1-icx^3}\right)-ib^2c\left(a+b\text{A}\right.$$

```
[Out] -1/3*I*c*(a+b*arctan(c*x^3))^3-1/3*(a+b*arctan(c*x^3))^3/x^3+b*c*(a+b*arcta
n(c*x^3))^2*ln(2-2/(1-I*c*x^3))-I*b^2*c*(a+b*arctan(c*x^3))*polylog(2,-1+2/
(1-I*c*x^3))+1/2*b^3*c*polylog(3,-1+2/(1-I*c*x^3))
```

Rubi [A]
time = 0.22, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4948, 4946, 5044, 4988, 5004, 5112, 6745}

$$-ib^2c\text{Li}_2\left(\frac{2}{1-icx^3}-1\right)\left(a+b\text{ArcTan}\left(cx^3\right)\right)-\frac{1}{3}ic\left(a+b\text{ArcTan}\left(cx^3\right)\right)^3-\frac{\left(a+b\text{ArcTan}\left(cx^3\right)\right)^3}{3x^3}+bc\log\left(2-\frac{2}{1-icx^3}\right)\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2+\frac{1}{2}b^3c\text{Li}_3\left(\frac{2}{1-icx^3}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])^3/x^4,x]
```

```
[Out] (-1/3*I)*c*(a + b*ArcTan[c*x^3])^3 - (a + b*ArcTan[c*x^3])^3/(3*x^3) + b*c*
(a + b*ArcTan[c*x^3])^2*Log[2 - 2/(1 - I*c*x^3)] - I*b^2*c*(a + b*ArcTan[c*
x^3])*PolyLog[2, -1 + 2/(1 - I*c*x^3)] + (b^3*c*PolyLog[3, -1 + 2/(1 - I*c*
x^3)])/2
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
```

```
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{(a+b\tan^{-1}(cx^3))^3}{x^4}\,dx = \int \left(\frac{(2a+ib\log(1-icx^3))^3}{8x^4} + \frac{3ib(-2ia+b\log(1-icx^3))^2\log(1+icx^3)}{8x^4} - \frac{3i}{}\right.$$

$$= \frac{1}{8}\int \frac{(2a+ib\log(1-icx^3))^3}{x^4}\,dx + \frac{1}{8}(3ib)\int \frac{(-2ia+b\log(1-icx^3))^2\log(1+}{x^4}$$

$$= \frac{1}{24}\text{Subst}\left(\int \frac{(2a+ib\log(1-icx))^3}{x^2}\,dx, x, x^3\right) + \frac{1}{8}(ib)\text{Subst}\left(\int \frac{(-2ia+b\log(}{}\right.$$

$$= -\frac{(1-icx^3)(2a+ib\log(1-icx^3))^3}{24x^3} - \frac{ib^3(1+icx^3)\log^3(1+icx^3)}{24x^3} + \frac{1}{8}(ib)\text{Sub}$$

$$= \frac{1}{8}bc\log(icx^3)(2a+ib\log(1-icx^3))^2 - \frac{(1-icx^3)(2a+ib\log(1-icx^3))^3}{24x^3} -$$

$$= \frac{1}{8}bc\log(icx^3)(2a+ib\log(1-icx^3))^2 - \frac{(1-icx^3)(2a+ib\log(1-icx^3))^3}{24x^3} -$$

$$= \frac{1}{8}bc\log(icx^3)(2a+ib\log(1-icx^3))^2 - \frac{(1-icx^3)(2a+ib\log(1-icx^3))^3}{24x^3} -$$

$$= \frac{1}{8}bc\log(icx^3)(2a+ib\log(1-icx^3))^2 - \frac{(1-icx^3)(2a+ib\log(1-icx^3))^3}{24x^3} -$$

Mathematica [A]
time = 0.27, size = 240, normalized size = 1.80

$$-\frac{a^3}{3x^3} - \frac{a^2b\text{ArcTan}(cx^3)}{x^3} + 3a^2bc\log(x) - \frac{1}{2}a^2bc\log(1+c^2x^6) + ab^2c\left(\text{ArcTan}(cx^3)\left(\left(-i-\frac{1}{cx^3}\right)\text{ArcTan}(cx^3) + 2\log\left(1-e^{2i\text{ArcTan}(cx^3)}\right)\right) - i\text{PolyLog}\left(2, e^{2i\text{ArcTan}(cx^3)}\right)\right) + \frac{1}{3}b^3c\left(-\frac{i\pi^3}{8} + i\text{ArcTan}(cx^3)^3 - \frac{\text{ArcTan}(cx^3)^3}{cx^3} + 3\text{ArcTan}(cx^3)^2\log\left(1-e^{-2i\text{ArcTan}(cx^3)}\right) + 3i\text{ArcTan}(cx^3)\text{PolyLog}\left(2, e^{-2i\text{ArcTan}(cx^3)}\right) + \frac{3}{2}\text{PolyLog}\left(3, e^{-2i\text{ArcTan}(cx^3)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 - (a^2*b*ArcTan[c*x^3])/x^3 + 3*a^2*b*c*Log[x] - (a^2*b*c*Log[
1 + c^2*x^6])/2 + a*b^2*c*(ArcTan[c*x^3]*((-I - 1/(c*x^3))*ArcTan[c*x^3] +
2*Log[1 - E^((2*I)*ArcTan[c*x^3])]) - I*PolyLog[2, E^((2*I)*ArcTan[c*x^3])]
) + (b^3*c*((-1/8*I)*Pi^3 + I*ArcTan[c*x^3]^3 - ArcTan[c*x^3]^3/(c*x^3) + 3
*ArcTan[c*x^3]^2*Log[1 - E^((-2*I)*ArcTan[c*x^3])] + (3*I)*ArcTan[c*x^3]*Po
lyLog[2, E^((-2*I)*ArcTan[c*x^3])] + (3*PolyLog[3, E^((-2*I)*ArcTan[c*x^3])
])/2))/3
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\arctan(c\,x^3))^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^3/x^4,x)
```

```
[Out] int((a+b*arctan(c*x^3))^3/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x^4,x, algorithm="maxima")
```

```
[Out] -1/2*(c*(log(c^2*x^6 + 1) - log(x^6)) + 2*arctan(c*x^3)/x^3)*a^2*b - 1/3*a^
3/x^3 - 1/96*(4*b^3*arctan(c*x^3)^3 - 3*b^3*arctan(c*x^3)*log(c^2*x^6 + 1)^
2 - 96*x^3*integrate(-1/32*(12*b^3*c^2*x^6*arctan(c*x^3)*log(c^2*x^6 + 1) -
 28*(b^3*c^2*x^6 + b^3)*arctan(c*x^3)^3 - 12*(8*a*b^2*c^2*x^6 + b^3*c*x^3 +
 8*a*b^2)*arctan(c*x^3)^2 + 3*(b^3*c*x^3 - (b^3*c^2*x^6 + b^3)*arctan(c*x^3
))*log(c^2*x^6 + 1)^2)/(c^2*x^10 + x^4), x))/x^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x^4,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c*
x^3) + a^3)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}{\left(c x^{3} \right)}\right)^{3}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**3/x**4,x)
```

```
[Out] Integral((a + b*atan(c*x**3))**3/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^3/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{atan}(c\,x^3))^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))^3/x^4,x)
```

```
[Out] int((a + b*atan(c*x^3))^3/x^4, x)
```

3.126 $\int \frac{\left(a+b\textbf{ArcTan}(cx^3)\right)^3}{x^7}\,dx$

Optimal. Leaf size=146

$$-\frac{1}{2}ibc^2\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2-\frac{bc\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2}{2x^3}-\frac{1}{6}c^2\left(a+b\text{ArcTan}\left(cx^3\right)\right)^3-\frac{\left(a+b\text{ArcTan}\left(cx^3\right)\right)^3}{6x^6}+b^2$$

```
[Out] -1/2*I*b*c^2*(a+b*arctan(c*x^3))^2-1/2*b*c*(a+b*arctan(c*x^3))^2/x^3-1/6*c^
2*(a+b*arctan(c*x^3))^3-1/6*(a+b*arctan(c*x^3))^3/x^6+b^2*c^2*(a+b*arctan(c
*x^3))*ln(2-2/(1-I*c*x^3))-1/2*I*b^3*c^2*polylog(2,-1+2/(1-I*c*x^3))
```

Rubi [A]
time = 0.23, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {4948, 4946, 5038, 5044, 4988, 2497, 5004}

$$b^2c^2\log\left(2-\frac{2}{1-icx^3}\right)\left(a+b\text{ArcTan}\left(cx^3\right)\right)-\frac{1}{2}ibc^2\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2-\frac{1}{6}c^2\left(a+b\text{ArcTan}\left(cx^3\right)\right)^3-\frac{bc\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2}{2x^3}-\frac{\left(a+b\text{ArcTan}\left(cx^3\right)\right)^3}{6x^6}-\frac{1}{2}ib^3c^2\text{Li}_2\left(\frac{2}{1-icx^3}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c*x^3])^3/x^7,x]
```

```
[Out] (-1/2*I)*b*c^2*(a + b*ArcTan[c*x^3])^2 - (b*c*(a + b*ArcTan[c*x^3])^2)/(2*x
^3) - (c^2*(a + b*ArcTan[c*x^3])^3)/6 - (a + b*ArcTan[c*x^3])^3/(6*x^6) + b
^2*c^2*(a + b*ArcTan[c*x^3])*Log[2 - 2/(1 - I*c*x^3)] - (I/2)*b^3*c^2*PolyL
og[2, -1 + 2/(1 - I*c*x^3)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
```

```
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(cx^3\right)\right)^3}{x^7}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{8x^7} + \frac{3ib\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(1+icx^3\right)}{8x^7} - \right.$$

$$= \frac{1}{8}\int \frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{x^7}\,dx + \frac{1}{8}(3ib)\int \frac{\left(-2ia+b\log\left(1-icx^3\right)\right)^2\log\left(1+\right.}{x^7}$$

$$= \frac{1}{24}\text{Subst}\left(\int \frac{\left(2a+ib\log(1-icx)\right)^3}{x^3}\,dx, x, x^3\right) + \frac{1}{8}(ib)\text{Subst}\left(\int \frac{\left(-2ia+b\log\right.}{}\right.$$

$$= -\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{48x^6} - \frac{ib^3\log^3\left(1+icx^3\right)}{48x^6} + \frac{1}{8}(ib)\text{Subst}\left(\int \frac{\left(-2ia+b\log\right.}{}\right.$$

$$= -\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{48x^6} - \frac{ib^3\log^3\left(1+icx^3\right)}{48x^6} + \frac{1}{16}(ib)\text{Subst}\left(\int \frac{\left(2a+ib\log\right.}{x\left(-\frac{i}{c}+\right.}\right.$$

$$= -\frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{48x^6} - \frac{ib^3\log^3\left(1+icx^3\right)}{48x^6} + \frac{1}{16}(ib)\text{Subst}\left(\int \frac{\left(2a+ib\log\right.}{\left(-\frac{i}{c}+\right.}\right.$$

$$= -\frac{bc\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{16x^3} - \frac{\left(2a+ib\log\left(1-icx^3\right)\right)^3}{48x^6} + \frac{b^3c(1+i}{}$$

$$= \frac{3}{4}ab^2c^2\log(x) - \frac{bc\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{16x^3} + \frac{1}{16}ibc^2\log\left(icx^3\right)\left(2a+\right.$$

$$= \frac{3}{4}ab^2c^2\log(x) - \frac{bc\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{16x^3} + \frac{1}{16}ibc^2\log\left(icx^3\right)\left(2a+\right.$$

$$= \frac{3}{4}ab^2c^2\log(x) - \frac{bc\left(1-icx^3\right)\left(2a+ib\log\left(1-icx^3\right)\right)^2}{16x^3} + \frac{1}{16}ibc^2\log\left(icx^3\right)\left(2a+\right.$$

Mathematica [**A**]
time = 0.22, size = 196, normalized size = 1.34

$$-\frac{3b^2\left(a+ac^2x^6+bcx^3\left(1+icx^3\right)\right)\text{ArcTan}\left(cx^3\right)^2+b^3\left(1+c^2x^6\right)\text{ArcTan}\left(cx^3\right)^3+3b\text{ArcTan}\left(cx^3\right)\left(a\left(a+2bcx^3+ac^2x^6\right)-2b^2c^2x^6\log\left(1-e^{2i\text{ArcTan}\left(cx^3\right)}\right)\right)+a\left(a\left(a+3bcx^3\right)-6b^2c^2x^6\log\left(\frac{cx^3}{\sqrt{1+c^2x^6}}\right)\right)+3ib^3c^2x^6\text{PolyLog}\left(2,e^{2i\text{ArcTan}\left(cx^3\right)}\right)}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c*x^3])^3/x^7,x]
```

```
[Out] -1/6*(3*b^2*(a + a*c^2*x^6 + b*c*x^3*(1 + I*c*x^3))*ArcTan[c*x^3]^2 + b^3*(
1 + c^2*x^6)*ArcTan[c*x^3]^3 + 3*b*ArcTan[c*x^3]*(a*(a + 2*b*c*x^3 + a*c^2*
x^6) - 2*b^2*c^2*x^6*Log[1 - E^((2*I)*ArcTan[c*x^3])]) + a*(a*(a + 3*b*c*x^
3) - 6*b^2*c^2*x^6*Log[(c*x^3)/Sqrt[1 + c^2*x^6]]) + (3*I)*b^3*c^2*x^6*Poly
Log[2, E^((2*I)*ArcTan[c*x^3])])/x^6
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\arctan\left(c\,x^3\right)\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c*x^3))^3/x^7,x)
```

```
[Out] int((a+b*arctan(c*x^3))^3/x^7,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x^7,x, algorithm="maxima")
```

```
[Out] -1/2*((c*arctan(c*x^3) + 1/x^3)*c + arctan(c*x^3)/x^6)*a^2*b + 1/2*((arctan
(c*x^3)^2 - log(c^2*x^6 + 1) + 6*log(x))*c^2 - 2*(c*arctan(c*x^3) + 1/x^3)*
c*arctan(c*x^3))*a*b^2 - 1/2*a*b^2*arctan(c*x^3)^2/x^6 + 1/192*(192*x^6*int
egrate(-1/64*(12*c^2*x^6*arctan(c*x^3)*log(c^2*x^6 + 1) - 12*c*x^3*arctan(c
*x^3)^2 - 56*(c^2*x^6 + 1)*arctan(c*x^3)^3 + 3*(c*x^3 - 2*(c^2*x^6 + 1)*arc
tan(c*x^3))*log(c^2*x^6 + 1)^2)/(c^2*x^13 + x^7), x) - 4*arctan(c*x^3)^3 +
3*arctan(c*x^3)*log(c^2*x^6 + 1)^2)*b^3/x^6 - 1/6*a^3/x^6
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x^7,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c*
x^3) + a^3)/x^7, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}\left(c x^{3}\right)\right)^{3}}{x^{7}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c*x**3))**3/x**7,x)
```

```
[Out] Integral((a + b*atan(c*x**3))**3/x**7, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c*x^3))^3/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^3/x^7, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^3}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c*x^3))^3/x^7,x)
```

```
[Out] int((a + b*atan(c*x^3))^3/x^7, x)
```

3.127 $\int (dx)^m \left(a+b\mathbf{ArcTan}(cx^3)\right)^3 dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m \left(a+b\text{ArcTan}\left(cx^3\right)\right)^3,x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x^3))^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\text{ArcTan}\left(cx^3\right)\right)^3 dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^3])^3,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x^3])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^3 dx = \int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^3 dx$$

Mathematica [A]
time = 1.34, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\text{ArcTan}\left(cx^3\right)\right)^3 dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^3,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^3, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\arctan\left(c\,x^3\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x^3))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x^3))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^3,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^3/(d*(m + 1)) + 1/32*(4*b^3*d^m*x*x^m*arctan(c*x^3)^3 - 3*b
^3*d^m*x*x^m*arctan(c*x^3)*log(c^2*x^6 + 1)^2 + 32*(m + 1)*integrate(1/32*(
36*b^3*c^2*d^m*x^6*x^m*arctan(c*x^3)*log(c^2*x^6 + 1) + 28*((b^3*c^2*d^m*m
+ b^3*c^2*d^m)*x^6 + b^3*d^m*m + b^3*d^m)*x^m*arctan(c*x^3)^3 - 12*(3*b^3*c
*d^m*x^3 - 8*(a*b^2*c^2*d^m*m + a*b^2*c^2*d^m)*x^6 - 8*a*b^2*d^m*m - 8*a*b^
2*d^m)*x^m*arctan(c*x^3)^2 + 96*((a^2*b*c^2*d^m*m + a^2*b*c^2*d^m)*x^6 + a^
2*b*d^m*m + a^2*b*d^m)*x^m*arctan(c*x^3) + 3*(3*b^3*c*d^m*x^3*x^m + ((b^3*c
^2*d^m*m + b^3*c^2*d^m)*x^6 + b^3*d^m*m + b^3*d^m)*x^m*arctan(c*x^3))*log(c
^2*x^6 + 1)^2)/((c^2*m + c^2)*x^6 + m + 1), x))/(m + 1)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arctan(c*x^3)^3 + 3*a*b^2*arctan(c*x^3)^2 + 3*a^2*b*arctan(c*
x^3) + a^3)*(d*x)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x**3))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^3*(d*x)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,{\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atan(c*x^3))^3,x)
```

```
[Out] int((d*x)^m*(a + b*atan(c*x^3))^3, x)
```

3.128 $\int (dx)^m \left(a+b\text{ArcTan}(cx^3)\right)^2 \, dx$

Optimal. Leaf size=21

$$\text{Int}\left((dx)^m \left(a+b\text{ArcTan}\left(cx^3\right)\right)^2, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arctan(c*x^3))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\text{ArcTan}\left(cx^3\right)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^3])^2,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcTan[c*x^3])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^2 \, dx = \int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right)^2 \, dx$$

Mathematica [A]
time = 0.87, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\text{ArcTan}\left(cx^3\right)\right)^2 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcTan[c*x^3])^2, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\arctan\left(c\,x^3\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x^3))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arctan(c*x^3))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] (d*x)^(m + 1)*a^2/(d*(m + 1)) + 1/16*(4*b^2*d^m*x*x^m*arctan(c*x^3)^2 - b^2
*d^m*x*x^m*log(c^2*x^6 + 1)^2 + 16*(m + 1)*integrate(1/16*(12*b^2*c^2*d^m*x
^6*x^m*log(c^2*x^6 + 1) + 12*((b^2*c^2*d^m*m + b^2*c^2*d^m)*x^6 + b^2*d^m*m
 + b^2*d^m)*x^m*arctan(c*x^3)^2 + ((b^2*c^2*d^m*m + b^2*c^2*d^m)*x^6 + b^2*
d^m*m + b^2*d^m)*x^m*log(c^2*x^6 + 1)^2 - 8*(3*b^2*c*d^m*x^3 - 4*(a*b*c^2*d
^m*m + a*b*c^2*d^m)*x^6 - 4*a*b*d^m*m - 4*a*b*d^m)*x^m*arctan(c*x^3))/((c^2
*m + c^2)*x^6 + m + 1), x))/(m + 1)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2)*(d*x)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x**3))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)^2*(d*x)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atan(c*x^3))^2,x)
[Out] int((d*x)^m*(a + b*atan(c*x^3))^2, x)
```

3.129 $\int (dx)^m (a + b\mathbf{ArcTan}(cx^3)) \, dx$

Optimal. Leaf size=75

$$\frac{(dx)^{1+m} (a + b\text{ArcTan}(cx^3))}{d(1+m)} - \frac{3bc(dx)^{4+m} \, {}_2F_1\left(1, \frac{4+m}{6}; \frac{10+m}{6}; -c^2x^6\right)}{d^4(1+m)(4+m)}$$

```
[Out] (d*x)^(1+m)*(a+b*arctan(c*x^3))/d/(1+m)-3*b*c*(d*x)^(4+m)*hypergeom([1, 2/3
+1/6*m],[5/3+1/6*m],-c^2*x^6)/d^4/(1+m)/(4+m)
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4958, 371}

$$\frac{(dx)^{m+1} (a + b\text{ArcTan}(cx^3))}{d(m+1)} - \frac{3bc(dx)^{m+4} \, {}_2F_1\left(1, \frac{m+4}{6}; \frac{m+10}{6}; -c^2x^6\right)}{d^4(m+1)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcTan[c*x^3]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcTan[c*x^3]))/(d*(1 + m)) - (3*b*c*(d*x)^(4 + m)*Hy
pergeometric2F1[1, (4 + m)/6, (10 + m)/6, -(c^2*x^6)])/(d^4*(1 + m)*(4 + m)
)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4958

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :>
 Simp[(d*x)^(m + 1)*((a + b*ArcTan[c*x^n])/(d*(m + 1))), x] - Dist[b*c*(n/(
d^n*(m + 1))), Int[(d*x)^(m + n)/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{a, b,
c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

Rubi steps

$$\int (dx)^m \left(a+b\tan^{-1}\left(cx^3\right)\right) dx = \frac{(dx)^{1+m}\left(a+b\tan^{-1}\left(cx^3\right)\right)}{d(1+m)} - \frac{(3bc)\int \frac{x^2(dx)^{1+m}}{1+c^2x^6}dx}{d(1+m)}$$
$$= \frac{(dx)^{1+m}\left(a+b\tan^{-1}\left(cx^3\right)\right)}{d(1+m)} - \frac{(3bc)\int \frac{(dx)^{3+m}}{1+c^2x^6}dx}{d^3(1+m)}$$
$$= \frac{(dx)^{1+m}\left(a+b\tan^{-1}\left(cx^3\right)\right)}{d(1+m)} - \frac{3bc(dx)^{4+m}\,{}_2F_1\left(1,\frac{4+m}{6};\frac{10+m}{6};-c^2x^6\right)}{d^4(1+m)(4+m)}$$

Mathematica [A]
time = 0.04, size = 65, normalized size = 0.87

$$-\frac{x(dx)^m\left(-((4+m)\left(a+b\text{ArcTan}(cx^3)\right))+3bcx^3\,{}_2F_1\left(1,\frac{4+m}{6};\frac{10+m}{6};-c^2x^6\right)\right)}{(1+m)(4+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcTan[c*x^3]),x]
[Out] -((x*(d*x)^m*(-((4 + m)*(a + b*ArcTan[c*x^3])) + 3*b*c*x^3*Hypergeometric2F
1[1, (4 + m)/6, (10 + m)/6, -(c^2*x^6)]))/((1 + m)*(4 + m)))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\arctan\left(c\,x^3\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arctan(c*x^3)),x)
[Out] int((d*x)^m*(a+b*arctan(c*x^3)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3)),x, algorithm="maxima")
[Out] (d^m*x*x^m*arctan(c*x^3) - 3*(c*d^m*m + c*d^m)*integrate(x^3*x^m/((c^2*m +
c^2)*x^6 + m + 1), x))*b/(m + 1) + (d*x)^(m + 1)*a/(d*(m + 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3)),x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c*x^3) + a)*(d*x)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*atan(c*x**3)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arctan(c*x^3)),x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c*x^3) + a)*(d*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m\,\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*atan(c*x^3)),x)
```

```
[Out] int((d*x)^m*(a + b*atan(c*x^3)), x)
```

3.130 $\int \frac{(dx)^m}{a+b\textbf{ArcTan}(cx^3)}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(dx)^m}{a+b\text{ArcTan}\left(cx^3\right)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctan(c*x^3)),x)
```

Rubi [A]

time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{a+b\text{ArcTan}\left(cx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTan[c*x^3]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcTan[c*x^3]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\tan^{-1}\left(cx^3\right)}\,dx=\int \frac{(dx)^m}{a+b\tan^{-1}\left(cx^3\right)}\,dx$$

Mathematica [A]

time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\text{ArcTan}\left(cx^3\right)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTan[c*x^3]),x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcTan[c*x^3]), x]
```

Maple [A]

time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\arctan\left(c\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctan(c*x^3)),x)
[Out] int((d*x)^m/(a+b*arctan(c*x^3)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arctan(c*x^3) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arctan(c*x^3) + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atan(c*x**3)),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3)),x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arctan(c*x^3) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{a+b\,\mathrm{atan}\left(c\,x^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atan(c*x^3)),x)
[Out] int((d*x)^m/(a + b*atan(c*x^3)), x)
```

3.131 $\int \frac{(dx)^m}{\left(a+b\mathbf{ArcTan}\left(cx^3\right)\right)^2}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{(dx)^m}{\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arctan(c*x^3))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcTan[c*x^3])^2,x]
[Out] Defer[Int][(d*x)^m/(a + b*ArcTan[c*x^3])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}\,dx=\int \frac{(dx)^m}{\left(a+b\tan^{-1}\left(cx^3\right)\right)^2}\,dx$$

Mathematica [A]
time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\text{ArcTan}\left(cx^3\right)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcTan[c*x^3])^2,x]
[Out] Integrate[(d*x)^m/(a + b*ArcTan[c*x^3])^2, x]
```

Maple [A]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\arctan\left(c\,x^3\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arctan(c*x^3))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arctan(c*x^3))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3))^2,x, algorithm="maxima")
```

```
[Out] -1/3*((c^2*d^m*x^6 + d^m)*x^m - 3*(b^2*c*x^2*arctan(c*x^3) + a*b*c*x^2)*int
egrate(1/3*((c^2*d^m*m + 4*c^2*d^m)*x^6 + d^m*m - 2*d^m)*x^m/(b^2*c*x^3*arc
tan(c*x^3) + a*b*c*x^3), x))/(b^2*c*x^2*arctan(c*x^3) + a*b*c*x^2)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*arctan(c*x^3)^2 + 2*a*b*arctan(c*x^3) + a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*atan(c*x**3))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arctan(c*x^3))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arctan(c*x^3) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{\left(a+b\,\mathrm{atan}\left(c\,x^3\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*atan(c*x^3))^2,x)
[Out] int((d*x)^m/(a + b*atan(c*x^3))^2, x)
```

3.132 $\int x^3\left(a+b\mathbf{ArcTan}\left(\frac{c}{x}\right)\right)\,dx$

Optimal. Leaf size=50

$$-\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}x^4\left(a+b\text{ArcTan}\left(\frac{c}{x}\right)\right)+\frac{1}{4}bc^4\text{ArcTan}\left(\frac{x}{c}\right)$$

```
[Out] -1/4*b*c^3*x+1/12*b*c*x^3+1/4*x^4*(a+b*arctan(c/x))+1/4*b*c^4*arctan(x/c)
```

Rubi [A]
time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {4946, 269, 308, 209}

$$\frac{1}{4}x^4\left(a+b\text{ArcTan}\left(\frac{c}{x}\right)\right)+\frac{1}{4}bc^4\text{ArcTan}\left(\frac{x}{c}\right)-\frac{1}{4}bc^3x+\frac{1}{12}bcx^3$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c/x]),x]
```

```
[Out] -1/4*(b*c^3*x) + (b*c*x^3)/12 + (x^4*(a + b*ArcTan[c/x]))/4 + (b*c^4*ArcTan
[x/c])/4
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)\,dx &= \frac{1}{4}x^4\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\frac{x^2}{1+\frac{c^2}{x^2}}\,dx\\
&= \frac{1}{4}x^4\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\frac{x^4}{c^2+x^2}\,dx\\
&= \frac{1}{4}x^4\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}(bc)\int\left(-c^2+x^2+\frac{c^4}{c^2+x^2}\right)\,dx\\
&= -\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}\left(bc^5\right)\int\frac{1}{c^2+x^2}\,dx\\
&= -\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}x^4\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{4}bc^4\tan^{-1}\left(\frac{x}{c}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 55, normalized size = 1.10

$$-\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{ax^4}{4}-\frac{1}{4}bc^4\text{ArcTan}\left(\frac{c}{x}\right)+\frac{1}{4}bx^4\text{ArcTan}\left(\frac{c}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c/x]),x]
```

```
[Out] -1/4*(b*c^3*x) + (b*c*x^3)/12 + (a*x^4)/4 - (b*c^4*ArcTan[c/x])/4 + (b*x^4*
ArcTan[c/x])/4
```

Maple [A]
time = 0.11, size = 56, normalized size = 1.12

method	result	size
derivativedivides	$-c^4\left(-\frac{a\,x^4}{4c^4}-\frac{b\,x^4\arctan\left(\frac{c}{x}\right)}{4c^4}+\frac{b\arctan\left(\frac{c}{x}\right)}{4}-\frac{b\,x^3}{12c^3}+\frac{bx}{4c}\right)$	56
default	$-c^4\left(-\frac{a\,x^4}{4c^4}-\frac{b\,x^4\arctan\left(\frac{c}{x}\right)}{4c^4}+\frac{b\arctan\left(\frac{c}{x}\right)}{4}-\frac{b\,x^3}{12c^3}+\frac{bx}{4c}\right)$	56
risch	Expression too large to display	697

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c/x)),x,method=_RETURNVERBOSE)
```

```
[Out] -c^4*(-1/4*a/c^4*x^4-1/4*b/c^4*x^4*arctan(c/x)+1/4*b*arctan(c/x)-1/12*b*x^3
/c^3+1/4*b*x/c)
```

Maxima [A]
time = 0.47, size = 45, normalized size = 0.90

$$\frac{1}{4}\,ax^4+\frac{1}{12}\left(3\,x^4\arctan\left(\frac{c}{x}\right)+\left(3\,c^3\arctan\left(\frac{x}{c}\right)-3\,c^2x+x^3\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/12*(3*x^4*arctan(c/x) + (3*c^3*arctan(x/c) - 3*c^2*x + x^3)*c
)*b
```

Fricas [A]
time = 1.87, size = 41, normalized size = 0.82

$$-\frac{1}{4}\,bc^3x+\frac{1}{12}\,bcx^3+\frac{1}{4}\,ax^4-\frac{1}{4}\left(bc^4-bx^4\right)\arctan\left(\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x)),x, algorithm="fricas")
```

```
[Out] -1/4*b*c^3*x + 1/12*b*c*x^3 + 1/4*a*x^4 - 1/4*(b*c^4 - b*x^4)*arctan(c/x)
```

Sympy [A]
time = 0.15, size = 46, normalized size = 0.92

$$\frac{ax^4}{4}-\frac{bc^4\operatorname{atan}\left(\frac{c}{x}\right)}{4}-\frac{bc^3x}{4}+\frac{bcx^3}{12}+\frac{bx^4\operatorname{atan}\left(\frac{c}{x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c/x)),x)
```

```
[Out] a*x**4/4 - b*c**4*atan(c/x)/4 - b*c**3*x/4 + b*c*x**3/12 + b*x**4*atan(c/x)
/4
```

Giac [C] Result contains complex when optimal does not.
time = 0.45, size = 81, normalized size = 1.62

$$\frac{\left(6\,bc^5\arctan\left(\frac{c}{x}\right)-\frac{3i\,bc^9\log\left(\frac{i\,c}{x}-1\right)}{x^4}+\frac{3i\,bc^9\log\left(-\frac{i\,c}{x}-1\right)}{x^4}+6\,ac^5-\frac{6\,bc^8}{x^3}+\frac{2\,bc^6}{x}\right)x^4}{24\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x)),x, algorithm="giac")
```

```
[Out] 1/24*(6*b*c^5*arctan(c/x) - 3*I*b*c^9*log(I*c/x - 1)/x^4 + 3*I*b*c^9*log(-I
*c/x - 1)/x^4 + 6*a*c^5 - 6*b*c^8/x^3 + 2*b*c^6/x)*x^4/c^5
```

Mupad [B]
time = 0.41, size = 45, normalized size = 0.90

$$\frac{a\,x^4}{4}-\frac{b\,c^4\,\mathrm{atan}\left(\frac{c}{x}\right)}{4}+\frac{b\,x^4\,\mathrm{atan}\left(\frac{c}{x}\right)}{4}+\frac{b\,c\,x^3}{12}-\frac{b\,c^3\,x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c/x)),x)
[Out] (a*x^4)/4 - (b*c^4*atan(c/x))/4 + (b*x^4*atan(c/x))/4 + (b*c*x^3)/12 - (b*c
^3*x)/4
```

3.133 $\int x^2\left(a+b\text{ArcTan}\left(\frac{c}{x}\right)\right)\,dx$

Optimal. Leaf size=43

$$\frac{1}{6}bcx^2+\frac{1}{3}x^3\left(a+b\text{ArcTan}\left(\frac{c}{x}\right)\right)-\frac{1}{6}bc^3\log\left(c^2+x^2\right)$$

```
[Out] 1/6*b*c*x^2+1/3*x^3*(a+b*arctan(c/x))-1/6*b*c^3*ln(c^2+x^2)
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4946, 269, 272, 45}

$$\frac{1}{3}x^3\left(a+b\text{ArcTan}\left(\frac{c}{x}\right)\right)-\frac{1}{6}bc^3\log\left(c^2+x^2\right)+\frac{1}{6}bcx^2$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c/x]),x]
```

```
[Out] (b*c*x^2)/6 + (x^3*(a + b*ArcTan[c/x]))/3 - (b*c^3*Log[c^2 + x^2])/6
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)\,dx &= \frac{1}{3}x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{3}(bc)\int\frac{x}{1+\frac{c^2}{x^2}}\,dx\\ &= \frac{1}{3}x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{3}(bc)\int\frac{x^3}{c^2+x^2}\,dx\\ &= \frac{1}{3}x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{6}(bc)\text{Subst}\left(\int\frac{x}{c^2+x}\,dx,x,x^2\right)\\ &= \frac{1}{3}x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{6}(bc)\text{Subst}\left(\int\left(1-\frac{c^2}{c^2+x}\right)\,dx,x,x^2\right)\\ &= \frac{1}{6}bcx^2+\frac{1}{3}x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{6}bc^3\log\left(c^2+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 1.12

$$\frac{1}{6}bcx^2+\frac{ax^3}{3}+\frac{1}{3}bx^3\text{ArcTan}\left(\frac{c}{x}\right)-\frac{1}{6}bc^3\log\left(c^2+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTan[c/x]),x]
[Out] (b*c*x^2)/6 + (a*x^3)/3 + (b*x^3*ArcTan[c/x])/3 - (b*c^3*Log[c^2 + x^2])/6
```

Maple [A]
time = 0.05, size = 62, normalized size = 1.44

method	result	size
derivativedivides	$-c^3\left(-\frac{a\,x^3}{3c^3}-\frac{b\,x^3\arctan\left(\frac{c}{x}\right)}{3c^3}+\frac{b\ln\left(1+\frac{c^2}{x^2}\right)}{6}-\frac{b\,x^2}{6c^2}-\frac{b\ln\left(\frac{c}{x}\right)}{3}\right)$	62
default	$-c^3\left(-\frac{a\,x^3}{3c^3}-\frac{b\,x^3\arctan\left(\frac{c}{x}\right)}{3c^3}+\frac{b\ln\left(1+\frac{c^2}{x^2}\right)}{6}-\frac{b\,x^2}{6c^2}-\frac{b\ln\left(\frac{c}{x}\right)}{3}\right)$	62
risch	Expression too large to display	692

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c/x)),x,method=_RETURNVERBOSE)
[Out] -c^3*(-1/3*a/c^3*x^3-1/3*b/c^3*x^3*arctan(c/x)+1/6*b*ln(1+c^2/x^2)-1/6*b/c^
2*x^2-1/3*b*ln(c/x))
```

Maxima [A]
time = 0.26, size = 43, normalized size = 1.00

$$\frac{1}{3}\,ax^3+\frac{1}{6}\left(2\,x^3\arctan\left(\frac{c}{x}\right)-\left(c^2\log\left(c^2+x^2\right)-x^2\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/6*(2*x^3*arctan(c/x) - (c^2*log(c^2 + x^2) - x^2)*c)*b
```

Fricas [A]
time = 0.80, size = 40, normalized size = 0.93

$$\frac{1}{3}\,bx^3\arctan\left(\frac{c}{x}\right)-\frac{1}{6}\,bc^3\log\left(c^2+x^2\right)+\frac{1}{6}\,bcx^2+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x)),x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3*arctan(c/x) - 1/6*b*c^3*log(c^2 + x^2) + 1/6*b*c*x^2 + 1/3*a*x^3
```

Sympy [A]
time = 0.13, size = 41, normalized size = 0.95

$$\frac{ax^3}{3}-\frac{bc^3\log\left(c^2+x^2\right)}{6}+\frac{bcx^2}{6}+\frac{bx^3\operatorname{atan}\left(\frac{c}{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c/x)),x)
```

```
[Out] a*x**3/3 - b*c**3*log(c**2 + x**2)/6 + b*c*x**2/6 + b*x**3*atan(c/x)/3
```

Giac [A]
time = 0.41, size = 69, normalized size = 1.60

$$\frac{\left(2\,bc^4\arctan\left(\frac{c}{x}\right)-\frac{bc^7\log\left(\frac{c^2}{x^2}+1\right)}{x^3}+\frac{2\,bc^7\log\left(\frac{c}{x}\right)}{x^3}+2\,ac^4+\frac{bc^5}{x}\right)x^3}{6\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x)),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*c^4*arctan(c/x) - b*c^7*log(c^2/x^2 + 1)/x^3 + 2*b*c^7*log(c/x)/x^
3 + 2*a*c^4 + b*c^5/x)*x^3/c^4
```

Mupad [B]
time = 0.35, size = 40, normalized size = 0.93

$$\frac{a\,x^3}{3}+\frac{b\,x^3\,\mathrm{atan}\left(\frac{c}{x}\right)}{3}-\frac{b\,c^3\,\ln\left(c^2+x^2\right)}{6}+\frac{b\,c\,x^2}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c/x)),x)
```

```
[Out] (a*x^3)/3 + (b*x^3*atan(c/x))/3 - (b*c^3*log(c^2 + x^2))/6 + (b*c*x^2)/6
```

3.134 $\int x\left(a+b\text{ArcTan}\left(\frac{c}{x}\right)\right)\,dx$

Optimal. Leaf size=39

$$\frac{bcx}{2}+\frac{1}{2}x^2\left(a+b\text{ArcTan}\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\text{ArcTan}\left(\frac{x}{c}\right)$$

```
[Out] 1/2*b*c*x+1/2*x^2*(a+b*arctan(c/x))-1/2*b*c^2*arctan(x/c)
```

Rubi [A]
time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {4946, 199, 327, 209}

$$\frac{1}{2}x^2\left(a+b\text{ArcTan}\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\text{ArcTan}\left(\frac{x}{c}\right)+\frac{bcx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c/x]),x]
```

```
[Out] (b*c*x)/2 + (x^2*(a + b*ArcTan[c/x]))/2 - (b*c^2*ArcTan[x/c])/2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && LtQ[n, 0] && IntegerQ[p]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
```

```
IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)\,dx &= \frac{1}{2}x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{2}(bc)\int\frac{1}{1+\frac{c^2}{x^2}}\,dx\\
&= \frac{1}{2}x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)+\frac{1}{2}(bc)\int\frac{x^2}{c^2+x^2}\,dx\\
&= \frac{bcx}{2}+\frac{1}{2}x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{2}\left(bc^3\right)\int\frac{1}{c^2+x^2}\,dx\\
&= \frac{bcx}{2}+\frac{1}{2}x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\tan^{-1}\left(\frac{x}{c}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 1.13

$$\frac{bcx}{2}+\frac{ax^2}{2}+\frac{1}{2}bc^2\text{ArcTan}\left(\frac{c}{x}\right)+\frac{1}{2}bx^2\text{ArcTan}\left(\frac{c}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c/x]),x]
[Out] (b*c*x)/2 + (a*x^2)/2 + (b*c^2*ArcTan[c/x])/2 + (b*x^2*ArcTan[c/x])/2
```

Maple [A]
time = 0.06, size = 47, normalized size = 1.21

method	result	size
derivativedivides	$-c^2\left(-\frac{a\,x^2}{2c^2}-\frac{b\,x^2\arctan\left(\frac{c}{x}\right)}{2c^2}-\frac{b\arctan\left(\frac{c}{x}\right)}{2}-\frac{bx}{2c}\right)$	47
default	$-c^2\left(-\frac{a\,x^2}{2c^2}-\frac{b\,x^2\arctan\left(\frac{c}{x}\right)}{2c^2}-\frac{b\arctan\left(\frac{c}{x}\right)}{2}-\frac{bx}{2c}\right)$	47
risch	Expression too large to display	688

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c/x)),x,method=_RETURNVERBOSE)
[Out] -c^2*(-1/2*a/c^2*x^2-1/2*b/c^2*x^2*arctan(c/x)-1/2*b*arctan(c/x)-1/2*b*x/c)
```

Maxima [A]
time = 0.47, size = 36, normalized size = 0.92

$$\frac{1}{2}\,ax^2+\frac{1}{2}\left(x^2\arctan\left(\frac{c}{x}\right)-\left(c\arctan\left(\frac{x}{c}\right)-x\right)c\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/2*(x^2*arctan(c/x) - (c*arctan(x/c) - x)*c)*b
```

Fricas [A]
time = 0.66, size = 31, normalized size = 0.79

$$\frac{1}{2}\,bcx+\frac{1}{2}\,ax^2+\frac{1}{2}\,\left(bc^2+bx^2\right)\arctan\left(\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x)),x, algorithm="fricas")
```

```
[Out] 1/2*b*c*x + 1/2*a*x^2 + 1/2*(b*c^2 + b*x^2)*arctan(c/x)
```

Sympy [A]
time = 0.11, size = 36, normalized size = 0.92

$$\frac{ax^2}{2}+\frac{bc^2\operatorname{atan}\left(\frac{c}{x}\right)}{2}+\frac{bcx}{2}+\frac{bx^2\operatorname{atan}\left(\frac{c}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c/x)),x)
```

```
[Out] a*x**2/2 + b*c**2*atan(c/x)/2 + b*c*x/2 + b*x**2*atan(c/x)/2
```

Giac [C] Result contains complex when optimal does not.
time = 0.43, size = 72, normalized size = 1.85

$$\frac{\left(2\,bc^3\arctan\left(\frac{c}{x}\right)-\frac{i\,bc^5\log\left(\frac{i\,c}{x}+1\right)}{x^2}+\frac{i\,bc^5\log\left(-\frac{i\,c}{x}+1\right)}{x^2}+2\,ac^3+\frac{2\,bc^4}{x}\right)x^2}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x)),x, algorithm="giac")
```

```
[Out] 1/4*(2*b*c^3*arctan(c/x) - I*b*c^5*log(I*c/x + 1)/x^2 + I*b*c^5*log(-I*c/x
+ 1)/x^2 + 2*a*c^3 + 2*b*c^4/x)*x^2/c^3
```

Mupad [B]
time = 0.34, size = 36, normalized size = 0.92

$$\frac{a\,x^2}{2}+\frac{b\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}+\frac{b\,x^2\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}+\frac{b\,c\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c/x)),x)
```

```
[Out] (a*x^2)/2 + (b*c^2*atan(c/x))/2 + (b*x^2*atan(c/x))/2 + (b*c*x)/2
```

3.135 $\int \left(a+b\textbf{ArcTan}\left(\frac{c}{x}\right)\right)\,dx$

Optimal. Leaf size=27

$$ax+bx\text{ArcTan}\left(\frac{c}{x}\right)+\frac{1}{2}bc\log\left(c^2+x^2\right)$$

```
[Out] a*x+b*x*arctan(c/x)+1/2*b*c*ln(c^2+x^2)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {4930, 269, 266}

$$ax+bx\text{ArcTan}\left(\frac{c}{x}\right)+\frac{1}{2}bc\log\left(c^2+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcTan[c/x],x]
```

```
[Out] a*x + b*x*ArcTan[c/x] + (b*c*Log[c^2 + x^2])/2
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\int \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)dx = ax+b\int \tan^{-1}\left(\frac{c}{x}\right)dx$$
$$= ax+bx\tan^{-1}\left(\frac{c}{x}\right)+(bc)\int \frac{1}{\left(1+\frac{c^2}{x^2}\right)x}dx$$
$$= ax+bx\tan^{-1}\left(\frac{c}{x}\right)+(bc)\int \frac{x}{c^2+x^2}dx$$
$$= ax+bx\tan^{-1}\left(\frac{c}{x}\right)+\frac{1}{2}bc\log\left(c^2+x^2\right)$$

Mathematica [A]
time = 0.00, size = 27, normalized size = 1.00

$$ax+bx\text{ArcTan}\left(\frac{c}{x}\right)+\frac{1}{2}bc\log\left(c^2+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcTan[c/x],x]
[Out] a*x + b*x*ArcTan[c/x] + (b*c*Log[c^2 + x^2])/2
```

Maple [A]
time = 0.07, size = 38, normalized size = 1.41

method	result	size
default	$ax+bx\arctan\left(\frac{c}{x}\right)+\frac{bc\ln\left(1+\frac{c^2}{x^2}\right)}{2}-bc\ln\left(\frac{c}{x}\right)$	38
derivativedivides	$-c\left(-\frac{ax}{c}-\frac{bx\arctan\left(\frac{c}{x}\right)}{c}-\frac{b\ln\left(1+\frac{c^2}{x^2}\right)}{2}+b\ln\left(\frac{c}{x}\right)\right)$	46
risch	Expression too large to display	642

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arctan(c/x),x,method=_RETURNVERBOSE)
[Out] a*x+b*x*arctan(c/x)+1/2*b*c*ln(1+c^2/x^2)-b*c*ln(c/x)
```

Maxima [A]
time = 0.27, size = 27, normalized size = 1.00

$$\frac{1}{2}\left(2x\arctan\left(\frac{c}{x}\right)+c\log\left(c^2+x^2\right)\right)b+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c/x),x, algorithm="maxima")
```

```
[Out] 1/2*(2*x*arctan(c/x) + c*log(c^2 + x^2))*b + a*x
```

Fricas [A]
time = 0.76, size = 25, normalized size = 0.93

$$bx\arctan\left(\frac{c}{x}\right)+\frac{1}{2}bc\log\left(c^2+x^2\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c/x),x, algorithm="fricas")
```

```
[Out] b*x*arctan(c/x) + 1/2*b*c*log(c^2 + x^2) + a*x
```

Sympy [A]
time = 0.08, size = 22, normalized size = 0.81

$$ax+b\left(\frac{c\log\left(c^2+x^2\right)}{2}+x\operatorname{atan}\left(\frac{c}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*atan(c/x),x)
```

```
[Out] a*x + b*(c*log(c**2 + x**2)/2 + x*atan(c/x))
```

Giac [A]
time = 0.45, size = 46, normalized size = 1.70

$$ax+\frac{\left(c^2\left(\log\left(\frac{c^2}{x^2}+1\right)-\log\left(\frac{c^2}{x^2}\right)\right)+2\,cx\arctan\left(\frac{c}{x}\right)\right)b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arctan(c/x),x, algorithm="giac")
```

```
[Out] a*x + 1/2*(c^2*(log(c^2/x^2 + 1) - log(c^2/x^2)) + 2*c*x*arctan(c/x))*b/c
```

Mupad [B]
time = 0.30, size = 25, normalized size = 0.93

$$a\,x+b\,x\,\operatorname{atan}\left(\frac{c}{x}\right)+\frac{b\,c\,\ln\left(c^2+x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*atan(c/x),x)
```

```
[Out] a*x + b*x*atan(c/x) + (b*c*log(c^2 + x^2))/2
```

3.136 $\int \frac{a+b\text{ArcTan}\left(\frac{c}{x}\right)}{x}\,dx$

Optimal. Leaf size=39

$$a\log(x)-\frac{1}{2}ib\text{PolyLog}\left(2,-\frac{ic}{x}\right)+\frac{1}{2}ib\text{PolyLog}\left(2,\frac{ic}{x}\right)$$

```
[Out] a*ln(x)-1/2*I*b*polylog(2,-I*c/x)+1/2*I*b*polylog(2,I*c/x)
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4944, 4940, 2438}

$$a\log(x)-\frac{1}{2}ib\text{Li}_2\left(-\frac{ic}{x}\right)+\frac{1}{2}ib\text{Li}_2\left(\frac{ic}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])/x,x]
```

```
[Out] a*Log[x] - (I/2)*b*PolyLog[2, ((-I)*c)/x] + (I/2)*b*PolyLog[2, (I*c)/x]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x}\,dx = -\text{Subst}\left(\int \frac{a+b\tan^{-1}(cx)}{x}\,dx, x, \frac{1}{x}\right)$$
$$= a\log(x) - \frac{1}{2}(ib)\text{Subst}\left(\int \frac{\log(1-icx)}{x}\,dx, x, \frac{1}{x}\right) + \frac{1}{2}(ib)\text{Subst}\left(\int \frac{\log(1+icx)}{x}\,dx\right.$$
$$= a\log(x) - \frac{1}{2}ib\text{Li}_2\left(-\frac{ic}{x}\right) + \frac{1}{2}ib\text{Li}_2\left(\frac{ic}{x}\right)$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.00

$$a\log(x) - \frac{1}{2}ib\text{PolyLog}\left(2, -\frac{ic}{x}\right) + \frac{1}{2}ib\text{PolyLog}\left(2, \frac{ic}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])/x,x]
```

```
[Out] a*Log[x] - (I/2)*b*PolyLog[2, ((-I)*c)/x] + (I/2)*b*PolyLog[2, (I*c)/x]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(31) = 62.
time = 0.09, size = 94, normalized size = 2.41

method	result
derivativedivides	$-a\ln\left(\frac{c}{x}\right) - b\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right) - \frac{ib\ln\left(\frac{c}{x}\right)\ln\left(1+\frac{ic}{x}\right)}{2} + \frac{ib\ln\left(\frac{c}{x}\right)\ln\left(1-\frac{ic}{x}\right)}{2} - \frac{ib\,\text{dilog}\left(1+\frac{ic}{x}\right)}{2} + \frac{ib\,\text{dilog}}{}$
default	$-a\ln\left(\frac{c}{x}\right) - b\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right) - \frac{ib\ln\left(\frac{c}{x}\right)\ln\left(1+\frac{ic}{x}\right)}{2} + \frac{ib\ln\left(\frac{c}{x}\right)\ln\left(1-\frac{ic}{x}\right)}{2} - \frac{ib\,\text{dilog}\left(1+\frac{ic}{x}\right)}{2} + \frac{ib\,\text{dilog}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))/x,x,method=_RETURNVERBOSE)
```

```
[Out] -a*ln(c/x)-b*ln(c/x)*arctan(c/x)-1/2*I*b*ln(c/x)*ln(1+I*c/x)+1/2*I*b*ln(c/x
)*ln(1-I*c/x)-1/2*I*b*dilog(1+I*c/x)+1/2*I*b*dilog(1-I*c/x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x,x, algorithm="maxima")
```

```
[Out] b*integrate(arctan2(c, x)/x, x) + a*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arctan(c/x) + a)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{atan}\left(\frac{c}{x}\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))/x,x)
```

```
[Out] Integral((a + b*atan(c/x))/x, x)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(25) = 50$.
time = 0.43, size = 72, normalized size = 1.85

$$-\frac{\left(2\, bc^4 \arctan\left(\frac{c}{x}\right) - \frac{i\, bc^6 \log\left(\frac{i\, c}{x}+1\right)}{x^2} + \frac{i\, bc^6 \log\left(-\frac{i\, c}{x}+1\right)}{x^2} + 2\, ac^4 + \frac{2\, bc^5}{x}\right) x^2}{4\, c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x,x, algorithm="giac")
```

```
[Out] -1/4*(2*b*c^4*arctan(c/x) - I*b*c^6*log(I*c/x + 1)/x^2 + I*b*c^6*log(-I*c/x
 + 1)/x^2 + 2*a*c^4 + 2*b*c^5/x)*x^2/c^5
```

Mupad [B]
time = 0.34, size = 32, normalized size = 0.82

$$a\, \ln\left(x\right) + \frac{b\, \left(\mathrm{Li}_2\left(1 - \frac{c\, 1\mathrm{i}}{x}\right) - \mathrm{Li}_2\left(1 + \frac{c\, 1\mathrm{i}}{x}\right)\right)\, 1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))/x,x)
```

```
[Out] (b*(dilog(1 - (c*1i)/x) - dilog((c*1i)/x + 1))*1i)/2 + a*log(x)
```

3.137 $\int \frac{a+b\textbf{ArcTan}\left(\frac{c}{x}\right)}{x^2}\,dx$

Optimal. Leaf size=34

$$-\frac{a+b\text{ArcTan}\left(\frac{c}{x}\right)}{x}+\frac{b\log\left(1+\frac{c^2}{x^2}\right)}{2c}$$

```
[Out] (-a-b*arctan(c/x))/x+1/2*b*ln(1+c^2/x^2)/c
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4946, 266}

$$\frac{b\log\left(\frac{c^2}{x^2}+1\right)}{2c}-\frac{a+b\text{ArcTan}\left(\frac{c}{x}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])/x^2,x]
```

```
[Out] -((a + b*ArcTan[c/x])/x) + (b*Log[1 + c^2/x^2])/(2*c)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x^2}\,dx &= -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x}-(bc)\int \frac{1}{\left(1+\frac{c^2}{x^2}\right)x^3}\,dx\\ &= -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x}+\frac{b\log\left(1+\frac{c^2}{x^2}\right)}{2c}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 1.09

$$-\frac{a}{x}-\frac{b\text{ArcTan}\left(\frac{c}{x}\right)}{x}+\frac{b\log\left(1+\frac{c^2}{x^2}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])/x^2,x]
[Out] -(a/x) - (b*ArcTan[c/x])/x + (b*Log[1 + c^2/x^2])/(2*c)
```

Maple [A]
time = 0.05, size = 38, normalized size = 1.12

method	result	size
derivativedivides	$-\frac{\frac{ca}{x}+\frac{bc\arctan\left(\frac{c}{x}\right)}{x}-\frac{b\ln\left(1+\frac{c^2}{x^2}\right)}{2}}{c}$	38
default	$-\frac{\frac{ca}{x}+\frac{bc\arctan\left(\frac{c}{x}\right)}{x}-\frac{b\ln\left(1+\frac{c^2}{x^2}\right)}{2}}{c}$	38
risch	Expression too large to display	652

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))/x^2,x,method=_RETURNVERBOSE)
[Out] -1/c*(c/x*a+b*c/x*arctan(c/x)-1/2*b*ln(1+c^2/x^2))
```

Maxima [A]
time = 0.27, size = 38, normalized size = 1.12

$$-\frac{b\left(\frac{2\,c\arctan\left(\frac{c}{x}\right)}{x}-\log\left(\frac{c^2}{x^2}+1\right)\right)}{2\,c}-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^2,x, algorithm="maxima")
[Out] -1/2*b*(2*c*arctan(c/x)/x - log(c^2/x^2 + 1))/c - a/x
```

Fricas [A]
time = 0.66, size = 41, normalized size = 1.21

$$-\frac{2\,bc\arctan\left(\frac{c}{x}\right)-bx\log\left(c^2+x^2\right)+2\,bx\log\left(x\right)+2\,ac}{2\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*c*arctan(c/x) - b*x*log(c^2 + x^2) + 2*b*x*log(x) + 2*a*c)/(c*x)
```

Sympy [A]
time = 0.30, size = 36, normalized size = 1.06

$$\begin{cases} -\frac{a}{x} - \frac{b\operatorname{atan}\left(\frac{c}{x}\right)}{x} - \frac{b\log\left(x\right)}{c} + \frac{b\log\left(c^2+x^2\right)}{2c} & \text{for}\ c \neq 0 \\ -\frac{a}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))/x**2,x)
```

```
[Out] Piecewise((-a/x - b*atan(c/x)/x - b*log(x)/c + b*log(c**2 + x**2)/(2*c), Ne
(c, 0)), (-a/x, True))
```

Giac [A]
time = 0.45, size = 39, normalized size = 1.15

$$-\frac{\frac{2\,bc\arctan\left(\frac{c}{x}\right)}{x} - b\log\left(\frac{c^2}{x^2}+1\right) + \frac{2\,ac}{x}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*c*arctan(c/x)/x - b*log(c^2/x^2 + 1) + 2*a*c/x)/c
```

Mupad [B]
time = 0.34, size = 43, normalized size = 1.26

$$\frac{\frac{b\,\ln\left(c^2+x^2\right)}{2} - b\,\ln\left(x\right)}{c} - \frac{a\,c + b\,c\,\mathrm{atan}\left(\frac{c}{x}\right)}{c\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))/x^2,x)
```

```
[Out] ((b*log(c^2 + x^2))/2 - b*log(x))/c - (a*c + b*c*atan(c/x))/(c*x)
```

3.138 $\int \frac{a+b\mathbf{ArcTan}\left(\frac{c}{x}\right)}{x^3}\,dx$

Optimal. Leaf size=43

$$\frac{b}{2cx}-\frac{a+b\text{ArcTan}\left(\frac{c}{x}\right)}{2x^2}+\frac{b\text{ArcTan}\left(\frac{x}{c}\right)}{2c^2}$$

```
[Out] 1/2*b/c/x+1/2*(-a-b*arctan(c/x))/x^2+1/2*b*arctan(x/c)/c^2
```

Rubi [A]
time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {4946, 269, 331, 209}

$$-\frac{a+b\text{ArcTan}\left(\frac{c}{x}\right)}{2x^2}+\frac{b\text{ArcTan}\left(\frac{x}{c}\right)}{2c^2}+\frac{b}{2cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])/x^3,x]
```

```
[Out] b/(2*c*x) - (a + b*ArcTan[c/x])/(2*x^2) + (b*ArcTan[x/c])/(2*c^2)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
```

```
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x^3}\,dx &= -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{2x^2}-\frac{1}{2}(bc)\int \frac{1}{\left(1+\frac{c^2}{x^2}\right)x^4}\,dx\\ &= -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{2x^2}-\frac{1}{2}(bc)\int \frac{1}{x^2\left(c^2+x^2\right)}\,dx\\ &= \frac{b}{2cx}-\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{2x^2}+\frac{b\int \frac{1}{c^2+x^2}\,dx}{2c}\\ &= \frac{b}{2cx}-\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{2x^2}+\frac{b\tan^{-1}\left(\frac{x}{c}\right)}{2c^2}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 48, normalized size = 1.12

$$-\frac{a}{2x^2}+\frac{b}{2cx}-\frac{b\text{ArcTan}\left(\frac{c}{x}\right)}{2x^2}+\frac{b\text{ArcTan}\left(\frac{x}{c}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])/x^3,x]
[Out] -1/2*a/x^2 + b/(2*c*x) - (b*ArcTan[c/x])/(2*x^2) + (b*ArcTan[x/c])/(2*c^2)
```

Maple [**A**]
time = 0.07, size = 47, normalized size = 1.09

method	result	size
derivativedivides	$-\frac{\frac{a\,c^2}{2x^2}+\frac{b\,c^2\arctan\left(\frac{c}{x}\right)}{2x^2}-\frac{bc}{2x}+\frac{b\arctan\left(\frac{c}{x}\right)}{2}}{c^2}$	47
default	$-\frac{\frac{a\,c^2}{2x^2}+\frac{b\,c^2\arctan\left(\frac{c}{x}\right)}{2x^2}-\frac{bc}{2x}+\frac{b\arctan\left(\frac{c}{x}\right)}{2}}{c^2}$	47
risch	Expression too large to display	709

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))/x^3,x,method=_RETURNVERBOSE)
[Out] -1/c^2*(1/2*a*c^2/x^2+1/2*b*c^2/x^2*arctan(c/x)-1/2*b*c/x+1/2*b*arctan(c/x)
)
```

Maxima [A]
time = 0.47, size = 42, normalized size = 0.98

$$\frac{1}{2}\left(c\left(\frac{\arctan\left(\frac{x}{c}\right)}{c^3}+\frac{1}{c^2x}\right)-\frac{\arctan\left(\frac{c}{x}\right)}{x^2}\right)b-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(c*(arctan(x/c)/c^3 + 1/(c^2*x)) - arctan(c/x)/x^2)*b - 1/2*a/x^2
```

Fricas [A]
time = 0.72, size = 37, normalized size = 0.86

$$-\frac{ac^2-bcx+(bc^2+bx^2)\arctan\left(\frac{c}{x}\right)}{2\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*c^2 - b*c*x + (b*c^2 + b*x^2)*arctan(c/x))/(c^2*x^2)
```

Sympy [A]
time = 0.33, size = 44, normalized size = 1.02

$$\begin{cases}-\frac{a}{2x^2}-\frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{2x^2}+\frac{b}{2cx}-\frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{2c^2} & \text{for } c\neq 0\\ -\frac{a}{2x^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))/x**3,x)
```

```
[Out] Piecewise((-a/(2*x**2) - b*atan(c/x)/(2*x**2) + b/(2*c*x) - b*atan(c/x)/(2*
c**2), Ne(c, 0)), (-a/(2*x**2), True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.45, size = 60, normalized size = 1.40

$$-\frac{i\left(-\frac{2i\,bc^2\arctan\left(\frac{c}{x}\right)}{x^2}+b\log\left(\frac{i\,c}{x}-1\right)-b\log\left(-\frac{i\,c}{x}-1\right)-\frac{2i\,ac^2}{x^2}+\frac{2i\,bc}{x}\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^3,x, algorithm="giac")
```

```
[Out] -1/4*I*(-2*I*b*c^2*arctan(c/x)/x^2 + b*log(I*c/x - 1) - b*log(-I*c/x - 1) -
 2*I*a*c^2/x^2 + 2*I*b*c/x)/c^2
```

Mupad [B]
time = 0.38, size = 50, normalized size = 1.16

$$\frac{b\,c\,\mathrm{atan}\left(\frac{x}{\sqrt{c^2}}\right)}{2\,{\left(c^2\right)}^{3/2}}-\frac{\frac{a\,c^2}{2}+\frac{b\,c^2\,\mathrm{atan}\left(\frac{c}{x}\right)}{2}-\frac{b\,c\,x}{2}}{c^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))/x^3,x)
```

```
[Out] (b*c*atan(x/(c^2)^(1/2)))/(2*(c^2)^(3/2)) - ((a*c^2)/2 + (b*c^2*atan(c/x))/
2 - (b*c*x)/2)/(c^2*x^2)
```

3.139 $\int \frac{a+b\text{ArcTan}\left(\frac{c}{x}\right)}{x^4}\,dx$

Optimal. Leaf size=55

$$\frac{b}{6cx^2}-\frac{a+b\text{ArcTan}\left(\frac{c}{x}\right)}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3}$$

```
[Out] 1/6*b/c/x^2+1/3*(-a-b*arctan(c/x))/x^3+1/3*b*ln(x)/c^3-1/6*b*ln(c^2+x^2)/c^
3
```

Rubi [A]
time = 0.03, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4946, 269, 272, 46}

$$-\frac{a+b\text{ArcTan}\left(\frac{c}{x}\right)}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3}+\frac{b}{6cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])/x^4,x]
```

```
[Out] b/(6*c*x^2) - (a + b*ArcTan[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) - (b*Log[c^2
 + x^2])/(6*c^3)
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 269

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
```

```
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{x^4}\,dx &= -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{3}(bc)\int \frac{1}{\left(1+\frac{c^2}{x^2}\right)x^5}\,dx\\ &= -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{3}(bc)\int \frac{1}{x^3\left(c^2+x^2\right)}\,dx\\ &= -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{6}(bc)\text{Subst}\left(\int \frac{1}{x^2\left(c^2+x\right)}\,dx,x,x^2\right)\\ &= -\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{6}(bc)\text{Subst}\left(\int \left(\frac{1}{c^2x^2}-\frac{1}{c^4x}+\frac{1}{c^4\left(c^2+x\right)}\right)\,dx,x,x^2\right)\\ &= \frac{b}{6cx^2}-\frac{a+b\tan^{-1}\left(\frac{c}{x}\right)}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 60, normalized size = 1.09

$$-\frac{a}{3x^3}+\frac{b}{6cx^2}-\frac{b\text{ArcTan}\left(\frac{c}{x}\right)}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])/x^4,x]
[Out] -1/3*a/x^3 + b/(6*c*x^2) - (b*ArcTan[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) - (
b*Log[c^2 + x^2])/(6*c^3)
```

Maple [A]
time = 0.08, size = 53, normalized size = 0.96

method	result	size
derivativedivides	$-\frac{\frac{a\,c^3}{3x^3}+\frac{b\,c^3\arctan\left(\frac{c}{x}\right)}{3x^3}-\frac{b\,c^2}{6x^2}+\frac{b\ln\left(1+\frac{c^2}{x^2}\right)}{6}}{c^3}$	53
default	$-\frac{\frac{a\,c^3}{3x^3}+\frac{b\,c^3\arctan\left(\frac{c}{x}\right)}{3x^3}-\frac{b\,c^2}{6x^2}+\frac{b\ln\left(1+\frac{c^2}{x^2}\right)}{6}}{c^3}$	53
risch	Expression too large to display	705

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^3*(1/3*a*c^3/x^3+1/3*b*c^3/x^3*arctan(c/x)-1/6*b*c^2/x^2+1/6*b*ln(1+c^
2/x^2))
```

Maxima [A]
time = 0.27, size = 54, normalized size = 0.98

$$-\frac{1}{6}\left(c\left(\frac{\log\left(c^2+x^2\right)}{c^4}-\frac{\log\left(x^2\right)}{c^4}-\frac{1}{c^2x^2}\right)+\frac{2\arctan\left(\frac{c}{x}\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*(c*(log(c^2 + x^2)/c^4 - log(x^2)/c^4 - 1/(c^2*x^2)) + 2*arctan(c/x)/x
^3)*b - 1/3*a/x^3
```

Fricas [A]
time = 0.70, size = 55, normalized size = 1.00

$$-\frac{2\,bc^3\arctan\left(\frac{c}{x}\right)+bx^3\log\left(c^2+x^2\right)-2\,bx^3\log\left(x\right)+2\,ac^3-bc^2x}{6\,c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*c^3*arctan(c/x) + b*x^3*log(c^2 + x^2) - 2*b*x^3*log(x) + 2*a*c^3
 - b*c^2*x)/(c^3*x^3)
```

Sympy [A]
time = 0.41, size = 60, normalized size = 1.09

$$\begin{cases}-\frac{a}{3x^3}-\frac{b\operatorname{atan}\left(\frac{c}{x}\right)}{3x^3}+\frac{b}{6cx^2}+\frac{b\log\left(x\right)}{3c^3}-\frac{b\log\left(c^2+x^2\right)}{6c^3} & \text{for}\ c\neq 0\\ -\frac{a}{3x^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))/x**4,x)
```

```
[Out] Piecewise((-a/(3*x**3) - b*atan(c/x)/(3*x**3) + b/(6*c*x**2) + b*log(x)/(3*
c**3) - b*log(c**2 + x**2)/(6*c**3), Ne(c, 0)), (-a/(3*x**3), True))
```

Giac [A]
time = 0.44, size = 51, normalized size = 0.93

$$-\frac{\frac{2\,bc^3\arctan\left(\frac{c}{x}\right)}{x^3}+b\log\left(\frac{c^2}{x^2}+1\right)+\frac{2\,ac^3}{x^3}-\frac{bc^2}{x^2}}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(2*b*c^3*arctan(c/x)/x^3 + b*log(c^2/x^2 + 1) + 2*a*c^3/x^3 - b*c^2/x^
2)/c^3
```

Mupad [B]
time = 0.37, size = 56, normalized size = 1.02

$$\frac{\frac{b\,x^3\,\ln(x)}{3}+\frac{b\,c^2\,x}{6}-\frac{b\,x^3\,\ln\left(c^2+x^2\right)}{6}}{c^3\,x^3}-\frac{\frac{a}{3}+\frac{b\,\mathrm{atan}\left(\frac{c}{x}\right)}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))/x^4,x)
```

```
[Out] ((b*x^3*log(x))/3 + (b*c^2*x)/6 - (b*x^3*log(c^2 + x^2))/6)/(c^3*x^3) - (a/
3 + (b*atan(c/x))/3)/x^3
```

3.140 $\int x^3\left(a+b\textbf{ArcTan}\left(\frac{c}{x}\right)\right)^2\,dx$

Optimal. Leaf size=122

$$\frac{1}{12}b^2c^2x^2-\frac{1}{2}bc^3x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{6}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{4}c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right.$$

```
[Out] 1/12*b^2*c^2*x^2-1/2*b*c^3*x*(a+b*arccot(x/c))+1/6*b*c*x^3*(a+b*arccot(x/c)
)-1/4*c^4*(a+b*arccot(x/c))^2+1/4*x^4*(a+b*arccot(x/c))^2-1/3*b^2*c^4*ln(1+
c^2/x^2)-2/3*b^2*c^4*ln(x)
```

Rubi [A]
time = 0.18, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$,
Rules used = {4948, 4946, 5038, 272, 46, 36, 29, 31, 5004}

$$-\frac{1}{4}c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{1}{2}bc^3x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{6}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-\frac{2}{3}b^2c^4\log(x)+\frac{1}{12}b^2c^2x^2-\frac{1}{3}b^2c^4\log\left(\frac{c^2}{x^2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] (b^2*c^2*x^2)/12 - (b*c^3*x*(a + b*ArcCot[x/c]))/2 + (b*c*x^3*(a + b*ArcCot
[x/c]))/6 - (c^4*(a + b*ArcCot[x/c])^2)/4 + (x^4*(a + b*ArcCot[x/c])^2)/4 -
 (b^2*c^4*Log[1 + c^2/x^2])/3 - (2*b^2*c^4*Log[x])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2\,dx=\int\left(\frac{1}{4}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{2}bx^3\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\right.\right.$$

$$=\frac{1}{4}\int x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2\,dx+\frac{1}{2}b\int x^3\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log$$

$$=-\left(\frac{1}{4}\mathrm{Subst}\left(\int\frac{(2a+ib\log(1-icx))^2}{x^5}\,dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}b\int\left(-2iax^3\log\left(1+\right.\right.$$

$$=\frac{1}{16}x^4\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{16}b^2x^4\log^2\left(1+\frac{ic}{x}\right)-(iab)\int x^3\log\left(1+\right.$$

$$=\frac{1}{16}x^4\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{4}iabx^4\log\left(1+\frac{ic}{x}\right)+\frac{1}{8}b^2x^4\log\left(1-\frac{ic}{x}\right)$$

$$=\frac{1}{16}x^4\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{4}iabx^4\log\left(1+\frac{ic}{x}\right)+\frac{1}{8}b^2x^4\log\left(1-\frac{ic}{x}\right)$$

$$=\frac{1}{24}bcx^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)+\frac{1}{16}x^4\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{8}ib^2c^3x\,l$$

$$=-\frac{1}{4}abc^3x-\frac{1}{8}iabc^2x^2+\frac{1}{12}abcx^3+\frac{1}{16}ibc^2x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)+\frac{1}{24}bc$$

$$=-\frac{1}{4}abc^3x-\frac{1}{16}ib^2c^3x-\frac{1}{8}iabc^2x^2+\frac{1}{24}b^2c^2x^2+\frac{1}{12}abcx^3-\frac{11}{48}b^2c^4\log\left(i-\frac{c}{x}\right.$$

$$=-\frac{1}{4}abc^3x-\frac{1}{8}iabc^2x^2+\frac{1}{24}b^2c^2x^2+\frac{1}{12}abcx^3-\frac{11}{48}b^2c^4\log\left(i-\frac{c}{x}\right)-\frac{5}{48}b^2c^4\,l$$

$$=-\frac{1}{4}abc^3x-\frac{1}{8}iabc^2x^2+\frac{1}{24}b^2c^2x^2+\frac{1}{12}abcx^3-\frac{11}{48}b^2c^4\log\left(i-\frac{c}{x}\right)-\frac{5}{48}b^2c^4\,l$$

$$=-\frac{1}{4}abc^3x-\frac{1}{8}iabc^2x^2+\frac{1}{12}b^2c^2x^2+\frac{1}{12}abcx^3-\frac{11}{48}b^2c^4\log\left(i-\frac{c}{x}\right)-\frac{5}{48}b^2c^4\,l$$

$$=-\frac{1}{4}abc^3x-\frac{1}{8}iabc^2x^2+\frac{1}{12}b^2c^2x^2+\frac{1}{12}abcx^3-\frac{11}{48}b^2c^4\log\left(i-\frac{c}{x}\right)-\frac{5}{48}b^2c^4\,l$$

$$=-\frac{1}{4}abc^3x-\frac{1}{8}iabc^2x^2+\frac{1}{12}b^2c^2x^2+\frac{1}{12}abcx^3-\frac{11}{48}b^2c^4\log\left(i-\frac{c}{x}\right)-\frac{5}{48}b^2c^4\,l$$

Mathematica [A]
time = 0.05, size = 111, normalized size = 0.91

$$\frac{1}{12}\left(x\left(b^2c^2x+3a^2x^3+2abc\left(-3c^2+x^2\right)\right)+2b\left(bcx\left(-3c^2+x^2\right)+3a\left(-c^4+x^4\right)\right)\mathrm{ArcTan}\left(\frac{c}{x}\right)+3b^2\left(-c^4+x^4\right)\mathrm{ArcTan}\left(\frac{c}{x}\right)^2-4b^2c^4\log\left(c^2+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] (x*(b^2*c^2*x + 3*a^2*x^3 + 2*a*b*c*(-3*c^2 + x^2)) + 2*b*(b*c*x*(-3*c^2 +
x^2) + 3*a*(-c^4 + x^4))*ArcTan[c/x] + 3*b^2*(-c^4 + x^4)*ArcTan[c/x]^2 - 4
*b^2*c^4*Log[c^2 + x^2])/12
```

Maple [A]
time = 0.42, size = 163, normalized size = 1.34

method	result
derivativedivides	$-c^4\left(-\frac{a^2x^4}{4c^4}-\frac{b^2x^4\arctan\left(\frac{c}{x}\right)^2}{4c^4}+\frac{b^2\arctan\left(\frac{c}{x}\right)^2}{4}-\frac{b^2\arctan\left(\frac{c}{x}\right)x^3}{6c^3}+\frac{b^2\arctan\left(\frac{c}{x}\right)x}{2c}+\frac{b^2\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\right.$
default	$-c^4\left(-\frac{a^2x^4}{4c^4}-\frac{b^2x^4\arctan\left(\frac{c}{x}\right)^2}{4c^4}+\frac{b^2\arctan\left(\frac{c}{x}\right)^2}{4}-\frac{b^2\arctan\left(\frac{c}{x}\right)x^3}{6c^3}+\frac{b^2\arctan\left(\frac{c}{x}\right)x}{2c}+\frac{b^2\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c/x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -c^4*(-1/4*a^2/c^4*x^4-1/4*b^2/c^4*x^4*arctan(c/x)^2+1/4*b^2*arctan(c/x)^2-
1/6*b^2*arctan(c/x)/c^3*x^3+1/2*b^2*arctan(c/x)/c*x+1/3*b^2*ln(1+c^2/x^2)-1
/12*b^2*x^2/c^2-2/3*b^2*ln(c/x)-1/2*a*b/c^4*x^4*arctan(c/x)+1/2*a*b*arctan(
c/x)-1/6*a*b*x^3/c^3+1/2*a*b*x/c)
```

Maxima [A]
time = 0.48, size = 134, normalized size = 1.10

$$\frac{1}{4}b^2x^4\arctan\left(\frac{c}{x}\right)^2+\frac{1}{4}a^2x^4+\frac{1}{6}\left(3x^4\arctan\left(\frac{c}{x}\right)+\left(3c^3\arctan\left(\frac{x}{c}\right)-3c^2x+x^3\right)c\right)ab+\frac{1}{12}\left(\left(3c^2\arctan\left(\frac{x}{c}\right)^2-4c^2\log\left(c^2+x^2\right)+x^2\right)c^2+2\left(3c^3\arctan\left(\frac{x}{c}\right)-3c^2x+x^3\right)c\arctan\left(\frac{c}{x}\right)\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4*arctan(c/x)^2 + 1/4*a^2*x^4 + 1/6*(3*x^4*arctan(c/x) + (3*c^3*a
rctan(x/c) - 3*c^2*x + x^3)*c)*a*b + 1/12*((3*c^2*arctan(x/c)^2 - 4*c^2*log
(c^2 + x^2) + x^2)*c^2 + 2*(3*c^3*arctan(x/c) - 3*c^2*x + x^3)*c*arctan(c/x
))*b^2
```

Fricas [A]
time = 1.88, size = 125, normalized size = 1.02

$$\frac{1}{2}abc^4\arctan\left(\frac{x}{c}\right)-\frac{1}{3}b^2c^4\log\left(c^2+x^2\right)-\frac{1}{2}abc^3x+\frac{1}{12}b^2c^2x^2+\frac{1}{6}abcx^3+\frac{1}{4}a^2x^4-\frac{1}{4}\left(b^2c^4-b^2x^4\right)\arctan\left(\frac{c}{x}\right)^2-\frac{1}{6}\left(3b^2c^3x-b^2cx^3-3abx^4\right)\arctan\left(\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*a*b*c^4*arctan(x/c) - 1/3*b^2*c^4*log(c^2 + x^2) - 1/2*a*b*c^3*x + 1/12
*b^2*c^2*x^2 + 1/6*a*b*c*x^3 + 1/4*a^2*x^4 - 1/4*(b^2*c^4 - b^2*x^4)*arctan
(c/x)^2 - 1/6*(3*b^2*c^3*x - b^2*c*x^3 - 3*a*b*x^4)*arctan(c/x)
```

Sympy [**A**]
time = 0.21, size = 144, normalized size = 1.18

$$\frac{a^2x^4}{4} - \frac{abc^4 \operatorname{atan}\left(\frac{c}{x}\right)}{2} - \frac{abc^3x}{2} + \frac{abcx^3}{6} + \frac{abx^4 \operatorname{atan}\left(\frac{c}{x}\right)}{2} - \frac{b^2c^4 \log\left(c^2+x^2\right)}{3} - \frac{b^2c^4 \operatorname{atan}^2\left(\frac{c}{x}\right)}{4} - \frac{b^2c^3x \operatorname{atan}\left(\frac{c}{x}\right)}{2} + \frac{b^2c^2x^2}{12} + \frac{b^2cx^3 \operatorname{atan}\left(\frac{c}{x}\right)}{6} + \frac{b^2x^4 \operatorname{atan}^2\left(\frac{c}{x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c/x))**2,x)
```

```
[Out] a**2*x**4/4 - a*b*c**4*atan(c/x)/2 - a*b*c**3*x/2 + a*b*c*x**3/6 + a*b*x**4
*atan(c/x)/2 - b**2*c**4*log(c**2 + x**2)/3 - b**2*c**4*atan(c/x)**2/4 - b*
*2*c**3*x*atan(c/x)/2 + b**2*c**2*x**2/12 + b**2*c*x**3*atan(c/x)/6 + b**2*
x**4*atan(c/x)**2/4
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^2*x^3, x)
```

Mupad [**B**]
time = 0.47, size = 140, normalized size = 1.15

$$\frac{a^2\,x^4}{4} - \frac{b^2\,c^4\,\operatorname{atan}\left(\frac{c}{x}\right)^2}{4} - \frac{b^2\,c^4\,\ln\left(c^2+x^2\right)}{3} + \frac{b^2\,x^4\,\operatorname{atan}\left(\frac{c}{x}\right)^2}{4} + \frac{b^2\,c^2\,x^2}{12} + \frac{b^2\,c\,x^3\,\operatorname{atan}\left(\frac{c}{x}\right)}{6} - \frac{b^2\,c^3\,x\,\operatorname{atan}\left(\frac{c}{x}\right)}{2} + \frac{a\,b\,c\,x^3}{6} - \frac{a\,b\,c^3\,x}{2} - \frac{a\,b\,c^4\,\operatorname{atan}\left(\frac{c}{x}\right)}{2} + \frac{a\,b\,x^4\,\operatorname{atan}\left(\frac{c}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c/x))^2,x)
```

```
[Out] (a^2*x^4)/4 - (b^2*c^4*atan(c/x)^2)/4 - (b^2*c^4*log(c^2 + x^2))/3 + (b^2*x
^4*atan(c/x)^2)/4 + (b^2*c^2*x^2)/12 + (b^2*c*x^3*atan(c/x))/6 - (b^2*c^3*x
*atan(c/x))/2 + (a*b*c*x^3)/6 - (a*b*c^3*x)/2 - (a*b*c^4*atan(c/x))/2 + (a*
b*x^4*atan(c/x))/2
```

3.141 $\int x^2\left(a+b\mathbf{ArcTan}\left(\frac{c}{x}\right)\right)^2\,dx$

Optimal. Leaf size=152

$$\frac{1}{3}b^2c^2x+\frac{1}{3}b^2c^3\cot^{-1}\left(\frac{x}{c}\right)+\frac{1}{3}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{2}{3}$$

```
[Out] 1/3*b^2*c^2*x+1/3*b^2*c^3*arccot(x/c)+1/3*b*c*x^2*(a+b*arccot(x/c))-1/3*I*c
^3*(a+b*arccot(x/c))^2+1/3*x^3*(a+b*arccot(x/c))^2+2/3*b*c^3*(a+b*arccot(x/
c))*ln(2-2/(1-I*c/x))-1/3*I*b^2*c^3*polylog(2,-1+2/(1-I*c/x))
```

Rubi [A]
time = 0.18, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {4948, 4946, 5038, 331, 209, 5044, 4988, 2497}

$$-\frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{2}{3}bc^3\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{3}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{3}ib^2c^3\text{Li}_2\left(\frac{2}{1-\frac{ic}{x}}-1\right)+\frac{1}{3}b^2c^3\cot^{-1}\left(\frac{x}{c}\right)+\frac{1}{3}b^2c^2x$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] (b^2*c^2*x)/3 + (b^2*c^3*ArcCot[x/c])/3 + (b*c*x^2*(a + b*ArcCot[x/c]))/3 -
 (I/3)*c^3*(a + b*ArcCot[x/c])^2 + (x^3*(a + b*ArcCot[x/c])^2)/3 + (2*b*c^3
*(a + b*ArcCot[x/c])*Log[2 - 2/(1 - (I*c)/x)])/3 - (I/3)*b^2*c^3*PolyLog[2,
 -1 + 2/(1 - (I*c)/x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\int x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2 dx = \int \left(\frac{1}{4}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{2}bx^2\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1\right.\right.$$

$$= \frac{1}{4}\int x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2 dx+\frac{1}{2}b\int x^2\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\text{lo}$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int\frac{(2a+ib\log(1-icx))^2}{x^4}dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}b\int\left(-2iax^2\log\left(1\right.\right.$$

$$= \frac{1}{12}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{12}b^2x^3\log^2\left(1+\frac{ic}{x}\right)-(iab)\int x^2\log\left(1\right.$$

$$= \frac{1}{12}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{3}iabx^3\log\left(1+\frac{ic}{x}\right)+\frac{1}{6}b^2x^3\log\left(1-\frac{ic}{x}\right.$$

$$= \frac{1}{12}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{3}iabx^3\log\left(1+\frac{ic}{x}\right)+\frac{1}{6}b^2x^3\log\left(1-\frac{ic}{x}\right.$$

$$= \frac{1}{12}bcx^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)+\frac{1}{12}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{6}b^2c^2x$$

$$= -\frac{1}{3}iabc^2x+\frac{1}{6}abcx^2+\frac{1}{6}ibc^2\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)+\frac{1}{12}bcx^2\left(\right.$$

$$= -\frac{1}{3}iabc^2x+\frac{1}{6}b^2c^2x+\frac{1}{6}abcx^2-\frac{1}{4}ib^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{12}ib^2c^3\log\left(1-\frac{ic}{x}\right)$$

$$= -\frac{1}{3}iabc^2x+\frac{1}{6}b^2c^2x+\frac{1}{6}abcx^2-\frac{1}{4}ib^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{12}ib^2c^3\log\left(1-\frac{ic}{x}\right)$$

$$= -\frac{1}{3}iabc^2x+\frac{1}{6}b^2c^2x+\frac{1}{6}abcx^2-\frac{1}{4}ib^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{12}ib^2c^3\log\left(1-\frac{ic}{x}\right)$$

$$= -\frac{1}{3}iabc^2x+\frac{1}{3}b^2c^2x+\frac{1}{6}abcx^2-\frac{1}{4}ib^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{12}ib^2c^3\log\left(1-\frac{ic}{x}\right)$$

$$= -\frac{1}{3}iabc^2x+\frac{1}{3}b^2c^2x+\frac{1}{6}abcx^2-\frac{1}{4}ib^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{12}ib^2c^3\log\left(1-\frac{ic}{x}\right)$$

$$= -\frac{1}{3}iabc^2x+\frac{1}{3}b^2c^2x+\frac{1}{6}abcx^2-\frac{1}{4}ib^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{12}ib^2c^3\log\left(1-\frac{ic}{x}\right)$$

Mathematica [A]
time = 0.24, size = 152, normalized size = 1.00

$$\frac{1}{3}\left(b^2c^2x+abcx^2+a^2x^3+b^2(-ic^3+x^3)\,\text{ArcTan}\left(\frac{c}{x}\right)^2+b\text{ArcTan}\left(\frac{c}{x}\right)\left(2ax^3+bc(c^2+x^2)+2bc^3\log\left(1-e^{2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)\right)-abc^3\log\left(1+\frac{c^2}{x^2}\right)+2abc^3\log\left(\frac{c}{x}\right)-ib^2c^3\text{PolyLog}\left(2,e^{2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] (b^2*c^2*x + a*b*c*x^2 + a^2*x^3 + b^2*((-I)*c^3 + x^3)*ArcTan[c/x]^2 + b*A
rcTan[c/x]*(2*a*x^3 + b*c*(c^2 + x^2) + 2*b*c^3*Log[1 - E^((2*I)*ArcTan[c/x
])]) - a*b*c^3*Log[1 + c^2/x^2] + 2*a*b*c^3*Log[c/x] - I*b^2*c^3*PolyLog[2,
 E^((2*I)*ArcTan[c/x])])/3
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(134) = 268$.
time = 0.38, size = 412, normalized size = 2.71

method	result
derivativedivides	$-c^3\left(-\frac{a^2x^3}{3c^3}-\frac{b^2x^3\arctan\left(\frac{c}{x}\right)^2}{3c^3}+\frac{b^2\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\frac{b^2\arctan\left(\frac{c}{x}\right)x^2}{3c^2}-\frac{2b^2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)}{3}+\frac{ib^2 \mathrm{d}}{}\right.$
default	$-c^3\left(-\frac{a^2x^3}{3c^3}-\frac{b^2x^3\arctan\left(\frac{c}{x}\right)^2}{3c^3}+\frac{b^2\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{3}-\frac{b^2\arctan\left(\frac{c}{x}\right)x^2}{3c^2}-\frac{2b^2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)}{3}+\frac{ib^2 \mathrm{d}}{}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c/x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -c^3*(-1/3*a^2/c^3*x^3-1/3*b^2/c^3*x^3*arctan(c/x)^2+1/3*b^2*arctan(c/x)*ln
(1+c^2/x^2)-1/3*b^2*arctan(c/x)/c^2*x^2-2/3*b^2*ln(c/x)*arctan(c/x)-1/6*I*b
^2*dilog(-1/2*I*(c/x+I))-1/6*I*b^2*ln(c/x-I)*ln(-1/2*I*(c/x+I))-1/12*I*b^2*
ln(c/x-I)^2+1/3*I*b^2*dilog(1-I*c/x)+1/6*I*b^2*dilog(1/2*I*(c/x-I))-1/6*I*b
^2*ln(c/x+I)*ln(1+c^2/x^2)+1/6*I*b^2*ln(c/x+I)*ln(1/2*I*(c/x-I))+1/6*I*b^2*
ln(c/x-I)*ln(1+c^2/x^2)-1/3*b^2*arctan(c/x)-1/3*b^2/c*x+1/12*I*b^2*ln(c/x+I
)^2+1/3*I*b^2*ln(c/x)*ln(1-I*c/x)-1/3*I*b^2*ln(c/x)*ln(1+I*c/x)-1/3*I*b^2*d
ilog(1+I*c/x)-2/3*a*b/c^3*x^3*arctan(c/x)+1/3*a*b*ln(1+c^2/x^2)-1/3*a*b/c^2
*x^2-2/3*a*b*ln(c/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/3*(2*x^3*arctan(c/x) - (c^2*log(c^2 + x^2) - x^2)*c)*a*b +
1/48*(4*x^3*arctan2(c, x)^2 - x^3*log(c^2 + x^2)^2 + 48*integrate(1/48*(36*
c^2*x^2*arctan2(c, x)^2 + 36*x^4*arctan2(c, x)^2 + 8*c*x^3*arctan2(c, x) +
```

```
4*x^4*log(c^2 + x^2) + 3*(c^2*x^2 + x^4)*log(c^2 + x^2)^2)/(c^2 + x^2), x))
*b^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x))^2,x, algorithm="fricas")
[Out] integral(b^2*x^2*arctan(c/x)^2 + 2*a*b*x^2*arctan(c/x) + a^2*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \operatorname{atan}\left(\frac{c}{x}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c/x))**2,x)
[Out] Integral(x**2*(a + b*atan(c/x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x))^2,x, algorithm="giac")
[Out] integrate((b*arctan(c/x) + a)^2*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(a + b \operatorname{atan}\left(\frac{c}{x}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c/x))^2,x)
[Out] int(x^2*(a + b*atan(c/x))^2, x)
```

3.142 $\int x\left(a+b\mathbf{ArcTan}\left(\frac{c}{x}\right)\right)^2\,dx$

Optimal. Leaf size=82

$$bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}b^2c^2\log\left(1+\frac{c^2}{x^2}\right)+b^2c^2\log(x)$$

```
[Out] b*c*x*(a+b*arccot(x/c))+1/2*c^2*(a+b*arccot(x/c))^2+1/2*x^2*(a+b*arccot(x/c
))^2+1/2*b^2*c^2*ln(1+c^2/x^2)+b^2*c^2*ln(x)
```

Rubi [A]
time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$,
Rules used = {4948, 4946, 5038, 272, 36, 29, 31, 5004}

$$\frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}b^2c^2\log\left(\frac{c^2}{x^2}+1\right)+b^2c^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] b*c*x*(a + b*ArcCot[x/c]) + (c^2*(a + b*ArcCot[x/c])^2)/2 + (x^2*(a + b*Arc
Cot[x/c])^2)/2 + (b^2*c^2*Log[1 + c^2/x^2])/2 + b^2*c^2*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int x\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2dx=\int\left(\frac{1}{4}x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{2}bx\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right.\right.$$

$$=\frac{1}{4}\int x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2dx+\frac{1}{2}b\int x\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1\right.$$

$$=-\left(\frac{1}{4}\text{Subst}\left(\int\frac{(2a+ib\log(1-icx))^2}{x^3}dx,x,\frac{1}{x}\right)\right)+\frac{1}{2}b\int\left(-2iax\log\left(1+\frac{i}{x}\right.\right.$$

$$=\frac{1}{8}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{8}b^2x^2\log^2\left(1+\frac{ic}{x}\right)-(iab)\int x\log\left(1+\frac{ic}{x}\right.$$

$$=\frac{1}{8}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{2}iabx^2\log\left(1+\frac{ic}{x}\right)+\frac{1}{4}b^2x^2\log\left(1-\frac{ic}{x}\right)\log$$

$$=\frac{1}{8}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{2}iabx^2\log\left(1+\frac{ic}{x}\right)+\frac{1}{4}b^2x^2\log\left(1-\frac{ic}{x}\right)\log$$

$$=\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)+\frac{1}{8}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-\frac{1}{4}i$$

$$=\frac{1}{2}abcx+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)+\frac{1}{8}c^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right.$$

$$=\frac{1}{2}abcx+\frac{1}{4}b^2c^2\log\left(i-\frac{c}{x}\right)+\frac{1}{4}ib^2cx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\right.$$

$$=\frac{1}{2}abcx+\frac{1}{4}b^2c^2\log\left(i-\frac{c}{x}\right)+\frac{1}{4}ib^2cx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\right.$$

$$=\frac{1}{2}abcx+\frac{1}{4}b^2c^2\log\left(i-\frac{c}{x}\right)+\frac{1}{4}ib^2cx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\right.$$

$$=\frac{1}{2}abcx+\frac{1}{4}b^2c^2\log\left(i-\frac{c}{x}\right)+\frac{1}{4}ib^2cx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\right.$$

$$=\frac{1}{2}abcx+\frac{1}{4}b^2c^2\log\left(i-\frac{c}{x}\right)+\frac{1}{4}ib^2cx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\right.$$

$$=\frac{1}{2}abcx+\frac{1}{4}b^2c^2\log\left(i-\frac{c}{x}\right)+\frac{1}{4}ib^2cx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\right.$$

Mathematica [A]
time = 0.03, size = 73, normalized size = 0.89

$$\frac{1}{2}\left(ax(2bc+ax)+2b\left(bcx+a\left(c^2+x^2\right)\right)\text{ArcTan}\left(\frac{c}{x}\right)+b^2\left(c^2+x^2\right)\text{ArcTan}\left(\frac{c}{x}\right)^2+b^2c^2\log\left(c^2+x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c/x])^2,x]
```

```
[Out] (a*x*(2*b*c + a*x) + 2*b*(b*c*x + a*(c^2 + x^2))*ArcTan[c/x] + b^2*(c^2 + x
^2)*ArcTan[c/x]^2 + b^2*c^2*Log[c^2 + x^2])/2
```

Maple [A]
time = 0.36, size = 124, normalized size = 1.51

method	result
derivativedivides	$-c^2\left(-\frac{a^2x^2}{2c^2}-\frac{b^2x^2\arctan\left(\frac{c}{x}\right)^2}{2c^2}-\frac{b^2\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{b^2\arctan\left(\frac{c}{x}\right)x}{c}-\frac{b^2\ln\left(1+\frac{c^2}{x^2}\right)}{2}+b^2\ln\left(\frac{c}{x}\right)-\frac{ab\,x^2}{}\right.$
default	$-c^2\left(-\frac{a^2x^2}{2c^2}-\frac{b^2x^2\arctan\left(\frac{c}{x}\right)^2}{2c^2}-\frac{b^2\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{b^2\arctan\left(\frac{c}{x}\right)x}{c}-\frac{b^2\ln\left(1+\frac{c^2}{x^2}\right)}{2}+b^2\ln\left(\frac{c}{x}\right)-\frac{ab\,x^2}{}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c/x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -c^2*(-1/2*a^2/c^2*x^2-1/2*b^2/c^2*x^2*arctan(c/x)^2-1/2*b^2*arctan(c/x)^2-
b^2*arctan(c/x)/c*x-1/2*b^2*ln(1+c^2/x^2)+b^2*ln(c/x)-a*b/c^2*x^2*arctan(c/
x)-a*b*arctan(c/x)-a*b*x/c)
```

Maxima [A]
time = 0.47, size = 104, normalized size = 1.27

$$\frac{1}{2}b^2x^2\arctan\left(\frac{c}{x}\right)^2+\frac{1}{2}a^2x^2+\left(x^2\arctan\left(\frac{c}{x}\right)-\left(c\arctan\left(\frac{x}{c}\right)-x\right)c\right)ab-\frac{1}{2}\left(\left(\arctan\left(\frac{x}{c}\right)^2-\log\left(c^2+x^2\right)\right)c^2+2\left(c\arctan\left(\frac{x}{c}\right)-x\right)c\arctan\left(\frac{c}{x}\right)\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arctan(c/x)^2 + 1/2*a^2*x^2 + (x^2*arctan(c/x) - (c*arctan(x/c)
 - x)*c)*a*b - 1/2*((arctan(x/c)^2 - log(c^2 + x^2))*c^2 + 2*(c*arctan(x/c)
 - x)*c*arctan(c/x))*b^2
```

Fricas [A]
time = 0.94, size = 88, normalized size = 1.07

$$-abc^2\arctan\left(\frac{x}{c}\right)+\frac{1}{2}b^2c^2\log\left(c^2+x^2\right)+abcx+\frac{1}{2}a^2x^2+\frac{1}{2}\left(b^2c^2+b^2x^2\right)\arctan\left(\frac{c}{x}\right)^2+\left(b^2cx+abx^2\right)\arctan\left(\frac{c}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x))^2,x, algorithm="fricas")
```

```
[Out] -a*b*c^2*arctan(x/c) + 1/2*b^2*c^2*log(c^2 + x^2) + a*b*c*x + 1/2*a^2*x^2 +
 1/2*(b^2*c^2 + b^2*x^2)*arctan(c/x)^2 + (b^2*c*x + a*b*x^2)*arctan(c/x)
```

Sympy [A]
time = 0.14, size = 97, normalized size = 1.18

$$\frac{a^2x^2}{2} + abc^2\operatorname{atan}\left(\frac{c}{x}\right) + abcx + abx^2\operatorname{atan}\left(\frac{c}{x}\right) + \frac{b^2c^2\log\left(c^2+x^2\right)}{2} + \frac{b^2c^2\operatorname{atan}^2\left(\frac{c}{x}\right)}{2} + b^2cx\operatorname{atan}\left(\frac{c}{x}\right) + \frac{b^2x^2\operatorname{atan}^2\left(\frac{c}{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c/x))**2,x)
```

```
[Out] a**2*x**2/2 + a*b*c**2*atan(c/x) + a*b*c*x + a*b*x**2*atan(c/x) + b**2*c**2
*log(c**2 + x**2)/2 + b**2*c**2*atan(c/x)**2/2 + b**2*c*x*atan(c/x) + b**2*
x**2*atan(c/x)**2/2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^2*x, x)
```

Mupad [B]
time = 0.40, size = 98, normalized size = 1.20

$$\frac{a^2\,x^2}{2} + \frac{b^2\,c^2\operatorname{atan}\left(\frac{c}{x}\right)^2}{2} + \frac{b^2\,c^2\,\ln\left(c^2+x^2\right)}{2} + \frac{b^2\,x^2\operatorname{atan}\left(\frac{c}{x}\right)^2}{2} + a\,b\,c^2\operatorname{atan}\left(\frac{c}{x}\right) + a\,b\,x^2\operatorname{atan}\left(\frac{c}{x}\right) + b^2\,c\,x\operatorname{atan}\left(\frac{c}{x}\right) + a\,b\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c/x))^2,x)
```

```
[Out] (a^2*x^2)/2 + (b^2*c^2*atan(c/x)^2)/2 + (b^2*c^2*log(c^2 + x^2))/2 + (b^2*x
^2*atan(c/x)^2)/2 + a*b*c^2*atan(c/x) + a*b*x^2*atan(c/x) + b^2*c*x*atan(c/
x) + a*b*c*x
```

3.143 $\int \left(a+b\mathbf{ArcTan}\left(\frac{c}{x}\right)\right)^2 dx$

Optimal. Leaf size=83

$$ic\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-2bc\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2c}{c+ix}\right)+ib^2c\text{PolyLog}\left(2,1-\frac{2c}{c+ix}\right)$$

```
[Out] I*c*(a+b*arccot(x/c))^2+x*(a+b*arccot(x/c))^2-2*b*c*(a+b*arccot(x/c))*ln(2*
c/(c+I*x))+I*b^2*c*polylog(2,1-2*c/(c+I*x))
```

Rubi [A]
time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4934, 4931, 5041, 4965, 2449, 2352}

$$ic\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-2bc\log\left(\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+ib^2c\text{Li}_2\left(1-\frac{2c}{c+ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])^2,x]
```

```
[Out] I*c*(a + b*ArcCot[x/c])^2 + x*(a + b*ArcCot[x/c])^2 - 2*b*c*(a + b*ArcCot[x
/c])*Log[(2*c)/(c + I*x)] + I*b^2*c*PolyLog[2, 1 - (2*c)/(c + I*x)]
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4934

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[(a + b*A
rcCot[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && ILtQ[n, 0]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2 dx = \int \left(a^2+iab\log\left(1-\frac{ic}{x}\right)-\frac{1}{4}b^2\log^2\left(1-\frac{ic}{x}\right)-iab\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}b^2\log\left(\right.\right.$$

$$= a^2x+(iab)\int\log\left(1-\frac{ic}{x}\right)dx-(iab)\int\log\left(1+\frac{ic}{x}\right)dx-\frac{1}{4}b^2\int\log^2\left(1-\right.$$

$$= a^2x+iabx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}b^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)-iabx\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}$$

$$= a^2x+iabx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}b^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)-iabx\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}$$

$$= a^2x+iabx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}b^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)-iabx\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}$$

$$= a^2x+iabx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}b^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)-iabx\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}$$

$$= a^2x+iabx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}b^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)-iabx\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}$$

$$= a^2x+iabx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}b^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)-iabx\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}$$

$$= a^2x+iabx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}b^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)-iabx\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}$$

$$= a^2x+iabx\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}b^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)-iabx\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}$$

Mathematica [A]
time = 0.07, size = 105, normalized size = 1.27

$$b^2(ic+x)\text{ArcTan}\left(\frac{c}{x}\right)^2+2b\text{ArcTan}\left(\frac{c}{x}\right)\left(ax-bc\log\left(1-e^{2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)\right)+a\left(ax+bc\log\left(1+\frac{c^2}{x^2}\right)-2bc\log\left(\frac{c}{x}\right)\right)+ib^2c\text{PolyLog}\left(2,e^{2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])^2,x]
```

```
[Out] b^2*(I*c + x)*ArcTan[c/x]^2 + 2*b*ArcTan[c/x]*(a*x - b*c*Log[1 - E^((2*I)*A
rcTan[c/x])]) + a*(a*x + b*c*Log[1 + c^2/x^2] - 2*b*c*Log[c/x]) + I*b^2*c*P
olyLog[2, E^((2*I)*ArcTan[c/x])]
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(79) = 158.
time = 0.21, size = 357, normalized size = 4.30

method	result
derivativedivides	$-c\left(-\frac{a^2x}{c}-\frac{b^2x\arctan\left(\frac{c}{x}\right)^2}{c}-b^2\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)+2b^2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)-\frac{ib^2\ln\left(\frac{c}{x}+\right.}{}\right.$
default	$-c\left(-\frac{a^2x}{c}-\frac{b^2x\arctan\left(\frac{c}{x}\right)^2}{c}-b^2\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)+2b^2\ln\left(\frac{c}{x}\right)\arctan\left(\frac{c}{x}\right)-\frac{ib^2\ln\left(\frac{c}{x}+\right.}{}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -c*(-a^2/c*x-b^2/c*x*arctan(c/x)^2-b^2*arctan(c/x)*ln(1+c^2/x^2)+2*b^2*ln(c
/x)*arctan(c/x)-1/2*I*b^2*ln(c/x+I)*ln(1/2*I*(c/x-I))+I*b^2*ln(c/x)*ln(1+I*
c/x)-1/2*I*b^2*dilog(1/2*I*(c/x-I))-I*b^2*ln(c/x)*ln(1-I*c/x)+1/4*I*b^2*ln(
c/x-I)^2-1/4*I*b^2*ln(c/x+I)^2-1/2*I*b^2*ln(c/x-I)*ln(1+c^2/x^2)+1/2*I*b^2*
ln(c/x+I)*ln(1+c^2/x^2)+I*b^2*dilog(1+I*c/x)-I*b^2*dilog(1-I*c/x)+1/2*I*b^2
*ln(c/x-I)*ln(-1/2*I*(c/x+I))+1/2*I*b^2*dilog(-1/2*I*(c/x+I))-2*a*b/c*x*arc
tan(c/x)-a*b*ln(1+c^2/x^2)+2*a*b*ln(c/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2,x, algorithm="maxima")
```

```
[Out] (2*x*arctan(c/x) + c*log(c^2 + x^2))*a*b + 1/16*(12*c*arctan(c/x)^2*arctan(
x/c) + 4*(3*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^3/c)*c^2 + 4*x*arctan
2(c, x)^2 + 16*c^2*integrate(1/16*log(c^2 + x^2)^2/(c^2 + x^2), x) - x*log(
c^2 + x^2)^2 + 128*c*integrate(1/16*x*arctan(c/x)/(c^2 + x^2), x) + 192*int
```

```
egrate(1/16*x^2*arctan(c/x)^2/(c^2 + x^2), x) + 16*integrate(1/16*x^2*log(c
^2 + x^2)^2/(c^2 + x^2), x) + 64*integrate(1/16*x^2*log(c^2 + x^2)/(c^2 + x
^2), x))*b^2 + a^2*x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2,x, algorithm="fricas")
[Out] integral(b^2*arctan(c/x)^2 + 2*a*b*arctan(c/x) + a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{atan}{\left(\frac{c}{x} \right)}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**2,x)
[Out] Integral((a + b*atan(c/x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2,x, algorithm="giac")
[Out] integrate((b*arctan(c/x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))^2,x)
[Out] int((a + b*atan(c/x))^2, x)
```

3.144 $\int \frac{\left(a+b\mathbf{ArcTan}\left(\frac{c}{x}\right)\right)^2}{x}\,dx$

Optimal. Leaf size=148

$$-2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)+ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)-ib\left(a+b\cot^{-}\right.$$

```
[Out] 2*(a+b*arccot(x/c))^2*arctanh(-1+2/(1+I*c/x))+I*b*(a+b*arccot(x/c))*polylog
(2,1-2/(1+I*c/x))-I*b*(a+b*arccot(x/c))*polylog(2,-1+2/(1+I*c/x))+1/2*b^2*p
olylog(3,1-2/(1+I*c/x))-1/2*b^2*polylog(3,-1+2/(1+I*c/x))
```

Rubi [A]
time = 0.20, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4944, 4942, 5108, 5004, 5114, 6745}

$$ib\text{Li}_2\left(1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-ib\text{Li}_2\left(\frac{2}{\frac{ic}{x}+1}-1\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-2\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}b^2\text{Li}_3\left(1-\frac{2}{\frac{ic}{x}+1}\right)-\frac{1}{2}b^2\text{Li}_3\left(\frac{2}{\frac{ic}{x}+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])^2/x,x]
```

```
[Out] -2*(a + b*ArcCot[x/c])^2*ArcTanh[1 - 2/(1 + (I*c)/x)] + I*b*(a + b*ArcCot[x
/c])*PolyLog[2, 1 - 2/(1 + (I*c)/x)] - I*b*(a + b*ArcCot[x/c])*PolyLog[2, -
1 + 2/(1 + (I*c)/x)] + (b^2*PolyLog[3, 1 - 2/(1 + (I*c)/x)])/2 - (b^2*PolyL
og[3, -1 + 2/(1 + (I*c)/x)])/2
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2}{x}\,dx = -\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^2}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)+(4bc)\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)}{1+\cdots}\right.$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)-(2bc)\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)}{1+c^2x\cdots}\right.$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)+ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{Li}_2\left(1-\frac{2}{1+\cdots}\right.$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)+ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{Li}_2\left(1-\frac{2}{1+\cdots}\right.$$

Mathematica [A]
time = 0.06, size = 148, normalized size = 1.00

$$-2\left(a+b\text{ArcTan}\left(\frac{c}{x}\right)\right)^2\tanh^{-1}\left(\frac{c+ix}{c-ix}\right)+\frac{1}{2}b\left(2i\left(a+b\text{ArcTan}\left(\frac{c}{x}\right)\right)\text{PolyLog}\left(2,\frac{c+ix}{c-ix}\right)-2i\left(a+b\text{ArcTan}\left(\frac{c}{x}\right)\right)\text{PolyLog}\left(2,\frac{-ic+x}{ic+x}\right)+b\left(\text{PolyLog}\left(3,\frac{c+ix}{c-ix}\right)-\text{PolyLog}\left(3,\frac{-ic+x}{ic+x}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])^2/x,x]
```

```
[Out] -2*(a + b*ArcTan[c/x])^2*ArcTanh[(c + I*x)/(c - I*x)] + (b*((2*I)*(a + b*Ar
cTan[c/x])*PolyLog[2, (c + I*x)/(c - I*x)] - (2*I)*(a + b*ArcTan[c/x])*Poly
Log[2, ((-I)*c + x)/(I*c + x)] + b*(PolyLog[3, (c + I*x)/(c - I*x)] - PolyL
og[3, ((-I)*c + x)/(I*c + x)])))/2
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.37, size = 1249, normalized size = 8.44

method	result	size
derivativedivides	Expression too large to display	1249
default	Expression too large to display	1249

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2*ln(c/x)+1/2*b^2*polylog(3,-(1+I*c/x)^2/(1+c^2/x^2))-2*b^2*polylog(3,-(
1+I*c/x)/(1+c^2/x^2)^(1/2))-2*b^2*polylog(3,(1+I*c/x)/(1+c^2/x^2)^(1/2))-1/
2*I*b^2*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I/(1+(1+I*c/x)^2/(1+c^2
/x^2)))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))*arc
tan(c/x)^2-2*a*b*ln(c/x)*arctan(c/x)-1/2*I*b^2*Pi*arctan(c/x)^2+2*I*b^2*arc
tan(c/x)*polylog(2,(1+I*c/x)/(1+c^2/x^2)^(1/2))-I*a*b*dilog(1+I*c/x)+I*a*b*
dilog(1-I*c/x)-I*b^2*arctan(c/x)*polylog(2,-(1+I*c/x)^2/(1+c^2/x^2))+2*I*b^
2*arctan(c/x)*polylog(2,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-b^2*arctan(c/x)^2*ln(
1+(1+I*c/x)/(1+c^2/x^2)^(1/2))-b^2*arctan(c/x)^2*ln(1-(1+I*c/x)/(1+c^2/x^2)
^(1/2))-b^2*ln(c/x)*arctan(c/x)^2+b^2*arctan(c/x)^2*ln((1+I*c/x)^2/(1+c^2/x
^2)-1)+1/2*I*b^2*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I*((1+I*c/x)^2
/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2*arctan(c/x)^2-1/2*I*b^2*Pi*c
sgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(((1+I*c
/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))*arctan(c/x)^2+1/2*I*b^2*P
i*csgn(I/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1
+(1+I*c/x)^2/(1+c^2/x^2)))^2*arctan(c/x)^2+1/2*I*b^2*Pi*csgn(I*((1+I*c/x)^2
/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-
1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2*arctan(c/x)^2+1/2*I*b^2*Pi*csgn(((1+I*c/x
)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2*arctan(c/x)^2-1/2*I*b^2*P
i*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^3*arctan(
c/x)^2-1/2*I*b^2*Pi*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/
x^2)))^3*arctan(c/x)^2-I*a*b*ln(c/x)*ln(1+I*c/x)+I*a*b*ln(c/x)*ln(1-I*c/x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + 1/16*integrate((12*b^2*arctan2(c, x)^2 + b^2*log(c^2 + x^2)^2
+ 32*a*b*arctan2(c, x))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x,x, algorithm="fricas")
```

```
[Out] integral((b^2*arctan(c/x)^2 + 2*a*b*arctan(c/x) + a^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}\left(\frac{c}{x}\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**2/x,x)
```

```
[Out] Integral((a + b*atan(c/x))**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)}^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))^2/x,x)
```

```
[Out] int((a + b*atan(c/x))^2/x, x)
```

3.145 $\int \frac{\left(a+b\operatorname{ArcTan}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$

Optimal. Leaf size=96

$$-\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{c}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{x}-\frac{2b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2}{1+\frac{ic}{x}}\right)}{c}-\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)}{c}$$

```
[Out] -I*(a+b*arccot(x/c))^2/c-(a+b*arccot(x/c))^2/x-2*b*(a+b*arccot(x/c))*ln(2/(
1+I*c/x))/c-I*b^2*polylog(2,1-2/(1+I*c/x))/c
```

Rubi [A]
time = 0.09, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4948, 4930, 5040, 4964, 2449, 2352}

$$-\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{c}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{x}-\frac{2b\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{c}-\frac{ib^2\text{Li}_2\left(1-\frac{2}{\frac{ic}{x}+1}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])^2/x^2,x]
```

```
[Out] ((-I)*(a + b*ArcCot[x/c])^2)/c - (a + b*ArcCot[x/c])^2/x - (2*b*(a + b*ArcC
ot[x/c])*Log[2/(1 + (I*c)/x)])/c - (I*b^2*PolyLog[2, 1 - 2/(1 + (I*c)/x)])/
c
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4x^2}+\frac{b\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{2x^2}-\frac{b^2\log^2\left(1+\right.}{4x^2}\right.$$

$$=\frac{1}{4}\int \frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{x^2}\,dx+\frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{x^2}\,dx-$$

$$=-\left(\frac{1}{4}\text{Subst}\left(\int (2a+ib\log(1-icx))^2\,dx,x,\frac{1}{x}\right)\right)-\frac{1}{2}b\text{Subst}\left(\int (-2ia+b\log(1\right.$$

$$=\frac{b\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{2x}-\frac{i\text{Subst}\left(\int (2a+ib\log(x))^2\,dx,x,1-\frac{ic}{x}\right)}{4c}-$$

$$=-\frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4c}+\frac{b\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{2x}-\frac{ib^2\left(1-\right.}{}$$

$$=\frac{iab}{x}+\frac{b^2}{2x}-\frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4c}+\frac{ib^2\left(1+\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)}{2c}+\frac{b\left(2ia-\right.}{}$$

$$=\frac{b^2}{x}-\frac{ib^2\left(1-\frac{ic}{x}\right)\log\left(1-\frac{ic}{x}\right)}{2c}-\frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4c}+\frac{ib^2\left(1+\frac{ic}{x}\right)\log}{2c}$$

$$=\frac{b^2}{2x}-\frac{ib^2\left(1-\frac{ic}{x}\right)\log\left(1-\frac{ic}{x}\right)}{2c}-\frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4c}+\frac{b\left(2ia-b\log\right.}{}$$

$$=-\frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4c}+\frac{b\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{2x}-\frac{ib^2\left(1-\right.}{}$$

Mathematica [A]
time = 0.09, size = 107, normalized size = 1.11

$$-\frac{b^2(c-ix)\text{ArcTan}\left(\frac{c}{x}\right)^2+2b\text{ArcTan}\left(\frac{c}{x}\right)\left(ac+bx\log\left(1+e^{2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)\right)+a\left(ac+2bx\log\left(\frac{1}{\sqrt{1+\frac{c^2}{x^2}}}\right)\right)-ib^2x\text{PolyLog}\left(2,-e^{2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)}{cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])^2/x^2,x]
```

```
[Out] -((b^2*(c - I*x)*ArcTan[c/x]^2 + 2*b*ArcTan[c/x]*(a*c + b*x*Log[1 + E^((2*I
)*ArcTan[c/x])]) + a*(a*c + 2*b*x*Log[1/Sqrt[1 + c^2/x^2]]) - I*b^2*x*PolyL
og[2, -E^((2*I)*ArcTan[c/x])])/(c*x))
```

Maple [A]
time = 0.51, size = 142, normalized size = 1.48

method	result
derivativedivides	$-\frac{\frac{c\,a^2}{x}-i\arctan\left(\frac{c}{x}\right)^2b^2+\frac{\arctan\left(\frac{c}{x}\right)^2b^2c}{x}-i\,\text{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right)b^2+2\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right)b^2+\frac{2abc\arctan\left(\frac{c}{x}\right)}{x}}{c}$
default	$-\frac{\frac{c\,a^2}{x}-i\arctan\left(\frac{c}{x}\right)^2b^2+\frac{\arctan\left(\frac{c}{x}\right)^2b^2c}{x}-i\,\text{polylog}\left(2,-\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right)b^2+2\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right)b^2+\frac{2abc\arctan\left(\frac{c}{x}\right)}{x}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c*(c/x*a^2-I*arctan(c/x)^2*b^2+arctan(c/x)^2*b^2*c/x-I*polylog(2,-(1+I*c
/x)^2/(1+c^2/x^2))*b^2+2*arctan(c/x)*ln(1+(1+I*c/x)^2/(1+c^2/x^2))*b^2+2*a*
b*c/x*arctan(c/x)-a*b*ln(1+c^2/x^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*(48*c^2*integrate(1/16*arctan(c/x)^2/(c^2*x^2 + x^4), x) + 4*c^2*in
tegrate(1/16*log(c^2 + x^2)^2/(c^2*x^2 + x^4), x) + 3*arctan(c/x)^2*arctan(
x/c)/c + 3*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^3/c - 32*c*integrate(1
/16*x*arctan(c/x)/(c^2*x^2 + x^4), x) + 4*integrate(1/16*x^2*log(c^2 + x^2)
^2/(c^2*x^2 + x^4), x) - 16*integrate(1/16*x^2*log(c^2 + x^2)/(c^2*x^2 + x^
```

```
4), x))*x - 4*arctan2(c, x)^2 + log(c^2 + x^2)^2)*b^2/x - a*b*(2*c*arctan(c
/x)/x - log(c^2/x^2 + 1))/c - a^2/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*arctan(c/x)^2 + 2*a*b*arctan(c/x) + a^2)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}{\left(\frac{c}{x} \right)}\right)^{2}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**2/x**2,x)
[Out] Integral((a + b*atan(c/x))**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x^2,x, algorithm="giac")
[Out] integrate((b*arctan(c/x) + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)}^2}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))^2/x^2,x)
[Out] int((a + b*atan(c/x))^2/x^2, x)
```

3.146 $\int \frac{\left(a+b\textbf{ArcTan}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$

Optimal. Leaf size=84

$$\frac{ab}{cx}+\frac{b^2\cot^{-1}\left(\frac{x}{c}\right)}{cx}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2x^2}-\frac{b^2\log\left(1+\frac{c^2}{x^2}\right)}{2c^2}$$

```
[Out] a*b/c/x+b^2*arccot(x/c)/c/x-1/2*(a+b*arccot(x/c))^2/c^2-1/2*(a+b*arccot(x/c
))^2/x^2-1/2*b^2*ln(1+c^2/x^2)/c^2
```

Rubi [A]
time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4948, 4946, 5036, 4930, 266, 5004}

$$-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2x^2}+\frac{ab}{cx}-\frac{b^2\log\left(\frac{c^2}{x^2}+1\right)}{2c^2}+\frac{b^2\cot^{-1}\left(\frac{x}{c}\right)}{cx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])^2/x^3,x]
```

```
[Out] (a*b)/(c*x) + (b^2*ArcCot[x/c])/(c*x) - (a + b*ArcCot[x/c])^2/(2*c^2) - (a
+ b*ArcCot[x/c])^2/(2*x^2) - (b^2*Log[1 + c^2/x^2])/(2*c^2)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4x^3} + \frac{b\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{2x^3} - \frac{b^2\log^2\left(1\right.}{4x^3}\right.$$

$$= \frac{1}{4}\int \frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{x^3}\,dx + \frac{1}{2}b\int \frac{\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)\log\left(1+\frac{ic}{x}\right)}{x^3}\,dx -$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int x(2a+ib\log(1-icx))^2\,dx, x, \frac{1}{x}\right)\right) + \frac{1}{2}b\int\left(-\frac{2ia\log\left(1+\frac{ic}{x}\right)}{x^3}\right.$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int\left(-\frac{i(2a+ib\log(1-icx))^2}{c} + \frac{i(1-icx)(2a+ib\log(1-icx))^2}{c}\right.\right.\right.$$

$$= -\frac{b^2\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)}{4x^2} + (iab)\text{Subst}\left(\int x\log(1+icx)\,dx, x, \frac{1}{x}\right) - \frac{1}{2}b^2\int$$

$$= \frac{iab\log\left(1+\frac{ic}{x}\right)}{2x^2} - \frac{b^2\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)}{4x^2} - \frac{\text{Subst}\left(\int(2a+ib\log(x))^2\,dx, x,\right.}{4c^2}$$

$$= -\frac{\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{4c^2} + \frac{\left(1-\frac{ic}{x}\right)^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{8c^2} + \frac{iab\log\left(\right.}{2x}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} + \frac{3ab}{2cx} + \frac{ib^2}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} - \frac{ib\left(1-\frac{ic}{x}\right)}{}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} + \frac{3ab}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} - \frac{b^2\left(1-\frac{ic}{x}\right)\log\left(1\right.}{2c^2}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} + \frac{3ab}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} - \frac{b^2\left(1-\frac{ic}{x}\right)\log\left(1\right.}{2c^2}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} + \frac{3ab}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} - \frac{3b^2\left(1-\frac{ic}{x}\right)\log\left(1\right.}{4c^2}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} - \frac{b^2}{8x^2} + \frac{3ab}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} + \frac{b^2\log\left(i-\right.}{8c^2}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} - \frac{b^2}{8x^2} + \frac{3ab}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} + \frac{b^2\log\left(i-\right.}{8c^2}$$

$$= -\frac{b^2\left(1-\frac{ic}{x}\right)^2}{16c^2} - \frac{b^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{iab}{4x^2} - \frac{b^2}{8x^2} + \frac{3ab}{2cx} + \frac{iab\log\left(i-\frac{c}{x}\right)}{2c^2} + \frac{b^2\log\left(i-\right.}{8c^2}$$

Mathematica [A]
time = 0.04, size = 99, normalized size = 1.18

$$-\frac{a^2c^2-2abcx+2bc(ac-bx)\text{ArcTan}\left(\frac{c}{x}\right)+b^2(c^2+x^2)\,\text{ArcTan}\left(\frac{c}{x}\right)^2-2abx^2\text{ArcTan}\left(\frac{x}{c}\right)-2b^2x^2\log(x)+b^2x^2\log\left(c^2+x^2\right)}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])^2/x^3,x]
```

```
[Out] -1/2*(a^2*c^2 - 2*a*b*c*x + 2*b*c*(a*c - b*x)*ArcTan[c/x] + b^2*(c^2 + x^2)
*ArcTan[c/x]^2 - 2*a*b*x^2*ArcTan[x/c] - 2*b^2*x^2*Log[x] + b^2*x^2*Log[c^2
 + x^2])/(c^2*x^2)
```

Maple [A]
time = 0.94, size = 112, normalized size = 1.33

method	result	size
derivativedivides	$-\frac{\frac{a^2c^2}{2x^2}+\frac{b^2c^2\arctan\left(\frac{c}{x}\right)^2}{2x^2}+\frac{b^2\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{b^2\arctan\left(\frac{c}{x}\right)c}{x}+\frac{b^2\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\frac{ab\,c^2\arctan\left(\frac{c}{x}\right)}{x^2}+ab\arctan\left(\frac{c}{x}\right)-\frac{abc}{x}}{c^2}$	112
default	$-\frac{\frac{a^2c^2}{2x^2}+\frac{b^2c^2\arctan\left(\frac{c}{x}\right)^2}{2x^2}+\frac{b^2\arctan\left(\frac{c}{x}\right)^2}{2}-\frac{b^2\arctan\left(\frac{c}{x}\right)c}{x}+\frac{b^2\ln\left(1+\frac{c^2}{x^2}\right)}{2}+\frac{ab\,c^2\arctan\left(\frac{c}{x}\right)}{x^2}+ab\arctan\left(\frac{c}{x}\right)-\frac{abc}{x}}{c^2}$	112
risch	Expression too large to display	59876

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^2/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^2*(1/2*a^2*c^2/x^2+1/2*b^2*c^2/x^2*arctan(c/x)^2+1/2*b^2*arctan(c/x)^2
-b^2*arctan(c/x)*c/x+1/2*b^2*ln(1+c^2/x^2)+a*b*c^2/x^2*arctan(c/x)+a*b*arct
an(c/x)-a*b*c/x)
```

Maxima [A]
time = 0.50, size = 120, normalized size = 1.43

$$\left(c\left(\frac{\arctan\left(\frac{x}{c}\right)}{c^3}+\frac{1}{c^2x}\right)-\frac{\arctan\left(\frac{c}{x}\right)}{x^2}\right)ab+\frac{1}{2}\left(2\,c\left(\frac{\arctan\left(\frac{x}{c}\right)}{c^3}+\frac{1}{c^2x}\right)\arctan\left(\frac{c}{x}\right)+\frac{\arctan\left(\frac{x}{c}\right)^2-\log\left(c^2+x^2\right)+2\,\log\left(x\right)}{c^2}\right)b^2-\frac{b^2\arctan\left(\frac{c}{x}\right)^2}{2\,x^2}-\frac{a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x^3,x, algorithm="maxima")
```

```
[Out] (c*(arctan(x/c)/c^3 + 1/(c^2*x)) - arctan(c/x)/x^2)*a*b + 1/2*(2*c*(arctan(
x/c)/c^3 + 1/(c^2*x))*arctan(c/x) + (arctan(x/c)^2 - log(c^2 + x^2) + 2*log
(x))/c^2)*b^2 - 1/2*b^2*arctan(c/x)^2/x^2 - 1/2*a^2/x^2
```

Fricas [A]
time = 0.76, size = 109, normalized size = 1.30

$$\frac{2\,abx^2\arctan\left(\frac{x}{c}\right)-b^2x^2\log\left(c^2+x^2\right)+2\,b^2x^2\log\left(x\right)-a^2c^2+2\,abcx-\left(b^2c^2+b^2x^2\right)\arctan\left(\frac{c}{x}\right)^2-2\,\left(abc^2-b^2cx\right)\arctan\left(\frac{c}{x}\right)}{2\,c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*x^2*arctan(x/c) - b^2*x^2*log(c^2 + x^2) + 2*b^2*x^2*log(x) - a^
2*c^2 + 2*a*b*c*x - (b^2*c^2 + b^2*x^2)*arctan(c/x)^2 - 2*(a*b*c^2 - b^2*c*
x)*arctan(c/x))/(c^2*x^2)
```

Sympy [A]
time = 0.37, size = 117, normalized size = 1.39

$$\begin{cases} -\frac{a^2}{2x^2} - \frac{ab\operatorname{atan}\left(\frac{c}{x}\right)}{x^2} + \frac{ab}{cx} - \frac{ab\operatorname{atan}\left(\frac{c}{x}\right)}{c^2} - \frac{b^2\operatorname{atan}^2\left(\frac{c}{x}\right)}{2x^2} + \frac{b^2\operatorname{atan}\left(\frac{c}{x}\right)}{cx} + \frac{b^2\log\left(x\right)}{c^2} - \frac{b^2\log\left(c^2+x^2\right)}{2c^2} - \frac{b^2\operatorname{atan}^2\left(\frac{c}{x}\right)}{2c^2} & \text{for}\ c \neq 0 \\ -\frac{a^2}{2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**2/x**3,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - a*b*atan(c/x)/x**2 + a*b/(c*x) - a*b*atan(c/x)/
c**2 - b**2*atan(c/x)**2/(2*x**2) + b**2*atan(c/x)/(c*x) + b**2*log(x)/c**2
 - b**2*log(c**2 + x**2)/(2*c**2) - b**2*atan(c/x)**2/(2*c**2), Ne(c, 0)),
(-a**2/(2*x**2), True))
```

Giac [C] Result contains complex when optimal does not.
time = 0.43, size = 137, normalized size = 1.63

$$-\frac{b^2\arctan\left(\frac{c}{x}\right)^2 + \frac{b^2c^2\arctan\left(\frac{c}{x}\right)^2}{x^2} + \frac{2\,abc^2\arctan\left(\frac{c}{x}\right)}{x^2} - \frac{2\,b^2c\arctan\left(\frac{c}{x}\right)}{x} + i\,ab\log\left(\frac{i\,c}{x} - 1\right) + b^2\log\left(\frac{i\,c}{x} - 1\right) - i\,ab\log\left(-\frac{i\,c}{x} - 1\right) + b^2\log\left(-\frac{i\,c}{x} - 1\right) + \frac{a^2c^2}{x^2} - \frac{2\,abc}{x}}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*arctan(c/x)^2 + b^2*c^2*arctan(c/x)^2/x^2 + 2*a*b*c^2*arctan(c/x)
/x^2 - 2*b^2*c*arctan(c/x)/x + I*a*b*log(I*c/x - 1) + b^2*log(I*c/x - 1) -
I*a*b*log(-I*c/x - 1) + b^2*log(-I*c/x - 1) + a^2*c^2/x^2 - 2*a*b*c/x)/c^2
```

Mupad [B]
time = 2.75, size = 143, normalized size = 1.70

$$\frac{b^2\,\ln\left(x\right) - \frac{b^2\,\ln(x+c\,1\mathrm{i})}{2} - \frac{b^2\,\operatorname{atan}\left(\frac{c}{x}\right)^2}{2} + \frac{b^2\,\ln\left(\frac{1}{-x+c\,1\mathrm{i}}\right)}{2} + \frac{a\,b\,\ln(x+c\,1\mathrm{i})\,1\mathrm{i}}{2} - \frac{a\,b\,\ln(-x+c\,1\mathrm{i})\,1\mathrm{i}}{2}}{c^2} - \frac{\frac{a^2\,c^2}{2} - x\,\left(c\,\operatorname{atan}\left(\frac{c}{x}\right)\,b^2 + a\,c\,b\right) + \frac{b^2\,c^2\,\operatorname{atan}\left(\frac{c}{x}\right)^2}{2} + a\,b\,c^2\,\operatorname{atan}\left(\frac{c}{x}\right)}{c^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))^2/x^3,x)
```

```
[Out] (b^2*log(x) - (b^2*log(c*1i + x))/2 - (b^2*atan(c/x)^2)/2 + (b^2*log(1/(c*1
i - x)))/2 + (a*b*log(c*1i + x)*1i)/2 - (a*b*log(c*1i - x)*1i)/2)/c^2 - ((a
^2*c^2)/2 - x*(b^2*c*atan(c/x) + a*b*c) + (b^2*c^2*atan(c/x)^2)/2 + a*b*c^2
*atan(c/x))/(c^2*x^2)
```

3.147 $\int x^3\left(a+b\text{ArcTan}\left(\frac{c}{x}\right)\right)^3 dx$

Optimal. Leaf size=214

$$\frac{1}{4}b^3c^3x+\frac{1}{4}b^3c^4\cot^{-1}\left(\frac{x}{c}\right)+\frac{1}{4}b^2c^2x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-ibc^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{3}{4}bc^3x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+$$

```
[Out] 1/4*b^3*c^3*x+1/4*b^3*c^4*arccot(x/c)+1/4*b^2*c^2*x^2*(a+b*arccot(x/c))-I*b
*c^4*(a+b*arccot(x/c))^2-3/4*b*c^3*x*(a+b*arccot(x/c))^2+1/4*b*c*x^3*(a+b*a
rccot(x/c))^2-1/4*c^4*(a+b*arccot(x/c))^3+1/4*x^4*(a+b*arccot(x/c))^3+2*b^2
*c^4*(a+b*arccot(x/c))*ln(2-2/(1-I*c/x))-I*b^3*c^4*polylog(2,-1+2/(1-I*c/x)
)
```

Rubi [A]
time = 0.42, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {4948, 4946, 5038, 331, 209, 5044, 4988, 2497, 5004}

$$2b^2c^4\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{4}b^2c^2x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{4}c^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3-ibc^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{3}{4}bc^3x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{4}x^4\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{4}bcx^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-ib^3c^4\text{Li}_2\left(\frac{2}{1-\frac{ic}{x}}-1\right)+\frac{1}{4}b^3c^4\cot^{-1}\left(\frac{x}{c}\right)+\frac{1}{4}b^3c^3x$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] (b^3*c^3*x)/4 + (b^3*c^4*ArcCot[x/c])/4 + (b^2*c^2*x^2*(a + b*ArcCot[x/c]))
/4 - I*b*c^4*(a + b*ArcCot[x/c])^2 - (3*b*c^3*x*(a + b*ArcCot[x/c])^2)/4 +
(b*c*x^3*(a + b*ArcCot[x/c])^2)/4 - (c^4*(a + b*ArcCot[x/c])^3)/4 + (x^4*(a
 + b*ArcCot[x/c])^3)/4 + 2*b^2*c^4*(a + b*ArcCot[x/c])*Log[2 - 2/(1 - (I*c)
/x)] - I*b^3*c^4*PolyLog[2, -1 + 2/(1 - (I*c)/x)]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\int x^3\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(\frac{1}{8}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3+\frac{3}{8}ibx^3\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1\right.\right.$$

$$=\frac{1}{8}\int x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3dx+\frac{1}{8}(3ib)\int x^3\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)$$

$$=-\left(\frac{1}{8}\text{Subst}\left(\int\frac{(2a+ib\log(1-icx))^3}{x^5}dx,x,\frac{1}{x}\right)\right)+\frac{1}{8}(3ib)\int\left(-4a^2x^3\log\left(\right.\right.$$

$$=\frac{1}{32}x^4\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3+\frac{1}{32}ib^3x^4\log^3\left(1+\frac{ic}{x}\right)-\frac{1}{2}(3ia^2b)\int x^3\log$$

$$=\frac{1}{32}x^4\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{3}{8}ia^2bx^4\log\left(1+\frac{ic}{x}\right)+\frac{3}{8}ab^2x^4\log\left(1-\frac{ic}{x}\right.$$

$$=\frac{1}{32}x^4\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{3}{8}ia^2bx^4\log\left(1+\frac{ic}{x}\right)+\frac{3}{8}ab^2x^4\log\left(1-\frac{ic}{x}\right.$$

$$=\frac{1}{32}bcx^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{32}x^4\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{3}{8}ia^2bx^4$$

$$=-\frac{3}{8}a^2bc^3x-\frac{3}{16}ia^2bc^2x^2+\frac{1}{8}a^2bcx^3+\frac{3}{64}ibc^2x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+$$

$$=-\frac{3}{8}a^2bc^3x-\frac{3}{16}ia^2bc^2x^2+\frac{1}{8}a^2bcx^3-\frac{3}{8}iab^2c^3x\log\left(1-\frac{ic}{x}\right)+\frac{3}{16}ab^2c^2x^2\log$$

$$=-\frac{3}{8}a^2bc^3x-\frac{3}{16}ia^2bc^2x^2+\frac{1}{8}a^2bcx^3-\frac{3}{8}iab^2c^3x\log\left(1-\frac{ic}{x}\right)+\frac{3}{16}ab^2c^2x^2\log$$

$$=-\frac{3}{8}a^2bc^3x-\frac{5}{16}iab^2c^3x+\frac{1}{16}b^3c^3x-\frac{3}{16}ia^2bc^2x^2+\frac{1}{16}ab^2c^2x^2+\frac{1}{8}a^2bcx^3-$$

$$=-\frac{3}{8}a^2bc^3x-\frac{5}{16}iab^2c^3x+\frac{1}{16}b^3c^3x-\frac{3}{16}ia^2bc^2x^2+\frac{3}{16}ab^2c^2x^2+\frac{1}{8}a^2bcx^3-$$

$$=-\frac{3}{8}a^2bc^3x-\frac{5}{16}iab^2c^3x+\frac{1}{16}b^3c^3x-\frac{3}{16}ia^2bc^2x^2+\frac{3}{16}ab^2c^2x^2+\frac{1}{8}a^2bcx^3-$$

$$=-\frac{3}{8}a^2bc^3x-\frac{5}{16}iab^2c^3x+\frac{1}{16}b^3c^3x-\frac{3}{16}ia^2bc^2x^2+\frac{3}{16}ab^2c^2x^2+\frac{1}{8}a^2bcx^3-$$

Mathematica [A]
time = 0.44, size = 253, normalized size = 1.18

$$\frac{1}{4}\left(ab^2c^4-3a^2bc^3x+b^3c^3x+ab^2c^2x^2+a^2bcx^3+a^3x^4+b^2\left(bc\left(-4ic^3-3c^2x+x^3\right)+3a\left(-c^4+x^4\right)\right)\text{ArcTan}\left(\frac{c}{x}\right)^2+b^3\left(-c^4+x^4\right)\text{ArcTan}\left(\frac{c}{x}\right)^3+b\text{ArcTan}\left(\frac{c}{x}\right)\left(2abcx\left(-3c^2+x^2\right)+b^2c^2\left(c^2+x^2\right)+3a^2\left(-c^4+x^4\right)+8b^2c^4\log\left(1-e^{2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)\right)+8ab^2c^4\log\left(\frac{c}{\sqrt{1+\frac{c^2}{x^2}}\,x}\right)-4ib^3c^4\text{PolyLog}\left(2,e^{2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] (a*b^2*c^4 - 3*a^2*b*c^3*x + b^3*c^3*x + a*b^2*c^2*x^2 + a^2*b*c*x^3 + a^3*
x^4 + b^2*(b*c*((-4*I)*c^3 - 3*c^2*x + x^3) + 3*a*(-c^4 + x^4))*ArcTan[c/x]
^2 + b^3*(-c^4 + x^4)*ArcTan[c/x]^3 + b*ArcTan[c/x]*(2*a*b*c*x*(-3*c^2 + x^
2) + b^2*c^2*(c^2 + x^2) + 3*a^2*(-c^4 + x^4) + 8*b^2*c^4*Log[1 - E^((2*I)*
ArcTan[c/x])]) + 8*a*b^2*c^4*Log[c/(Sqrt[1 + c^2/x^2]*x)] - (4*I)*b^3*c^4*P
olyLog[2, E^((2*I)*ArcTan[c/x])])/4
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(196) = 392$.
time = 6.78, size = 569, normalized size = 2.66

method	result
derivativedivides	$-c^4\left(-\frac{a^3x^4}{4c^4}-\frac{3b^2a\,x^4\arctan\left(\frac{c}{x}\right)^2}{4c^4}-\frac{b^2a\,x^3\arctan\left(\frac{c}{x}\right)}{2c^3}+\frac{3b^2ax\arctan\left(\frac{c}{x}\right)}{2c}-\frac{3a^2b\,x^4\arctan\left(\frac{c}{x}\right)}{4c^4}+ib^3\,\text{dil}\right.$
default	$-c^4\left(-\frac{a^3x^4}{4c^4}-\frac{3b^2a\,x^4\arctan\left(\frac{c}{x}\right)^2}{4c^4}-\frac{b^2a\,x^3\arctan\left(\frac{c}{x}\right)}{2c^3}+\frac{3b^2ax\arctan\left(\frac{c}{x}\right)}{2c}-\frac{3a^2b\,x^4\arctan\left(\frac{c}{x}\right)}{4c^4}+ib^3\,\text{dil}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arctan(c/x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -c^4*(-1/4*a^3/c^4*x^4-3/4*b^2*a/c^4*x^4*arctan(c/x)^2-1/2*b^2*a/c^3*x^3*ar
ctan(c/x)+3/2*b^2*a/c*x*arctan(c/x)-3/4*a^2*b/c^4*x^4*arctan(c/x)+1/4*b^3*a
rctan(c/x)^3-1/4*b^3*arctan(c/x)-1/4*b^3/c*x-2*b^3*ln(c/x)*arctan(c/x)+I*b^
3*dilog(1-I*c/x)+1/4*I*b^3*ln(c/x+I)^2-I*b^3*dilog(1+I*c/x)-1/2*I*b^3*dilog
(-1/2*I*(c/x+I))-1/4*I*b^3*ln(c/x-I)^2+1/2*I*b^3*dilog(1/2*I*(c/x-I))+3/4*b
^2*a*arctan(c/x)^2+b^2*a*ln(1+c^2/x^2)-2*b^2*a*ln(c/x)+3/4*a^2*b*arctan(c/x
)+b^3*arctan(c/x)*ln(1+c^2/x^2)-1/2*I*b^3*ln(c/x+I)*ln(1+c^2/x^2)+1/2*I*b^3
*ln(c/x-I)*ln(1+c^2/x^2)-I*b^3*ln(c/x)*ln(1+I*c/x)-1/2*I*b^3*ln(c/x-I)*ln(-
1/2*I*(c/x+I))+1/2*I*b^3*ln(c/x+I)*ln(1/2*I*(c/x-I))+I*b^3*ln(c/x)*ln(1-I*c
/x)-1/4*b^2*a/c^2*x^2-1/4*a^2*b/c^3*x^3+3/4*a^2*b/c*x-1/4*b^3/c^4*x^4*arcta
n(c/x)^3-1/4*b^3*arctan(c/x)^2/c^3*x^3+3/4*b^3*arctan(c/x)^2/c*x-1/4*b^3*ar
ctan(c/x)/c^2*x^2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x))^3,x, algorithm="maxima")
```

```
[Out] 3/4*a*b^2*x^4*arctan(c/x)^2 + 1/4*a^3*x^4 + 1/4*(3*x^4*arctan(c/x) + (3*c^3
*arctan(x/c) - 3*c^2*x + x^3)*c)*a^2*b + 1/4*((3*c^2*arctan(x/c)^2 - 4*c^2*
log(c^2 + x^2) + x^2)*c^2 + 2*(3*c^3*arctan(x/c) - 3*c^2*x + x^3)*c*arctan(
c/x))*a*b^2 - 1/64*(12*c^4*arctan(c/x)^2*arctan(x/c) + 8*c^4*arctan2(c, x)^
3 - 8*x^4*arctan2(c, x)^3 + 4*(3*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^
3/c)*c^5 + 12*c^3*x*arctan2(c, x)^2 - 4*c*x^3*arctan2(c, x)^2 + 192*c^5*int
egrate(1/64*log(c^2 + x^2)^2/(c^2 + x^2), x) + 1536*c^4*integrate(1/64*x*ar
ctan(c/x)/(c^2 + x^2), x) + 768*c^3*integrate(1/64*x^2*log(c^2 + x^2)/(c^2
+ x^2), x) - 2048*c^2*integrate(1/64*x^3*arctan(c/x)^3/(c^2 + x^2), x) - 51
2*c^2*integrate(1/64*x^3*arctan(c/x)/(c^2 + x^2), x) - (3*c^3*x - c*x^3)*lo
g(c^2 + x^2)^2 - 768*c*integrate(1/64*x^4*arctan(c/x)^2/(c^2 + x^2), x) - 1
92*c*integrate(1/64*x^4*log(c^2 + x^2)^2/(c^2 + x^2), x) - 256*c*integrate(
1/64*x^4*log(c^2 + x^2)/(c^2 + x^2), x) - 2048*integrate(1/64*x^5*arctan(c/
x)^3/(c^2 + x^2), x))*b^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arctan(c/x)^3 + 3*a*b^2*x^3*arctan(c/x)^2 + 3*a^2*b*x^3*ar
ctan(c/x) + a^3*x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(a+b\operatorname{atan}\left(\frac{c}{x}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*atan(c/x))**3,x)
```

```
[Out] Integral(x**3*(a + b*atan(c/x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arctan(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^3*x^3, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(a + b \operatorname{atan}\left(\frac{c}{x}\right)\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*atan(c/x))^3,x)
```

```
[Out] int(x^3*(a + b*atan(c/x))^3, x)
```

3.148 $\int x^2 \left(a+b\mathbf{ArcTan}\left(\frac{c}{x}\right)\right)^3 dx$

Optimal. Leaf size=229

$$b^2c^2x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{3}x$$

```
[Out] b^2*c^2*x*(a+b*arccot(x/c))+1/2*b*c^3*(a+b*arccot(x/c))^2+1/2*b*c*x^2*(a+b*
arccot(x/c))^2-1/3*I*c^3*(a+b*arccot(x/c))^3+1/3*x^3*(a+b*arccot(x/c))^3+b*
c^3*(a+b*arccot(x/c))^2*ln(2-2/(1-I*c/x))+1/2*b^3*c^3*ln(1+c^2/x^2)+b^3*c^3
*ln(x)-I*b^2*c^3*(a+b*arccot(x/c))*polylog(2,-1+2/(1-I*c/x))+1/2*b^3*c^3*po
lylog(3,-1+2/(1-I*c/x))
```

Rubi [A]
time = 0.35, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4948, 4946, 5038, 272, 36, 29, 31, 5004, 5044, 4988, 5112, 6745}

$$-ib^2c^3\text{Li}_2\left(\frac{2}{1-\frac{ic}{x}}-1\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+b^2c^2x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{3}ic^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{2}bc^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+bc^3\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{3}x^3\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{2}bcx^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}b^3c^3\text{Li}_3\left(\frac{2}{1-\frac{ic}{x}}-1\right)+b^3c^3\log(x)+\frac{1}{2}b^3c^3\log\left(\frac{c^2}{x^2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] b^2*c^2*x*(a + b*ArcCot[x/c]) + (b*c^3*(a + b*ArcCot[x/c])^2)/2 + (b*c*x^2*
(a + b*ArcCot[x/c])^2)/2 - (I/3)*c^3*(a + b*ArcCot[x/c])^3 + (x^3*(a + b*Ar
cCot[x/c])^3)/3 + b*c^3*(a + b*ArcCot[x/c])^2*Log[2 - 2/(1 - (I*c)/x)] + (b
^3*c^3*Log[1 + c^2/x^2])/2 + b^3*c^3*Log[x] - I*b^2*c^3*(a + b*ArcCot[x/c])
*PolyLog[2, -1 + 2/(1 - (I*c)/x)] + (b^3*c^3*PolyLog[3, -1 + 2/(1 - (I*c)/x
)])/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rule 5112

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d
] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int x^2\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3dx=\int\left(\frac{1}{8}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3+\frac{3}{8}ibx^2\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(\right.\right.$$

$$=\frac{1}{8}\int x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3dx+\frac{1}{8}(3ib)\int x^2\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right.$$

$$=-\left(\frac{1}{8}\text{Subst}\left(\int\frac{(2a+ib\log(1-icx))^3}{x^4}dx,x,\frac{1}{x}\right)\right)+\frac{1}{8}(3ib)\int\left(-4a^2x^2\log\right.$$

$$=\frac{1}{24}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3+\frac{1}{24}ib^3x^3\log^3\left(1+\frac{ic}{x}\right)-\frac{1}{2}(3ia^2b)\int x^2\text{lo}$$

$$=\frac{1}{24}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{1}{2}ia^2bx^3\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}ab^2x^3\log\left(1-\right.$$

$$=\frac{1}{24}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{1}{2}ia^2bx^3\log\left(1+\frac{ic}{x}\right)+\frac{1}{2}ab^2x^3\log\left(1-\right.$$

$$=\frac{1}{16}bcx^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{24}x^3\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{1}{2}ia^2ba$$

$$=-\frac{1}{2}ia^2bc^2x+\frac{1}{4}a^2bcx^2+\frac{1}{8}ibc^2\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{16}bc$$

$$=-\frac{1}{2}ia^2bc^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}iab^2cx^2\log\left(1-\frac{ic}{x}\right)+$$

$$=-\frac{1}{2}ia^2bc^2x+\frac{1}{4}a^2bcx^2+\frac{1}{2}ab^2c^2x\log\left(1-\frac{ic}{x}\right)+\frac{1}{4}iab^2cx^2\log\left(1-\frac{ic}{x}\right)+$$

$$=-\frac{1}{2}ia^2bc^2x+\frac{1}{4}ab^2c^2x+\frac{1}{4}a^2bcx^2-\frac{3}{4}iab^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{2}ab^2c^2x\log\left(1-\right.$$

$$=-\frac{1}{2}ia^2bc^2x+\frac{3}{4}ab^2c^2x+\frac{1}{4}a^2bcx^2-\frac{3}{4}iab^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{2}ab^2c^2x\log\left(1-\right.$$

$$=-\frac{1}{2}ia^2bc^2x+\frac{3}{4}ab^2c^2x+\frac{1}{4}a^2bcx^2-\frac{3}{4}iab^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{2}ab^2c^2x\log\left(1-\right.$$

$$=-\frac{1}{2}ia^2bc^2x+\frac{3}{4}ab^2c^2x+\frac{1}{4}a^2bcx^2-\frac{3}{4}iab^2c^3\log\left(i-\frac{c}{x}\right)+\frac{1}{2}ab^2c^2x\log\left(1-\right.$$

<u>**Mathematica**</u> [**A**]
time = 0.49, size = 330, normalized size = 1.44

$$\frac{1}{2}a^2bcx^2+\frac{a^3x^3}{3}+a^2bx^3\text{ArcTan}\left(\frac{c}{x}\right)-\frac{1}{2}a^2bc^3\log\left(c^2+x^2\right)+ab^2\left(c^2x+\left(-ic^3+x^3\right)\text{ArcTan}\left(\frac{c}{x}\right)^2+c\text{ArcTan}\left(\frac{c}{x}\right)\left(c^2+x^2+2c^2\log\left(1-e^{2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)\right)-ic^3\text{PolyLog}\left(2,e^{2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)\right)+\frac{1}{24}b^3\left(-ic^3\pi^3+24c^2x\text{ArcTan}\left(\frac{c}{x}\right)+12c^3\text{ArcTan}\left(\frac{c}{x}\right)^2+12cx^2\text{ArcTan}\left(\frac{c}{x}\right)^2+8ic^3\text{ArcTan}\left(\frac{c}{x}\right)^3+8x^3\text{ArcTan}\left(\frac{c}{x}\right)^3+24c^3\text{ArcTan}\left(\frac{c}{x}\right)^2\log\left(1-e^{-2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)-24c^3\log\left(\frac{c}{\sqrt{1+\frac{c^2}{x^2}}\,x}\right)+24ic^3\text{ArcTan}\left(\frac{c}{x}\right)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)+12c^3\text{PolyLog}\left(3,e^{-2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] (a^2*b*c*x^2)/2 + (a^3*x^3)/3 + a^2*b*x^3*ArcTan[c/x] - (a^2*b*c^3*Log[c^2
+ x^2])/2 + a*b^2*(c^2*x + ((-I)*c^3 + x^3)*ArcTan[c/x]^2 + c*ArcTan[c/x]*(
c^2 + x^2 + 2*c^2*Log[1 - E^((2*I)*ArcTan[c/x])]) - I*c^3*PolyLog[2, E^((2*
I)*ArcTan[c/x])]) + (b^3*((-I)*c^3*Pi^3 + 24*c^2*x*ArcTan[c/x] + 12*c^3*Arc
Tan[c/x]^2 + 12*c*x^2*ArcTan[c/x]^2 + (8*I)*c^3*ArcTan[c/x]^3 + 8*x^3*ArcTa
n[c/x]^3 + 24*c^3*ArcTan[c/x]^2*Log[1 - E^((-2*I)*ArcTan[c/x])] - 24*c^3*Lo
g[c/(Sqrt[1 + c^2/x^2]*x)] + (24*I)*c^3*ArcTan[c/x]*PolyLog[2, E^((-2*I)*Ar
cTan[c/x])] + 12*c^3*PolyLog[3, E^((-2*I)*ArcTan[c/x])]))/24
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 13.42, size = 6282, normalized size = 27.43

method	result	size
derivativedivides	Expression too large to display	6282
default	Expression too large to display	6282

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arctan(c/x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x))^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*x^3*arctan2(c, x)^3 - 1/32*b^3*x^3*arctan2(c, x)*log(c^2 + x^2)^2
+ 1/3*a^3*x^3 + 1/2*(2*x^3*arctan(c/x) - (c^2*log(c^2 + x^2) - x^2)*c)*a^2*
b + integrate(1/32*(4*b^3*c*x^3*arctan2(c, x)^2 + 4*b^3*x^4*arctan2(c, x)*l
og(c^2 + x^2) + 4*(7*b^3*arctan2(c, x)^3 + 24*a*b^2*arctan2(c, x)^2)*x^4 +
4*(7*b^3*c^2*arctan2(c, x)^3 + 24*a*b^2*c^2*arctan2(c, x)^2)*x^2 + (3*b^3*c
^2*x^2*arctan2(c, x) + 3*b^3*x^4*arctan2(c, x) - b^3*c*x^3)*log(c^2 + x^2)^
2)/(c^2 + x^2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arctan(c/x)^3 + 3*a*b^2*x^2*arctan(c/x)^2 + 3*a^2*b*x^2*ar
ctan(c/x) + a^3*x^2, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*atan(c/x))**3,x)
```

```
[Out] Integral(x**2*(a + b*atan(c/x))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arctan(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^3*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*atan(c/x))^3,x)
```

```
[Out] int(x^2*(a + b*atan(c/x))^3, x)
```

3.149 $\int x\left(a+b\mathbf{ArcTan}\left(\frac{c}{x}\right)\right)^3\,dx$

Optimal. Leaf size=145

$$\frac{3}{2}ibc^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{3}{2}bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3-3b^2c$$

```
[Out] 3/2*I*b*c^2*(a+b*arccot(x/c))^2+3/2*b*c*x*(a+b*arccot(x/c))^2+1/2*c^2*(a+b*
arccot(x/c))^3+1/2*x^2*(a+b*arccot(x/c))^3-3*b^2*c^2*(a+b*arccot(x/c))*ln(2
-2/(1-I*c/x))+3/2*I*b^3*c^2*polylog(2,-1+2/(1-I*c/x))
```

Rubi [A]
time = 0.21, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4948, 4946, 5038, 5044, 4988, 2497, 5004}

$$-3b^2c^2\log\left(2-\frac{2}{1-\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{3}{2}ibc^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}c^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{2}x^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{3}{2}bcx\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{3}{2}ib^3c^2\text{Li}_2\left(\frac{2}{1-\frac{ic}{x}}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] ((3*I)/2)*b*c^2*(a + b*ArcCot[x/c])^2 + (3*b*c*x*(a + b*ArcCot[x/c])^2)/2 +
 (c^2*(a + b*ArcCot[x/c])^3)/2 + (x^2*(a + b*ArcCot[x/c])^3)/2 - 3*b^2*c^2*
(a + b*ArcCot[x/c])*Log[2 - 2/(1 - (I*c)/x)] + ((3*I)/2)*b^3*c^2*PolyLog[2,
 -1 + 2/(1 - (I*c)/x)]
```

Rule 2497

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
```

```
[(m + 1)/n]]
```

Rule 4988

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Di
st[b*c*(p/d), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
+ c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5038

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 5044

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Di
st[I/d, Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

Rubi steps

$$\int x\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 dx = \int \left(\frac{1}{8}x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3+\frac{3}{8}ibx\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\right.\right.$$

$$= \frac{1}{8}\int x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3 dx+\frac{1}{8}(3ib)\int x\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2 \mathrm{lo}$$

$$= -\left(\frac{1}{8}\text{Subst}\left(\int \frac{(2a+ib\log(1-icx))^3}{x^3}dx,x,\frac{1}{x}\right)\right)+\frac{1}{8}(3ib)\int\left(-4a^2x\log\left(1\right.\right.$$

$$= \frac{1}{16}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3+\frac{1}{16}ib^3x^2\log^3\left(1+\frac{ic}{x}\right)-\frac{1}{2}\left(3ia^2b\right)\int x\log\left(\right.$$

$$= \frac{1}{16}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{3}{4}ia^2bx^2\log\left(1+\frac{ic}{x}\right)+\frac{3}{4}ab^2x^2\log\left(1-\frac{ic}{x}\right.$$

$$= \frac{1}{16}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-\frac{3}{4}ia^2bx^2\log\left(1+\frac{ic}{x}\right)+\frac{3}{4}ab^2x^2\log\left(1-\frac{ic}{x}\right.$$

$$= \frac{3}{16}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{16}x^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3-$$

$$= \frac{3}{4}a^2bcx+\frac{3}{16}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2+\frac{1}{16}x^2\left(2a+ib\log\left(1-\right.\right.$$

$$= \frac{3}{4}a^2bcx+\frac{3}{4}iab^2cx\log\left(1-\frac{ic}{x}\right)+\frac{3}{16}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-$$

$$= \frac{3}{4}a^2bcx+\frac{3}{4}iab^2cx\log\left(1-\frac{ic}{x}\right)+\frac{3}{16}bc\left(1-\frac{ic}{x}\right)x\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2-$$

$$= \frac{3}{4}a^2bcx+\frac{3}{4}ab^2c^2\log\left(i-\frac{c}{x}\right)+\frac{3}{4}iab^2cx\log\left(1-\frac{ic}{x}\right)+\frac{3}{16}bc\left(1-\frac{ic}{x}\right)x\left(2a\right.$$

$$= \frac{3}{4}a^2bcx+\frac{3}{4}ab^2c^2\log\left(i-\frac{c}{x}\right)+\frac{3}{4}iab^2cx\log\left(1-\frac{ic}{x}\right)+\frac{3}{16}bc\left(1-\frac{ic}{x}\right)x\left(2a\right.$$

$$= \frac{3}{4}a^2bcx+\frac{3}{4}ab^2c^2\log\left(i-\frac{c}{x}\right)+\frac{3}{4}iab^2cx\log\left(1-\frac{ic}{x}\right)+\frac{3}{16}bc\left(1-\frac{ic}{x}\right)x\left(2a\right.$$

$$= \frac{3}{4}a^2bcx+\frac{3}{4}ab^2c^2\log\left(i-\frac{c}{x}\right)+\frac{3}{4}iab^2cx\log\left(1-\frac{ic}{x}\right)+\frac{3}{16}bc\left(1-\frac{ic}{x}\right)x\left(2a\right.$$

Mathematica [A]
time = 0.19, size = 174, normalized size = 1.20

$$\frac{1}{2}\left(3b^2\left(bc(ic+x)+a\left(c^2+x^2\right)\right)\text{ArcTan}\left(\frac{c}{x}\right)^2+b^3\left(c^2+x^2\right)\text{ArcTan}\left(\frac{c}{x}\right)^3+3b\text{ArcTan}\left(\frac{c}{x}\right)\left(a\left(2bcx+a\left(c^2+x^2\right)\right)-2b^2c^2\log\left(1-e^{2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)\right)+a\left(ax(3bc+ax)-6b^2c^2\log\left(\frac{c}{\sqrt{1+\frac{c^2}{x^2}}\,x}\right)\right)+3ib^3c^2\text{PolyLog}\left(2,e^{2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcTan[c/x])^3,x]
```

```
[Out] (3*b^2*(b*c*(I*c + x) + a*(c^2 + x^2))*ArcTan[c/x]^2 + b^3*(c^2 + x^2)*ArcT
an[c/x]^3 + 3*b*ArcTan[c/x]*(a*(2*b*c*x + a*(c^2 + x^2)) - 2*b^2*c^2*Log[1
- E^((2*I)*ArcTan[c/x])]) + a*(a*x*(3*b*c + a*x) - 6*b^2*c^2*Log[c/(Sqrt[1
+ c^2/x^2]*x)]) + (3*I)*b^3*c^2*PolyLog[2, E^((2*I)*ArcTan[c/x])])/2
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(131) = 262$.
time = 2.49, size = 473, normalized size = 3.26

method	result
derivativedivides	$-c^2\left(-\frac{3b^2ax\arctan\left(\frac{c}{x}\right)}{c}-\frac{3ib^3\operatorname{dilog}\left(1-\frac{ic}{x}\right)}{2}+\frac{3ib^3\operatorname{dilog}\left(1+\frac{ic}{x}\right)}{2}+\frac{3ib^3\operatorname{dilog}\left(-\frac{i\left(\frac{c}{x}+i\right)}{2}\right)}{4}-\frac{3ib^3\operatorname{dilog}\left(\frac{i\left(\frac{c}{x}\right.}{2}\right.}{4}\right.$
default	$-c^2\left(-\frac{3b^2ax\arctan\left(\frac{c}{x}\right)}{c}-\frac{3ib^3\operatorname{dilog}\left(1-\frac{ic}{x}\right)}{2}+\frac{3ib^3\operatorname{dilog}\left(1+\frac{ic}{x}\right)}{2}+\frac{3ib^3\operatorname{dilog}\left(-\frac{i\left(\frac{c}{x}+i\right)}{2}\right)}{4}-\frac{3ib^3\operatorname{dilog}\left(\frac{i\left(\frac{c}{x}\right.}{2}\right.}{4}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arctan(c/x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -c^2*(-3/8*I*b^3*ln(c/x+I)^2-3*b^2*a/c*x*arctan(c/x)-3/2*b^2*a/c^2*x^2*arct
an(c/x)^2-3/2*a^2*b/c^2*x^2*arctan(c/x)-1/2*b^3*arctan(c/x)^3+3*b^3*ln(c/x)
*arctan(c/x)-3/2*b^2*a*arctan(c/x)^2-3/2*b^2*a*ln(1+c^2/x^2)+3*b^2*a*ln(c/x
)-3/2*a^2*b*arctan(c/x)-3/2*b^3*arctan(c/x)*ln(1+c^2/x^2)-3/2*a^2*b/c*x-3/2
*b^3*arctan(c/x)^2/c*x+3/2*I*b^3*ln(c/x)*ln(1+I*c/x)+3/4*I*b^3*ln(c/x-I)*ln
(-1/2*I*(c/x+I))-3/4*I*b^3*ln(c/x+I)*ln(1/2*I*(c/x-I))-1/2*b^3/c^2*x^2*arct
an(c/x)^3+3/4*I*b^3*ln(c/x+I)*ln(1+c^2/x^2)-3/4*I*b^3*ln(c/x-I)*ln(1+c^2/x^
2)-3/2*I*b^3*ln(c/x)*ln(1-I*c/x)+3/2*I*b^3*dilog(1+I*c/x)-3/2*I*b^3*dilog(1
-I*c/x)+3/4*I*b^3*dilog(-1/2*I*(c/x+I))+3/8*I*b^3*ln(c/x-I)^2-3/4*I*b^3*dil
og(1/2*I*(c/x-I))-1/2*a^3/c^2*x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a*b^2*x^2*arctan(c/x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arctan(c/x) - (c*arcta
n(x/c) - x)*c)*a^2*b - 3/2*((arctan(x/c)^2 - log(c^2 + x^2))*c^2 + 2*(c*arc
```

```
tan(x/c) - x)*c*arctan(c/x))*a*b^2 + 1/32*(12*c^2*arctan(c/x)^2*arctan(x/c)
 + 8*c^2*arctan2(c, x)^3 + 8*x^2*arctan2(c, x)^3 + 4*(3*arctan(c/x)*arctan(
x/c)^2/c + arctan(x/c)^3/c)*c^3 + 12*c*x*arctan2(c, x)^2 + 96*c^3*integrate
(1/32*log(c^2 + x^2)^2/(c^2 + x^2), x) - 3*c*x*log(c^2 + x^2)^2 + 512*c^2*i
ntegrate(1/32*x*arctan(c/x)^3/(c^2 + x^2), x) + 768*c^2*integrate(1/32*x*ar
ctan(c/x)/(c^2 + x^2), x) + 384*c*integrate(1/32*x^2*arctan(c/x)^2/(c^2 + x
^2), x) + 96*c*integrate(1/32*x^2*log(c^2 + x^2)^2/(c^2 + x^2), x) + 384*c*
integrate(1/32*x^2*log(c^2 + x^2)/(c^2 + x^2), x) + 512*integrate(1/32*x^3*
arctan(c/x)^3/(c^2 + x^2), x))*b^3
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x))^3,x, algorithm="fricas")
[Out] integral(b^3*x*arctan(c/x)^3 + 3*a*b^2*x*arctan(c/x)^2 + 3*a^2*b*x*arctan(c
/x) + a^3*x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a + b\operatorname{atan}\left(\frac{c}{x}\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*atan(c/x))**3,x)
[Out] Integral(x*(a + b*atan(c/x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arctan(c/x))^3,x, algorithm="giac")
[Out] integrate((b*arctan(c/x) + a)^3*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a + b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*atan(c/x))^3,x)
[Out] int(x*(a + b*atan(c/x))^3, x)
```

3.150 $\int \left(a+b\mathbf{ArcTan}\left(\frac{c}{x}\right)\right)^3 \, dx$

Optimal. Leaf size=119

$$ic\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3-3bc\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(\frac{2c}{c+ix}\right)+3ib^2c\left(a+b\cot^{-1}\left(\frac{x}{c}\right.\right.$$

```
[Out] I*c*(a+b*arccot(x/c))^3+x*(a+b*arccot(x/c))^3-3*b*c*(a+b*arccot(x/c))^2*ln(
2*c/(c+I*x))+3*I*b^2*c*(a+b*arccot(x/c))*polylog(2,1-2*c/(c+I*x))-3/2*b^3*c
*polylog(3,1-2*c/(c+I*x))
```

Rubi [A]
time = 0.16, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {4934, 4931, 5041, 4965, 5005, 5115, 6745}

$$3ib^2c\text{Li}_2\left(1-\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+ic\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3+x\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3-3bc\log\left(\frac{2c}{c+ix}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{3}{2}b^3c\text{Li}_3\left(1-\frac{2c}{c+ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])^3,x]
```

```
[Out] I*c*(a + b*ArcCot[x/c])^3 + x*(a + b*ArcCot[x/c])^3 - 3*b*c*(a + b*ArcCot[x
/c])^2*Log[(2*c)/(c + I*x)] + (3*I)*b^2*c*(a + b*ArcCot[x/c])*PolyLog[2, 1
- (2*c)/(c + I*x)] - (3*b^3*c*PolyLog[3, 1 - (2*c)/(c + I*x)])/2
```

Rule 4931

```
Int[((a_.) + ArcCot[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcCot[c*x^n])^p, x] + Dist[b*c*n*p, Int[x^n*((a + b*ArcCot[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4934

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[(a + b*A
rcCot[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && ILtQ[n, 0]
```

Rule 4965

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcCot[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] - Dist[b*c*(
p/e), Int[(a + b*ArcCot[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5005

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[-(a + b*ArcCot[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
 c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5041

```
Int[(((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[I*((a + b*ArcCot[c*x])^(p + 1)/(b*e*(p + 1))), x] - Dist[
1/(c*d), Int[(a + b*ArcCot[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5115

```
Int[(Log[u_]*((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcCot[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] - Dist[b*p*(I/2), Int[(a + b*ArcCot[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3 dx = \int \left(a^3+\frac{3}{2}ia^2b\log\left(1-\frac{ic}{x}\right)-\frac{3}{4}ab^2\log^2\left(1-\frac{ic}{x}\right)-\frac{1}{8}ib^3\log^3\left(1-\frac{ic}{x}\right)-\frac{3}{2}\right.$$

$$= a^3x+\frac{1}{2}(3ia^2b)\int\log\left(1-\frac{ic}{x}\right)dx-\frac{1}{2}(3ia^2b)\int\log\left(1+\frac{ic}{x}\right)dx-\frac{1}{4}(3ab^2$$

$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3$$

$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3$$

$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3$$

$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3$$

$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3$$

$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3$$

$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3$$

$$= a^3x+\frac{3}{2}ia^2bx\log\left(1-\frac{ic}{x}\right)+\frac{3}{4}ab^2(ic-x)\log^2\left(1-\frac{ic}{x}\right)+\frac{1}{8}ib^3(ic-x)\log^3$$

Mathematica [A]
time = 0.17, size = 215, normalized size = 1.81

$$a^3x+3a^2bx\text{ArcTan}\left(\frac{c}{x}\right)+\frac{3}{2}a^2bc\log\left(c^2+x^2\right)-3ab^2\left(-\left((ic+x)\text{ArcTan}\left(\frac{c}{x}\right)^2\right)+2c\text{ArcTan}\left(\frac{c}{x}\right)\log\left(1-e^{2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)-ic\text{PolyLog}\left(2,e^{2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)\right)-\frac{1}{8}b^3\left(-ic\pi^3+8ic\text{ArcTan}\left(\frac{c}{x}\right)^3-8x\text{ArcTan}\left(\frac{c}{x}\right)^3+24c\text{ArcTan}\left(\frac{c}{x}\right)^2\log\left(1-e^{-2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)+24ic\text{ArcTan}\left(\frac{c}{x}\right)\text{PolyLog}\left(2,e^{-2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)+12c\text{PolyLog}\left(3,e^{-2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])^3,x]
```

```
[Out] a^3*x + 3*a^2*b*x*ArcTan[c/x] + (3*a^2*b*c*Log[c^2 + x^2])/2 - 3*a*b^2*(-((
I*c + x)*ArcTan[c/x]^2) + 2*c*ArcTan[c/x]*Log[1 - E^((2*I)*ArcTan[c/x])] -
I*c*PolyLog[2, E^((2*I)*ArcTan[c/x])]) - (b^3*((-I)*c*Pi^3 + (8*I)*c*ArcTan
[c/x]^3 - 8*x*ArcTan[c/x]^3 + 24*c*ArcTan[c/x]^2*Log[1 - E^((-2*I)*ArcTan[c
/x])] + (24*I)*c*ArcTan[c/x]*PolyLog[2, E^((-2*I)*ArcTan[c/x])] + 12*c*Poly
Log[3, E^((-2*I)*ArcTan[c/x])]))/8
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 3.20, size = 2333, normalized size = 19.61

method	result	size
derivativedivides	Expression too large to display	2333
default	Expression too large to display	2333

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -c*(-a^3/c*x+3/4*I*b^3*arctan(c/x)^2*Pi*csgn(I/(1+(1+I*c/x)^2/(1+c^2/x^2))^
2)*csgn(I*(1+I*c/x)^2/(1+c^2/x^2)/(1+(1+I*c/x)^2/(1+c^2/x^2))^2)^2-6*I*b^3*
arctan(c/x)*polylog(2,-(1+I*c/x)/(1+c^2/x^2)^(1/2))+3/2*I*b^3*arctan(c/x)^2
*Pi-6*I*b^3*arctan(c/x)*polylog(2,(1+I*c/x)/(1+c^2/x^2)^(1/2))-b^3/c*x*arct
an(c/x)^3-3*I*b^2*a*dilog(1-I*c/x)+3/2*I*b^3*arctan(c/x)^2*Pi*csgn(I*((1+I*
c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(((1+I*c/x)^2/(1+c^2
/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))-3/2*I*b^3*arctan(c/x)^2*Pi*csgn(I*((1
+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2
/(1+c^2/x^2)))^2-3/4*I*b^3*arctan(c/x)^2*Pi*csgn(I*(1+I*c/x)/(1+c^2/x^2)^(1
/2))^2*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))-3/2*I*b^3*arctan(c/x)^2*Pi*csgn(I*((
1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(((1+I*c/x)^2/(1
+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2+3/2*I*b^3*arctan(c/x)^2*Pi*csgn
(I*(1+I*c/x)/(1+c^2/x^2)^(1/2))*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))^2-3/2*I*b^3
*arctan(c/x)^2*Pi*csgn(I/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(I*((1+I*c/x)^2/(
1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2-3/2*I*b^3*arctan(c/x)^2*Pi*csg
n(I*(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(I*(1+(1+I*c/x)^2/(1+c^2/x^2))^2)^2+3/
4*I*b^3*arctan(c/x)^2*Pi*csgn(I*(1+(1+I*c/x)^2/(1+c^2/x^2)))^2*csgn(I*(1+(1
+I*c/x)^2/(1+c^2/x^2))^2)+3/4*I*b^3*arctan(c/x)^2*Pi*csgn(I*(1+I*c/x)^2/(1+
c^2/x^2))*csgn(I*(1+I*c/x)^2/(1+c^2/x^2)/(1+(1+I*c/x)^2/(1+c^2/x^2))^2)^2-I
*b^3*arctan(c/x)^3-3*b^2*a*arctan(c/x)*ln(1+c^2/x^2)+6*b^2*a*ln(c/x)*arctan
(c/x)+3*b^3*arctan(c/x)^2*ln((1+I*c/x)/(1+c^2/x^2)^(1/2))-3*b^3*arctan(c/x)
^2*ln((1+I*c/x)^2/(1+c^2/x^2)-1)+3*b^3*arctan(c/x)^2*ln(1+(1+I*c/x)/(1+c^2/
x^2)^(1/2))+3*b^3*arctan(c/x)^2*ln(1-(1+I*c/x)/(1+c^2/x^2)^(1/2))-3/2*b^3*a
rctan(c/x)^2*ln(1+c^2/x^2)+3*b^3*ln(c/x)*arctan(c/x)^2+3*b^3*arctan(c/x)^2*
ln(2)-3/2*a^2*b*ln(1+c^2/x^2)+3*a^2*b*ln(c/x)+3/4*I*b^3*arctan(c/x)^2*Pi*cs
gn(I*(1+(1+I*c/x)^2/(1+c^2/x^2))^2)^3-3/4*I*b^3*arctan(c/x)^2*Pi*csgn(I*(1+
I*c/x)^2/(1+c^2/x^2)/(1+(1+I*c/x)^2/(1+c^2/x^2))^2)^3-3/2*I*b^3*arctan(c/x)
^2*Pi*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2+3/2*I
*b^3*arctan(c/x)^2*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+
c^2/x^2)))^3-3*a^2*b/c*x*arctan(c/x)-3/2*I*b^2*a*ln(c/x-I)*ln(1+c^2/x^2)+3/
2*I*b^2*a*ln(c/x-I)*ln(-1/2*I*(c/x+I))+3/2*I*b^2*a*ln(c/x+I)*ln(1+c^2/x^2)-
3/2*I*b^2*a*ln(c/x+I)*ln(1/2*I*(c/x-I))+3*I*b^2*a*ln(c/x)*ln(1+I*c/x)-3*I*b
^2*a*ln(c/x)*ln(1-I*c/x)-3/4*I*b^3*arctan(c/x)^2*Pi*csgn(I/(1+(1+I*c/x)^2/(
1+c^2/x^2))^2)*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))*csgn(I*(1+I*c/x)^2/(1+c^2/x^
2)/(1+(1+I*c/x)^2/(1+c^2/x^2))^2)+3/2*I*b^3*arctan(c/x)^2*Pi*csgn(I*((1+I*c
/x)^2/(1+c^2/x^2)-1))*csgn(I/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(I*((1+I*c/x)
```

```
^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))+6*b^3*polylog(3,(1+I*c/x)/(1
+c^2/x^2)^(1/2))+6*b^3*polylog(3,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-3*b^2*a/c*x*
arctan(c/x)^2-3/4*I*b^3*arctan(c/x)^2*Pi*csgn(I*(1+I*c/x)^2/(1+c^2/x^2))^3+
3/2*I*b^3*arctan(c/x)^2*Pi*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/
(1+c^2/x^2)))^3+3/4*I*b^2*a*ln(c/x-I)^2+3/2*I*b^2*a*dilog(-1/2*I*(c/x+I))-3
/4*I*b^2*a*ln(c/x+I)^2-3/2*I*b^2*a*dilog(1/2*I*(c/x-I))+3*I*b^2*a*dilog(1+I
*c/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3,x, algorithm="maxima")
```

```
[Out] 7/8*b^3*c*arctan(c/x)^3*arctan(x/c) + 3*a*b^2*c*arctan(c/x)^2*arctan(x/c) +
 1/8*b^3*x*arctan2(c, x)^3 - 3/32*b^3*x*arctan2(c, x)*log(c^2 + x^2)^2 + (3
*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^3/c)*a*b^2*c^2 + 7/32*(6*arctan(
c/x)^2*arctan(x/c)^2/c + 4*arctan(c/x)*arctan(x/c)^3/c + arctan(x/c)^4/c)*b
^3*c^2 + 3*b^3*c^2*integrate(1/32*arctan(c/x)*log(c^2 + x^2)^2/(c^2 + x^2),
 x) + 12*b^3*c*integrate(1/32*x*arctan(c/x)^2/(c^2 + x^2), x) - 3*b^3*c*int
egrate(1/32*x*log(c^2 + x^2)^2/(c^2 + x^2), x) + 3/2*(2*x*arctan(c/x) + c*l
og(c^2 + x^2))*a^2*b + a^3*x + 28*b^3*integrate(1/32*x^2*arctan(c/x)^3/(c^2
 + x^2), x) + 3*b^3*integrate(1/32*x^2*arctan(c/x)*log(c^2 + x^2)^2/(c^2 +
x^2), x) + 96*a*b^2*integrate(1/32*x^2*arctan(c/x)^2/(c^2 + x^2), x) + 12*b
^3*integrate(1/32*x^2*arctan(c/x)*log(c^2 + x^2)/(c^2 + x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x) +
a^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{atan}{\left(\frac{c}{x} \right)}\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**3,x)
```

```
[Out] Integral((a + b*atan(c/x))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arctan(c/x) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))^3,x)
```

```
[Out] int((a + b*atan(c/x))^3, x)
```

3.151 $\int \frac{\left(a+b\text{ArcTan}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$

Optimal. Leaf size=230

$$-2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)+\frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\text{PolyLog}\left(2,1-\frac{2}{1+\frac{ic}{x}}\right)-\frac{3}{2}ib\left(a+b\right.$$

```
[Out] 2*(a+b*arccot(x/c))^3*arctanh(-1+2/(1+I*c/x))+3/2*I*b*(a+b*arccot(x/c))^2*p
olylog(2,1-2/(1+I*c/x))-3/2*I*b*(a+b*arccot(x/c))^2*polylog(2,-1+2/(1+I*c/x
))+3/2*b^2*(a+b*arccot(x/c))*polylog(3,1-2/(1+I*c/x))-3/2*b^2*(a+b*arccot(x
/c))*polylog(3,-1+2/(1+I*c/x))-3/4*I*b^3*polylog(4,1-2/(1+I*c/x))+3/4*I*b^3
*polylog(4,-1+2/(1+I*c/x))
```

Rubi [A]

time = 0.31, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {4944, 4942, 5108, 5004, 5114, 5118, 6745}

$$\frac{3}{2}b^2\text{Li}_3\left(1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)-\frac{3}{2}b^2\text{Li}_3\left(\frac{2}{\frac{ic}{x}+1}-1\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)+\frac{3}{2}ib\text{Li}_2\left(1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{3}{2}ib\text{Li}_2\left(\frac{2}{\frac{ic}{x}+1}-1\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2-2\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3-\frac{3}{4}ib^3\text{Li}_4\left(1-\frac{2}{\frac{ic}{x}+1}\right)+\frac{3}{4}ib^3\text{Li}_4\left(\frac{2}{\frac{ic}{x}+1}-1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])^3/x,x]
```

```
[Out] -2*(a + b*ArcCot[x/c])^3*ArcTanh[1 - 2/(1 + (I*c)/x)] + ((3*I)/2)*b*(a + b*
ArcCot[x/c])^2*PolyLog[2, 1 - 2/(1 + (I*c)/x)] - ((3*I)/2)*b*(a + b*ArcCot[
x/c])^2*PolyLog[2, -1 + 2/(1 + (I*c)/x)] + (3*b^2*(a + b*ArcCot[x/c])*PolyL
og[3, 1 - 2/(1 + (I*c)/x)])/2 - (3*b^2*(a + b*ArcCot[x/c])*PolyLog[3, -1 +
2/(1 + (I*c)/x)])/2 - ((3*I)/4)*b^3*PolyLog[4, 1 - 2/(1 + (I*c)/x)] + ((3*I
)/4)*b^3*PolyLog[4, -1 + 2/(1 + (I*c)/x)]
```

Rule 4942

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Dist[2*b*c*p, Int[(a + b*
ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5108

```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Dist[1/2, Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e*
x^2)), x], x] - Dist[1/2, Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] && Eq
Q[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 5118

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3}{x}\,dx = -\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx)\right)^3}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right) + (6bc)\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx\right.}{1}\right.$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right) - (3bc)\text{Subst}\left(\int \frac{\left(a+b\tan^{-1}(cx\right.}{1+c^2}\right.$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right) + \frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\text{Li}_2\left(1-\right.$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right) + \frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\text{Li}_2\left(1-\right.$$

$$= -2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3\tanh^{-1}\left(1-\frac{2}{1+\frac{ic}{x}}\right) + \frac{3}{2}ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\text{Li}_2\left(1-\right.$$

Mathematica [A]
time = 0.14, size = 219, normalized size = 0.95

$$-2\left(a+b\text{ArcTan}\left(\frac{c}{x}\right)\right)^3\tanh^{-1}\left(\frac{c+ix}{c-ix}\right)+\frac{3}{4}ib\left(2\left(a+b\text{ArcTan}\left(\frac{c}{x}\right)\right)^2\text{PolyLog}\left(2,\frac{c+ix}{c-ix}\right)-2\left(a+b\text{ArcTan}\left(\frac{c}{x}\right)\right)^2\text{PolyLog}\left(2,\frac{-ic+x}{ic+x}\right)+b\left(-2i\left(a+b\text{ArcTan}\left(\frac{c}{x}\right)\right)\text{PolyLog}\left(3,\frac{c+ix}{c-ix}\right)+2i\left(a+b\text{ArcTan}\left(\frac{c}{x}\right)\right)\text{PolyLog}\left(3,\frac{-ic+x}{ic+x}\right)+b\left(-\text{PolyLog}\left(4,\frac{c+ix}{c-ix}\right)+\text{PolyLog}\left(4,\frac{-ic+x}{ic+x}\right)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])^3/x,x]
```

```
[Out] -2*(a + b*ArcTan[c/x])^3*ArcTanh[(c + I*x)/(c - I*x)] + ((3*I)/4)*b*(2*(a +
 b*ArcTan[c/x])^2*PolyLog[2, (c + I*x)/(c - I*x)] - 2*(a + b*ArcTan[c/x])^2
*PolyLog[2, ((-I)*c + x)/(I*c + x)] + b*((-2*I)*(a + b*ArcTan[c/x])*PolyLog
[3, (c + I*x)/(c - I*x)] + (2*I)*(a + b*ArcTan[c/x])*PolyLog[3, ((-I)*c + x
)/(I*c + x)] + b*(-PolyLog[4, (c + I*x)/(c - I*x)] + PolyLog[4, ((-I)*c + x
)/(I*c + x)])))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.87, size = 2542, normalized size = 11.05

method	result	size
derivativedivides	Expression too large to display	2542
default	Expression too large to display	2542

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^3/x,x,method=_RETURNVERBOSE)
```

[Out] 3/2*I*b^2*a*Pi*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2*arctan(c/x)^2-3/2*I*b^2*a*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^3*arctan(c/x)^2-3/2*I*b^2*a*Pi*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^3*arctan(c/x)^2-1/2*I*b^3*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))*arctan(c/x)^3+1/2*I*b^3*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2*arctan(c/x)^3+1/2*I*b^3*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2*arctan(c/x)^3+1/2*I*b^3*Pi*csgn(I/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2*arctan(c/x)^3-6*I*b^3*polylog(4,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-6*I*b^3*polylog(4,(1+I*c/x)/(1+c^2/x^2)^(1/2))-b^3*ln(c/x)*arctan(c/x)^3+b^3*arctan(c/x)^3*ln((1+I*c/x)^2/(1+c^2/x^2)-1)-b^3*arctan(c/x)^3*ln(1+(1+I*c/x)/(1+c^2/x^2)^(1/2))-6*b^3*arctan(c/x)*polylog(3,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-b^3*arctan(c/x)^3*ln(1-(1+I*c/x)/(1+c^2/x^2)^(1/2))-6*b^3*arctan(c/x)*polylog(3,(1+I*c/x)/(1+c^2/x^2)^(1/2))+3/2*b^3*arctan(c/x)*polylog(3,-(1+I*c/x)^2/(1+c^2/x^2))+3/2*b^2*a*polylog(3,-(1+I*c/x)^2/(1+c^2/x^2))-6*b^2*a*polylog(3,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-6*b^2*a*polylog(3,(1+I*c/x)/(1+c^2/x^2)^(1/2))-3/2*I*b^2*a*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))*arctan(c/x)^2-1/2*I*b^3*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))*arctan(c/x)^3+3/2*I*b^2*a*Pi*csgn(I/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2*arctan(c/x)^2+3/2*I*b^2*a*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2*arctan(c/x)^2+3/2*I*b^2*a*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1))*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2*arctan(c/x)^2-3/2*I*b^2*a*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))*arctan(c/x)^2+3/4*I*b^3*polylog(4,-(1+I*c/x)^2/(1+c^2/x^2))-a^3*ln(c/x)-1/2*I*b^3*Pi*csgn(I*((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^3*arctan(c/x)^3-1/2*I*b^3*Pi*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^3*arctan(c/x)^3+1/2*I*b^3*Pi*csgn(((1+I*c/x)^2/(1+c^2/x^2)-1)/(1+(1+I*c/x)^2/(1+c^2/x^2)))^2*arctan(c/x)^3-3/2*I*a^2*b*ln(c/x)*ln(1+I*c/x)+3/2*I*a^2*b*ln(c/x)*ln(1-I*c/x)+6*I*b^2*a*arctan(c/x)*polylog(2,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-3/2*I*b^2*a*Pi*arctan(c/x)^2-3*I*b^2*a*arctan(c/x)*polylog(2,-(1+I*c/x)^2/(1+c^2/x^2))+6*I*b^2*a*arctan(c/x)*polylog(2,(1+I*c/x)/(1+c^2/x^2)^(1/2))+3*I*b^3*arctan(c/x)^2*polylog(2,-(1+I*c/x)/(1+c^2/x^2)^(1/2))-3/2*I*b^3*arctan(c/x)^2*polylog(2,-(1+I*c/x)^2/(1+c^2/x^2))-3/2*I*a^2*b*dilog(1+I*c/x)+3/2*I*a^2*b*dilog(1-I*c/x)-3*b^2*a*ln(c/x)*arctan(c/x)^2+3*b^2*a*arctan(c/x)^2*ln((1+I*c/x)^2/(1+c^2/x^2)-1)-3*b^2*a*arctan(c/x)^2*ln(1+(1+I*c/x)/(1+c^2/x^2)^(1/2))-3*b^2*a*arctan(c/x)^2*ln(1-(1+I*c/x)/(1+c^2/x^2)^(1/2))-3*a^2*b*ln(c/x)*arctan(c/x)-1/2*I*b^3*Pi*arctan(c/x)^3+3*I*b^3*a

```
rctan(c/x)^2*polylog(2,(1+I*c/x)/(1+c^2/x^2)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x,x, algorithm="maxima")
[Out] a^3*log(x) + 1/32*integrate((28*b^3*arctan2(c, x)^3 + 3*b^3*arctan2(c, x)*l
og(c^2 + x^2)^2 + 96*a*b^2*arctan2(c, x)^2 + 96*a^2*b*arctan2(c, x))/x, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x,x, algorithm="fricas")
[Out] integral((b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x) +
 a^3)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}\left(\frac{c}{x}\right)\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**3/x,x)
[Out] Integral((a + b*atan(c/x))**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x,x, algorithm="giac")
[Out] integrate((b*arctan(c/x) + a)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))^3/x,x)
[Out] int((a + b*atan(c/x))^3/x, x)
```

3.152 $\int \frac{\left(a+b\,\mathbf{ArcTan}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$

Optimal. Leaf size=136

$$-\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{c}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{x}-\frac{3b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(\frac{2}{1+\frac{ic}{x}}\right)}{c}-\frac{3ib^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}}{c}$$

```
[Out] -I*(a+b*arccot(x/c))^3/c-(a+b*arccot(x/c))^3/x-3*b*(a+b*arccot(x/c))^2*ln(2
/(1+I*c/x))/c-3*I*b^2*(a+b*arccot(x/c))*polylog(2,1-2/(1+I*c/x))/c-3/2*b^3*
polylog(3,1-2/(1+I*c/x))/c
```

Rubi [A]
time = 0.16, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {4948, 4930, 5040, 4964, 5004, 5114, 6745}

$$-\frac{3ib^2\text{Li}_2\left(1-\frac{2}{\frac{ic}{x}+1}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{c}-\frac{i\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{c}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{x}-\frac{3b\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{c}-\frac{3b^3\text{Li}_3\left(1-\frac{2}{\frac{ic}{x}+1}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])^3/x^2,x]
```

```
[Out] ((-I)*(a + b*ArcCot[x/c])^3)/c - (a + b*ArcCot[x/c])^3/x - (3*b*(a + b*ArcC
ot[x/c])^2*Log[2/(1 + (I*c)/x)])/c - ((3*I)*b^2*(a + b*ArcCot[x/c])*PolyLog
[2, 1 - 2/(1 + (I*c)/x)])/c - (3*b^3*PolyLog[3, 1 - 2/(1 + (I*c)/x)])/(2*c)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
```

```
x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 5114

```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Dist[b*p*(I/2), Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^
2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

Rule 6745

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{8x^2} + \frac{3ib\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)}{8x^2} - \frac{3ib^2(-$$

$$= \frac{1}{8}\int \frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{x^2}\,dx + \frac{1}{8}(3ib)\int \frac{\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)}{x^2}$$

$$= -\left(\frac{1}{8}\text{Subst}\left(\int (2a+ib\log(1-icx))^3\,dx, x, \frac{1}{x}\right)\right) - \frac{1}{8}(3ib)\text{Subst}\left(\int (-2ia+b$$

$$= -\frac{3ib\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)}{8x} - \frac{3ib^2\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)\log^2\left(1+\frac{ic}{x}\right.}{8x}$$

$$= -\frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{8c} - \frac{3ib\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)}{8x} - 3i$$

$$= -\frac{3b\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{8c} - \frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{8c} - \frac{3ib(2i}{}$$

$$= \frac{3ab^2}{2x} + \frac{3ib^3}{4x} - \frac{3b\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{8c} - \frac{i\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right.\right.}{8c}$$

$$= \frac{3ab^2}{2x} - \frac{3b^3\left(1-\frac{ic}{x}\right)\log\left(1-\frac{ic}{x}\right)}{4c} - \frac{3b\left(1-\frac{ic}{x}\right)\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)^2}{8c} - \frac{3b\left(1-\frac{i}{x}\right.}{}$$

$$= -\frac{3ib^3}{4x} - \frac{3b^3\left(1-\frac{ic}{x}\right)\log\left(1-\frac{ic}{x}\right)}{4c} - \frac{3b\left(1-\frac{ic}{x}\right)\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)^2}{8c} - \frac{3b(1-}{}$$

$$= -\frac{3b\left(1-\frac{ic}{x}\right)\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)^2}{8c} - \frac{3b\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{8c} - \frac{i(1-}{}$$

$$= -\frac{3b\left(1-\frac{ic}{x}\right)\left(2ia-b\log\left(1-\frac{ic}{x}\right)\right)^2}{8c} - \frac{3b\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{8c} - \frac{i(1-}{}$$

<u>Mathematica</u> [A]
time = 0.07, size = 222, normalized size = 1.63

$$-\frac{2a^3c+6a^2bc\text{ArcTan}\left(\frac{c}{x}\right)+6ab^2c\text{ArcTan}\left(\frac{c}{x}\right)^2-6iab^2x\text{ArcTan}\left(\frac{c}{x}\right)^2+2b^3c\text{ArcTan}\left(\frac{c}{x}\right)^3-2ib^3x\text{ArcTan}\left(\frac{c}{x}\right)^3+12ab^2x\text{ArcTan}\left(\frac{c}{x}\right)\log\left(1+e^{2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)+6b^3x\text{ArcTan}\left(\frac{c}{x}\right)^2\log\left(1+e^{2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)-3a^2bx\log\left(1+\frac{c^2}{x^2}\right)-6ib^2x\left(a+b\text{ArcTan}\left(\frac{c}{x}\right)\right)\text{PolyLog}\left(2,-e^{2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)+3b^3x\text{PolyLog}\left(3,-e^{2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)}{2cx}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])^3/x^2,x]
```

```
[Out] -1/2*(2*a^3*c + 6*a^2*b*c*ArcTan[c/x] + 6*a*b^2*c*ArcTan[c/x]^2 - (6*I)*a*b
^2*x*ArcTan[c/x]^2 + 2*b^3*c*ArcTan[c/x]^3 - (2*I)*b^3*x*ArcTan[c/x]^3 + 12
*a*b^2*x*ArcTan[c/x]*Log[1 + E^((2*I)*ArcTan[c/x])] + 6*b^3*x*ArcTan[c/x]^2
*Log[1 + E^((2*I)*ArcTan[c/x])] - 3*a^2*b*x*Log[1 + c^2/x^2] - (6*I)*b^2*x*
```

```
(a + b*ArcTan[c/x])*PolyLog[2, -E^((2*I)*ArcTan[c/x])] + 3*b^3*x*PolyLog[3,
 -E^((2*I)*ArcTan[c/x])])/(c*x)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(129) = 258.
time = 1.47, size = 289, normalized size = 2.12

method	result
derivativedivides	$-\frac{\frac{c a^3}{x} - i b^3 \arctan\left(\frac{c}{x}\right)^3 + \frac{b^3 \arctan\left(\frac{c}{x}\right)^3 c}{x} + 3 b^3 \arctan\left(\frac{c}{x}\right)^2 \ln\left(1 + \frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right) - 3 i b^3 \arctan\left(\frac{c}{x}\right) \operatorname{polylog}\left(2, -\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right) + \frac{3b}{}}{}$
default	$-\frac{\frac{c a^3}{x} - i b^3 \arctan\left(\frac{c}{x}\right)^3 + \frac{b^3 \arctan\left(\frac{c}{x}\right)^3 c}{x} + 3 b^3 \arctan\left(\frac{c}{x}\right)^2 \ln\left(1 + \frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right) - 3 i b^3 \arctan\left(\frac{c}{x}\right) \operatorname{polylog}\left(2, -\frac{\left(1+\frac{ic}{x}\right)^2}{1+\frac{c^2}{x^2}}\right) + \frac{3b}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c*(c/x*a^3-I*b^3*arctan(c/x)^3+b^3*arctan(c/x)^3*c/x+3*b^3*arctan(c/x)^2
*ln(1+(1+I*c/x)^2/(1+c^2/x^2))-3*I*b^3*arctan(c/x)*polylog(2,-(1+I*c/x)^2/(
1+c^2/x^2))+3/2*b^3*polylog(3,-(1+I*c/x)^2/(1+c^2/x^2))+3*a^2*b*c/x*arctan(
c/x)-3/2*a^2*b*ln(1+c^2/x^2)-3*I*arctan(c/x)^2*a*b^2+3*arctan(c/x)^2*a*b^2*
c/x-3*I*polylog(2,-(1+I*c/x)^2/(1+c^2/x^2))*a*b^2+6*arctan(c/x)*ln(1+(1+I*c
/x)^2/(1+c^2/x^2))*a*b^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x^2,x, algorithm="maxima")
```

```
[Out] -3/2*a^2*b*(2*c*arctan(c/x)/x - log(c^2/x^2 + 1))/c - a^3/x - 1/32*(4*b^3*a
rctan2(c, x)^3 - 3*b^3*arctan2(c, x)*log(c^2 + x^2)^2 - (28*b^3*arctan(c/x)
^3*arctan(x/c)/c + 896*b^3*c^2*integrate(1/32*arctan(c/x)^3/(c^2*x^2 + x^4)
, x) + 96*b^3*c^2*integrate(1/32*arctan(c/x)*log(c^2 + x^2)^2/(c^2*x^2 + x^
4), x) + 3072*a*b^2*c^2*integrate(1/32*arctan(c/x)^2/(c^2*x^2 + x^4), x) +
96*a*b^2*arctan(c/x)^2*arctan(x/c)/c - 384*b^3*c*integrate(1/32*x*arctan(c/
x)^2/(c^2*x^2 + x^4), x) + 96*b^3*c*integrate(1/32*x*log(c^2 + x^2)^2/(c^2*
x^2 + x^4), x) + 32*(3*arctan(c/x)*arctan(x/c)^2/c + arctan(x/c)^3/c)*a*b^2
 + 7*(6*arctan(c/x)^2*arctan(x/c)^2/c + 4*arctan(c/x)*arctan(x/c)^3/c + arc
tan(x/c)^4/c)*b^3 + 96*b^3*integrate(1/32*x^2*arctan(c/x)*log(c^2 + x^2)^2/
(c^2*x^2 + x^4), x) - 384*b^3*integrate(1/32*x^2*arctan(c/x)*log(c^2 + x^2)
/(c^2*x^2 + x^4), x))*x)/x
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x^2,x, algorithm="fricas")
[Out] integral((b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x) +
 a^3)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}\left(\frac{c}{x}\right)\right)^{3}}{x^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**3/x**2,x)
[Out] Integral((a + b*atan(c/x))**3/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x^2,x, algorithm="giac")
[Out] integrate((b*arctan(c/x) + a)^3/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)}^3}{x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))^3/x^2,x)
[Out] int((a + b*atan(c/x))^3/x^2, x)
```

3.153 $\int \frac{\left(a+b\mathbf{ArcTan}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$

Optimal. Leaf size=147

$$\frac{3ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}+\frac{3b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2cx}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2c^2}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2x^2}+\frac{3b^2\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)\mathrm{l}}{c^2}$$

```
[Out] 3/2*I*b*(a+b*arccot(x/c))^2/c^2+3/2*b*(a+b*arccot(x/c))^2/c/x-1/2*(a+b*arcc
ot(x/c))^3/c^2-1/2*(a+b*arccot(x/c))^3/x^2+3*b^2*(a+b*arccot(x/c))*ln(2/(1+
I*c/x))/c^2+3/2*I*b^3*polylog(2,1-2/(1+I*c/x))/c^2
```

Rubi [A]
time = 0.19, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {4948, 4946, 5036, 4930, 5040, 4964, 2449, 2352, 5004}

$$\frac{3b^2\log\left(\frac{2}{1+\frac{ic}{x}}\right)\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)}{c^2}+\frac{3ib\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2c^2}-\frac{\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^3}{2x^2}+\frac{3b\left(a+b\cot^{-1}\left(\frac{x}{c}\right)\right)^2}{2cx}+\frac{3ib^3\mathrm{Li}_2\left(1-\frac{2}{\frac{ic}{x}+1}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcTan[c/x])^3/x^3,x]
```

```
[Out] (((3*I)/2)*b*(a + b*ArcCot[x/c])^2)/c^2 + (3*b*(a + b*ArcCot[x/c])^2)/(2*c*
x) - (a + b*ArcCot[x/c])^3/(2*c^2) - (a + b*ArcCot[x/c])^3/(2*x^2) + (3*b^2
*(a + b*ArcCot[x/c])*Log[2/(1 + (I*c)/x)])/c^2 + (((3*I)/2)*b^3*PolyLog[2,
1 - 2/(1 + (I*c)/x)])/c^2
```

Rule 2352

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2449

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Dist
[-e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTan[c*x])^p, x],
x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simplify
[(m + 1)/n]]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Dist[b*c*(
p/e), Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2)),
 x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 5004

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5036

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[d*(f^2/e), Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d +
e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 5040

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Di
st[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(a+b\tan^{-1}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx = \int \left(\frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{8x^3} + \frac{3ib\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)}{8x^3} - \frac{3ib^2(-2$$

$$= \frac{1}{8}\int \frac{\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{x^3}\,dx + \frac{1}{8}(3ib)\int \frac{\left(-2ia+b\log\left(1-\frac{ic}{x}\right)\right)^2\log\left(1+\frac{ic}{x}\right)}{x^3}\,d$$

$$= -\left(\frac{1}{8}\text{Subst}\left(\int x(2a+ib\log(1-icx))^3\,dx, x, \frac{1}{x}\right)\right) + \frac{1}{8}(3ib)\int\left(-\frac{4a^2\log\left(1+\frac{i}{}\right.}{x^3}\right.$$

$$= -\left(\frac{1}{8}\text{Subst}\left(\int\left(-\frac{i(2a+ib\log(1-icx))^3}{c} + \frac{i(1-icx)(2a+ib\log(1-icx))^3}{c}\right)\right.\right.$$

$$= -\frac{3ab^2\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)}{4x^2} + \frac{1}{2}\left(3ia^2b\right)\text{Subst}\left(\int x\log(1+icx)\,dx, x, \frac{1}{x}\right) + \frac{1}{4}$$

$$= \frac{3ia^2b\log\left(1+\frac{ic}{x}\right)}{4x^2} - \frac{3ab^2\log\left(1-\frac{ic}{x}\right)\log\left(1+\frac{ic}{x}\right)}{4x^2} + \frac{1}{4}\left(3ab^2\right)\text{Subst}\left(\int\left(\frac{i\log^2(1}{c}\right.\right.$$

$$= -\frac{\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{8c^2} + \frac{\left(1-\frac{ic}{x}\right)^2\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^3}{16c^2} + \frac{3ia^2b\log}{4x}$$

$$= -\frac{3ia^2b}{8x^2} + \frac{3a^2b}{4cx} + \frac{3ia^2b\log\left(i-\frac{c}{x}\right)}{4c^2} + \frac{3ib\left(1-\frac{ic}{x}\right)\left(2a+ib\log\left(1-\frac{ic}{x}\right)\right)^2}{8c^2} - \frac{3ib(1}{}$$

$$= \frac{3ib^3\left(1-\frac{ic}{x}\right)^2}{64c^2} - \frac{3ib^3\left(1+\frac{ic}{x}\right)^2}{64c^2} - \frac{3ia^2b}{8x^2} + \frac{3a^2b}{4cx} - \frac{3iab^2}{2cx} - \frac{3b^3}{4cx} + \frac{3ia^2b\log\left(i-\frac{c}{x}\right)}{4c^2}$$

$$= \frac{3ib^3\left(1-\frac{ic}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{3ib^3\left(1+\frac{ic}{x}\right)^2}{64c^2} - \frac{3ia^2b}{8x^2} + \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} + \frac{3ia^2b\log}{4}$$

$$= \frac{3ib^3\left(1-\frac{ic}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{3ib^3\left(1+\frac{ic}{x}\right)^2}{64c^2} - \frac{3ia^2b}{8x^2} + \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} + \frac{3ia^2b\log}{4}$$

$$= \frac{3ib^3\left(1-\frac{ic}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{3ib^3\left(1+\frac{ic}{x}\right)^2}{64c^2} - \frac{3ia^2b}{8x^2} - \frac{3ab^2}{8x^2} + \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} +$$

$$= \frac{3ib^3\left(1-\frac{ic}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{3ib^3\left(1+\frac{ic}{x}\right)^2}{64c^2} - \frac{3ia^2b}{8x^2} - \frac{3ab^2}{8x^2} + \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} +$$

$$= \frac{3ib^3\left(1-\frac{ic}{x}\right)^2}{64c^2} - \frac{3ab^2\left(1+\frac{ic}{x}\right)^2}{16c^2} - \frac{3ib^3\left(1+\frac{ic}{x}\right)^2}{64c^2} - \frac{3ia^2b}{8x^2} - \frac{3ab^2}{8x^2} + \frac{3a^2b}{4cx} - \frac{3b^3}{2cx} +$$

Mathematica [A]

time = 0.19, size = 178, normalized size = 1.21

$$\frac{3b^2(c-ix)(-a(c+ix)+bx)\text{ArcTan}\left(\frac{c}{x}\right)^2 - b^3(c^2+x^2)\,\text{ArcTan}\left(\frac{c}{x}\right)^3 - 3b\text{ArcTan}\left(\frac{c}{x}\right)\left(a(-2bcx+a(c^2+x^2)) - 2b^2x^2\log\left(1+e^{2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)\right) + a\left(ac(-ac+3bx) + 6b^2x^2\log\left(\frac{1}{\sqrt{1+\frac{c^2}{x^2}}}\right)\right) - 3ib^3x^2\text{PolyLog}\left(2,-e^{2i\text{ArcTan}\left(\frac{c}{x}\right)}\right)}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcTan[c/x])^3/x^3,x]
```

```
[Out] (3*b^2*(c - I*x)*(-(a*(c + I*x)) + b*x)*ArcTan[c/x]^2 - b^3*(c^2 + x^2)*Arc
Tan[c/x]^3 - 3*b*ArcTan[c/x]*(a*(-2*b*c*x + a*(c^2 + x^2)) - 2*b^2*x^2*Log[
1 + E^((2*I)*ArcTan[c/x])]) + a*(a*c*(-(a*c) + 3*b*x) + 6*b^2*x^2*Log[1/Sqr
t[1 + c^2/x^2]]) - (3*I)*b^3*x^2*PolyLog[2, -E^((2*I)*ArcTan[c/x])])/(2*c^2
*x^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(133) = 266$.
time = 4.39, size = 368, normalized size = 2.50

method	result
derivativedivides	$-\frac{a^3c^2}{2x^2}+\frac{b^3c^2\arctan\left(\frac{c}{x}\right)^3}{2x^2}+\frac{b^3\arctan\left(\frac{c}{x}\right)^3}{2}-\frac{3b^3\arctan\left(\frac{c}{x}\right)^2c}{2x}+\frac{3b^3\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{2}-\frac{3ib^3\ln\left(\frac{c}{x}+i\right)\ln\left(1+\frac{c^2}{x^2}\right)}{4}+\frac{3ib^3\ln}{}$
default	$-\frac{a^3c^2}{2x^2}+\frac{b^3c^2\arctan\left(\frac{c}{x}\right)^3}{2x^2}+\frac{b^3\arctan\left(\frac{c}{x}\right)^3}{2}-\frac{3b^3\arctan\left(\frac{c}{x}\right)^2c}{2x}+\frac{3b^3\arctan\left(\frac{c}{x}\right)\ln\left(1+\frac{c^2}{x^2}\right)}{2}-\frac{3ib^3\ln\left(\frac{c}{x}+i\right)\ln\left(1+\frac{c^2}{x^2}\right)}{4}+\frac{3ib^3\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arctan(c/x))^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^2*(1/2*a^3*c^2/x^2+1/2*b^3*c^2/x^2*arctan(c/x)^3+1/2*b^3*arctan(c/x)^3
-3/2*b^3*arctan(c/x)^2*c/x+3/2*b^3*arctan(c/x)*ln(1+c^2/x^2)+3/4*I*b^3*dilo
g(1/2*I*(c/x-I))+3/4*I*b^3*ln(c/x-I)*ln(1+c^2/x^2)+3/8*I*b^3*ln(c/x+I)^2-3/
4*I*b^3*dilog(-1/2*I*(c/x+I))-3/4*I*b^3*ln(c/x+I)*ln(1+c^2/x^2)-3/4*I*b^3*l
n(c/x-I)*ln(-1/2*I*(c/x+I))-3/8*I*b^3*ln(c/x-I)^2+3/4*I*b^3*ln(c/x+I)*ln(1/
2*I*(c/x-I))+3/2*a^2*b*c^2/x^2*arctan(c/x)-3/2*a^2*b*c/x+3/2*a^2*b*arctan(c
/x)+3/2*b^2*a*c^2/x^2*arctan(c/x)^2+3/2*b^2*a*arctan(c/x)^2-3*b^2*a*c/x*arc
tan(c/x)+3/2*b^2*a*ln(1+c^2/x^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x^3,x, algorithm="maxima")
```

```
[Out] 3/2*(c*(arctan(x/c)/c^3 + 1/(c^2*x)) - arctan(c/x)/x^2)*a^2*b + 3/2*(2*c*(a
rctan(x/c)/c^3 + 1/(c^2*x))*arctan(c/x) + (arctan(x/c)^2 - log(c^2 + x^2) +
 2*log(x))/c^2)*a*b^2 - 3/2*a*b^2*arctan(c/x)^2/x^2 - 1/2*a^3/x^2 + 1/32*(4
*(128*c^3*integrate(1/32*arctan(c/x)^3/(c^3*x^3 + c*x^5), x) - 96*c^2*integ
```

```
rate(1/32*x*arctan(c/x)^2/(c^3*x^3 + c*x^5), x) - 24*c^2*integrate(1/32*x*l
og(c^2 + x^2)^2/(c^3*x^3 + c*x^5), x) + 128*c*integrate(1/32*x^2*arctan(c/x
)^3/(c^3*x^3 + c*x^5), x) + 192*c*integrate(1/32*x^2*arctan(c/x)/(c^3*x^3 +
 c*x^5), x) - 3*arctan(c/x)^2*arctan(x/c)/c^2 - 3*arctan(c/x)*arctan(x/c)^2
/c^2 - arctan(x/c)^3/c^2 - 24*integrate(1/32*x^3*log(c^2 + x^2)^2/(c^3*x^3
+ c*x^5), x) + 96*integrate(1/32*x^3*log(c^2 + x^2)/(c^3*x^3 + c*x^5), x))*
c^2*x^2 - 8*c^2*arctan2(c, x)^3 - 8*x^2*arctan2(c, x)^3 + 12*c*x*arctan2(c,
 x)^2 - 3*c*x*log(c^2 + x^2)^2)*b^3/(c^2*x^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x^3,x, algorithm="fricas")
[Out] integral((b^3*arctan(c/x)^3 + 3*a*b^2*arctan(c/x)^2 + 3*a^2*b*arctan(c/x) +
 a^3)/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{atan}{\left(\frac{c}{x} \right)}\right)^{3}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*atan(c/x))**3/x**3,x)
[Out] Integral((a + b*atan(c/x))**3/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arctan(c/x))^3/x^3,x, algorithm="giac")
[Out] integrate((b*arctan(c/x) + a)^3/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{atan}\left(\frac{c}{x}\right)\right)}^3}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*atan(c/x))^3/x^3,x)
[Out] int((a + b*atan(c/x))^3/x^3, x)
```

3.154 $\int x^2 \textbf{ArcTan}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=51

$$-\frac{\sqrt{x}}{3}+\frac{x^{3/2}}{9}-\frac{x^{5/2}}{15}+\frac{\text{ArcTan}\left(\sqrt{x}\right)}{3}+\frac{1}{3}x^3\text{ArcTan}\left(\sqrt{x}\right)$$

```
[Out] 1/9*x^(3/2)-1/15*x^(5/2)+1/3*arctan(x^(1/2))+1/3*x^3*arctan(x^(1/2))-1/3*x^
(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4946, 52, 65, 209}

$$\frac{1}{3}x^3\text{ArcTan}\left(\sqrt{x}\right)+\frac{\text{ArcTan}\left(\sqrt{x}\right)}{3}-\frac{x^{5/2}}{15}+\frac{x^{3/2}}{9}-\frac{\sqrt{x}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[Sqrt[x]],x]
```

```
[Out] -1/3*Sqrt[x] + x^(3/2)/9 - x^(5/2)/15 + ArcTan[Sqrt[x]]/3 + (x^3*ArcTan[Sqr
t[x]])/3
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\tan^{-1}\left(\sqrt{x}\right)\,dx &= \frac{1}{3}x^3\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{6}\int\frac{x^{5/2}}{1+x}\,dx\\ &= -\frac{x^{5/2}}{15}+\frac{1}{3}x^3\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{6}\int\frac{x^{3/2}}{1+x}\,dx\\ &= \frac{x^{3/2}}{9}-\frac{x^{5/2}}{15}+\frac{1}{3}x^3\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{6}\int\frac{\sqrt{x}}{1+x}\,dx\\ &= -\frac{\sqrt{x}}{3}+\frac{x^{3/2}}{9}-\frac{x^{5/2}}{15}+\frac{1}{3}x^3\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{6}\int\frac{1}{\sqrt{x}\,(1+x)}\,dx\\ &= -\frac{\sqrt{x}}{3}+\frac{x^{3/2}}{9}-\frac{x^{5/2}}{15}+\frac{1}{3}x^3\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{3}\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\ &= -\frac{\sqrt{x}}{3}+\frac{x^{3/2}}{9}-\frac{x^{5/2}}{15}+\frac{1}{3}\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{3}x^3\tan^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 0.67

$$\frac{1}{45}\left(\sqrt{x}\left(-15+5x-3x^2\right)+15\left(1+x^3\right)\text{ArcTan}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[Sqrt[x]],x]
```

```
[Out] (Sqrt[x]*(-15 + 5*x - 3*x^2) + 15*(1 + x^3)*ArcTan[Sqrt[x]])/45
```

Maple [A]
time = 0.02, size = 32, normalized size = 0.63

method	result	size
meijerg	$-\frac{\sqrt{x}\,(21x^2-35x+105)}{315}+\frac{(7x^3+7)\arctan\left(\sqrt{x}\right)}{21}$	30
derivativedivides	$\frac{x^{\frac{3}{2}}}{9}-\frac{x^{\frac{5}{2}}}{15}+\frac{\arctan\left(\sqrt{x}\right)}{3}+\frac{x^3\arctan\left(\sqrt{x}\right)}{3}-\frac{\sqrt{x}}{3}$	32

default	$\frac{x^{\frac{3}{2}}}{9}-\frac{x^{\frac{5}{2}}}{15}+\frac{\arctan\left(\sqrt{x}\right)}{3}+\frac{x^{3}\arctan\left(\sqrt{x}\right)}{3}-\frac{\sqrt{x}}{3}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*x^(3/2)-1/15*x^(5/2)+1/3*arctan(x^(1/2))+1/3*x^3*arctan(x^(1/2))-1/3*x^
(1/2)
```

Maxima [A]
time = 0.47, size = 31, normalized size = 0.61

$$\frac{1}{3}\,x^{3}\arctan\left(\sqrt{x}\right)-\frac{1}{15}\,x^{\frac{5}{2}}+\frac{1}{9}\,x^{\frac{3}{2}}-\frac{1}{3}\,\sqrt{x}+\frac{1}{3}\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(sqrt(x)) - 1/15*x^(5/2) + 1/9*x^(3/2) - 1/3*sqrt(x) + 1/3*ar
ctan(sqrt(x))
```

Fricas [A]
time = 1.09, size = 27, normalized size = 0.53

$$\frac{1}{3}\left(x^{3}+1\right)\arctan\left(\sqrt{x}\right)-\frac{1}{45}\left(3\,x^{2}-5\,x+15\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + 1)*arctan(sqrt(x)) - 1/45*(3*x^2 - 5*x + 15)*sqrt(x)
```

Sympy [A]
time = 1.32, size = 39, normalized size = 0.76

$$-\frac{x^{\frac{5}{2}}}{15}+\frac{x^{\frac{3}{2}}}{9}-\frac{\sqrt{x}}{3}+\frac{x^{3}\operatorname{atan}\left(\sqrt{x}\right)}{3}+\frac{\operatorname{atan}\left(\sqrt{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(x**(1/2)),x)
```

```
[Out] -x**(5/2)/15 + x**(3/2)/9 - sqrt(x)/3 + x**3*atan(sqrt(x))/3 + atan(sqrt(x)
)/3
```

Giac [A]
time = 0.42, size = 31, normalized size = 0.61

$$\frac{1}{3}\,x^{3}\arctan\left(\sqrt{x}\right)-\frac{1}{15}\,x^{\frac{5}{2}}+\frac{1}{9}\,x^{\frac{3}{2}}-\frac{1}{3}\,\sqrt{x}+\frac{1}{3}\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arctan(sqrt(x)) - 1/15*x^(5/2) + 1/9*x^(3/2) - 1/3*sqrt(x) + 1/3*ar
ctan(sqrt(x))
```

Mupad [B]
time = 0.35, size = 31, normalized size = 0.61

$$\frac{\operatorname{atan}\left(\sqrt{x}\right)}{3}+\frac{x^3\operatorname{atan}\left(\sqrt{x}\right)}{3}-\frac{\sqrt{x}}{3}+\frac{x^{3/2}}{9}-\frac{x^{5/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*atan(x^(1/2)),x)
```

```
[Out] atan(x^(1/2))/3 + (x^3*atan(x^(1/2)))/3 - x^(1/2)/3 + x^(3/2)/9 - x^(5/2)/1
5
```

3.155 $\int x\textbf{ArcTan}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{x}}{2}-\frac{x^{3/2}}{6}-\frac{\text{ArcTan}\left(\sqrt{x}\right)}{2}+\frac{1}{2}x^2\text{ArcTan}\left(\sqrt{x}\right)$$

```
[Out] -1/6*x^(3/2)-1/2*arctan(x^(1/2))+1/2*x^2*arctan(x^(1/2))+1/2*x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.500$, Rules used = {4946, 52, 65, 209}

$$\frac{1}{2}x^2\text{ArcTan}\left(\sqrt{x}\right)-\frac{\text{ArcTan}\left(\sqrt{x}\right)}{2}-\frac{x^{3/2}}{6}+\frac{\sqrt{x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[Sqrt[x]],x]
```

```
[Out] Sqrt[x]/2 - x^(3/2)/6 - ArcTan[Sqrt[x]]/2 + (x^2*ArcTan[Sqrt[x]])/2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x\tan^{-1}\left(\sqrt{x}\right)\,dx &= \frac{1}{2}x^2\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int\frac{x^{3/2}}{1+x}\,dx\\
&= -\frac{x^{3/2}}{6}+\frac{1}{2}x^2\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{4}\int\frac{\sqrt{x}}{1+x}\,dx\\
&= \frac{\sqrt{x}}{2}-\frac{x^{3/2}}{6}+\frac{1}{2}x^2\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{4}\int\frac{1}{\sqrt{x}\,(1+x)}\,dx\\
&= \frac{\sqrt{x}}{2}-\frac{x^{3/2}}{6}+\frac{1}{2}x^2\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= \frac{\sqrt{x}}{2}-\frac{x^{3/2}}{6}-\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{2}x^2\tan^{-1}\left(\sqrt{x}\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 28, normalized size = 0.67

$$\frac{1}{6}\left(-\left((-3+x)\sqrt{x}\right)+3\left(-1+x^2\right)\text{ArcTan}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[Sqrt[x]],x]
[Out] (-((-3 + x)*Sqrt[x]) + 3*(-1 + x^2)*ArcTan[Sqrt[x]])/6
```

Maple [A]
time = 0.01, size = 27, normalized size = 0.64

method	result	size
meijerg	$\frac{\sqrt{x}\,(-5x+15)}{30}-\frac{(-5x^2+5)\arctan\left(\sqrt{x}\right)}{10}$	25
derivativedivides	$-\frac{x^{\frac{3}{2}}}{6}-\frac{\arctan\left(\sqrt{x}\right)}{2}+\frac{x^2\arctan\left(\sqrt{x}\right)}{2}+\frac{\sqrt{x}}{2}$	27
default	$-\frac{x^{\frac{3}{2}}}{6}-\frac{\arctan\left(\sqrt{x}\right)}{2}+\frac{x^2\arctan\left(\sqrt{x}\right)}{2}+\frac{\sqrt{x}}{2}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/6*x^(3/2)-1/2*arctan(x^(1/2))+1/2*x^2*arctan(x^(1/2))+1/2*x^(1/2)
```

Maxima [A]
time = 0.47, size = 26, normalized size = 0.62

$$\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)-\frac{1}{6}x^{\frac{3}{2}}+\frac{1}{2}\sqrt{x}-\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(sqrt(x)) - 1/6*x^(3/2) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

Fricas [A]
time = 0.67, size = 20, normalized size = 0.48

$$\frac{1}{2}\left(x^2-1\right)\arctan\left(\sqrt{x}\right)-\frac{1}{6}\left(x-3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 - 1)*arctan(sqrt(x)) - 1/6*(x - 3)*sqrt(x)
```

Sympy [A]
time = 0.78, size = 32, normalized size = 0.76

$$-\frac{x^{\frac{3}{2}}}{6}+\frac{\sqrt{x}}{2}+\frac{x^2\operatorname{atan}\left(\sqrt{x}\right)}{2}-\frac{\operatorname{atan}\left(\sqrt{x}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x**(1/2)),x)
```

```
[Out] -x**(3/2)/6 + sqrt(x)/2 + x**2*atan(sqrt(x))/2 - atan(sqrt(x))/2
```

Giac [A]
time = 0.41, size = 26, normalized size = 0.62

$$\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)-\frac{1}{6}x^{\frac{3}{2}}+\frac{1}{2}\sqrt{x}-\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2*arctan(sqrt(x)) - 1/6*x^(3/2) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

Mupad [B]
time = 0.37, size = 26, normalized size = 0.62

$$\frac{x^2\,\mathrm{atan}\left(\sqrt{x}\right)}{2}-\frac{\mathrm{atan}\left(\sqrt{x}\right)}{2}+\frac{\sqrt{x}}{2}-\frac{x^{3/2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*atan(x^(1/2)),x)
```

```
[Out] (x^2*atan(x^(1/2)))/2 - atan(x^(1/2))/2 + x^(1/2)/2 - x^(3/2)/6
```

3.156 $\int \mathbf{ArcTan}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=22

$$-\sqrt{x} + \text{ArcTan}\left(\sqrt{x}\right) + x\text{ArcTan}\left(\sqrt{x}\right)$$

```
[Out] arctan(x^(1/2))+x*arctan(x^(1/2))-x^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {4930, 52, 65, 209}

$$x\text{ArcTan}\left(\sqrt{x}\right) + \text{ArcTan}\left(\sqrt{x}\right) - \sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]],x]
```

```
[Out] -Sqrt[x] + ArcTan[Sqrt[x]] + x*ArcTan[Sqrt[x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Dist[b*c*n*p, Int[x^n*((a + b*ArcTan[c*x^n])^(p
```

```
- 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
(EqQ[n, 1] || EqQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}\left(\sqrt{x}\right)\,dx &= x\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{2}\int\frac{\sqrt{x}}{1+x}\,dx\\ &=-\sqrt{x}+x\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{2}\int\frac{1}{\sqrt{x}\,(1+x)}\,dx\\ &=-\sqrt{x}+x\tan^{-1}\left(\sqrt{x}\right)+\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\ &=-\sqrt{x}+\tan^{-1}\left(\sqrt{x}\right)+x\tan^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 0.82

$$-\sqrt{x}+(1+x)\text{ArcTan}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]],x]
```

```
[Out] -Sqrt[x] + (1 + x)*ArcTan[Sqrt[x]]
```

Maple [A]
time = 0.01, size = 17, normalized size = 0.77

method	result	size
derivativedivides	$\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)-\sqrt{x}$	17
default	$\arctan\left(\sqrt{x}\right)+x\arctan\left(\sqrt{x}\right)-\sqrt{x}$	17
meijerg	$-\sqrt{x}+\frac{(3x+3)\arctan\left(\sqrt{x}\right)}{3}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2)),x,method=_RETURNVERBOSE)
```

```
[Out] arctan(x^(1/2))+x*arctan(x^(1/2))-x^(1/2)
```

Maxima [A]
time = 0.47, size = 16, normalized size = 0.73

$$x\arctan\left(\sqrt{x}\right)-\sqrt{x}+\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2)),x, algorithm="maxima")
[Out] x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

Fricas [A]
time = 1.21, size = 14, normalized size = 0.64

$$(x+1)\arctan\left(\sqrt{x}\right)-\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2)),x, algorithm="fricas")
[Out] (x + 1)*arctan(sqrt(x)) - sqrt(x)
```

Sympy [A]
time = 0.69, size = 19, normalized size = 0.86

$$-\sqrt{x}+x\operatorname{atan}\left(\sqrt{x}\right)+\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2)),x)
[Out] -sqrt(x) + x*atan(sqrt(x)) + atan(sqrt(x))
```

Giac [A]
time = 0.43, size = 16, normalized size = 0.73

$$x\arctan\left(\sqrt{x}\right)-\sqrt{x}+\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2)),x, algorithm="giac")
[Out] x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

Mupad [B]
time = 0.07, size = 16, normalized size = 0.73

$$\operatorname{atan}\left(\sqrt{x}\right)+x\operatorname{atan}\left(\sqrt{x}\right)-\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x^(1/2)),x)
[Out] atan(x^(1/2)) + x*atan(x^(1/2)) - x^(1/2)
```

3.157 $\int \frac{\mathbf{ArcTan}(\sqrt{x})}{x} \, dx$

Optimal. Leaf size=31

$$i\text{PolyLog}(2, -i\sqrt{x}) - i\text{PolyLog}(2, i\sqrt{x})$$

```
[Out] I*polylog(2,-I*x^(1/2))-I*polylog(2,I*x^(1/2))
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4944, 4940, 2438}

$$i\text{Li}_2(-i\sqrt{x}) - i\text{Li}_2(i\sqrt{x})$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/x,x]
```

```
[Out] I*PolyLog[2, (-I)*Sqrt[x]] - I*PolyLog[2, I*Sqrt[x]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(\sqrt{x})}{x} \, dx &= 2\text{Subst}\left(\int \frac{\tan^{-1}(x)}{x} \, dx, x, \sqrt{x}\right) \\
&= i\text{Subst}\left(\int \frac{\log(1-ix)}{x} \, dx, x, \sqrt{x}\right) - i\text{Subst}\left(\int \frac{\log(1+ix)}{x} \, dx, x, \sqrt{x}\right) \\
&= i\text{Li}_2(-i\sqrt{x}) - i\text{Li}_2(i\sqrt{x})
\end{aligned}$$

Mathematica [**A**]
time = 0.00, size = 31, normalized size = 1.00

$$i\text{PolyLog}\left(2,-i\sqrt{x}\right)-i\text{PolyLog}\left(2,i\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/x,x]
```

```
[Out] I*PolyLog[2, (-I)*Sqrt[x]] - I*PolyLog[2, I*Sqrt[x]]
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(23) = 46.
time = 0.02, size = 61, normalized size = 1.97

method	result
meijerg	$i\,\mathrm{polylog}\left(2,-i\sqrt{x}\right)-i\,\mathrm{polylog}\left(2,i\sqrt{x}\right)$
derivativedivides	$\ln\left(x\right)\arctan\left(\sqrt{x}\right)+\frac{i\ln(x)\ln\left(1+i\sqrt{x}\right)}{2}-\frac{i\ln(x)\ln\left(1-i\sqrt{x}\right)}{2}+i\,\mathrm{dilog}\left(1+i\sqrt{x}\right)-i\,\mathrm{dilog}$
default	$\ln\left(x\right)\arctan\left(\sqrt{x}\right)+\frac{i\ln(x)\ln\left(1+i\sqrt{x}\right)}{2}-\frac{i\ln(x)\ln\left(1-i\sqrt{x}\right)}{2}+i\,\mathrm{dilog}\left(1+i\sqrt{x}\right)-i\,\mathrm{dilog}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*arctan(x^(1/2))+1/2*I*ln(x)*ln(1+I*x^(1/2))-1/2*I*ln(x)*ln(1-I*x^(1/2
))+I*dilog(1+I*x^(1/2))-I*dilog(1-I*x^(1/2))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(17) = 34.
time = 0.47, size = 35, normalized size = 1.13

$$-\frac{1}{2}\,\pi\log\left(x+1\right)+\arctan\left(\sqrt{x}\right)\log\left(x\right)-i\,\mathrm{Li}_2\left(i\,\sqrt{x}+1\right)+i\,\mathrm{Li}_2\left(-i\,\sqrt{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x,x, algorithm="maxima")
```

```
[Out] -1/2*pi*log(x + 1) + arctan(sqrt(x))*log(x) - I*dilog(I*sqrt(x) + 1) + I*di
log(-I*sqrt(x) + 1)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(sqrt(x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}\left(\sqrt{x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/x,x)
```

```
[Out] Integral(atan(sqrt(x))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(sqrt(x))/x, x)
```

Mupad [B]
time = 0.30, size = 24, normalized size = 0.77

$$-\mathrm{Li}_2\left(1-\sqrt{x}\ 1\mathrm{i}\right)\ 1\mathrm{i}+\mathrm{polylog}\left(2,-\sqrt{x}\ 1\mathrm{i}\right)\ 1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x^(1/2))/x,x)
```

```
[Out] polylog(2, -x^(1/2)*1i)*1i - dilog(1 - x^(1/2)*1i)*1i
```

3.158 $\int \frac{\mathbf{ArcTan}\left(\sqrt{x}\right)}{x^2}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{\sqrt{x}}-\text{ArcTan}\left(\sqrt{x}\right)-\frac{\text{ArcTan}\left(\sqrt{x}\right)}{x}$$

```
[Out] -arctan(x^(1/2))-arctan(x^(1/2))/x-1/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4946, 53, 65, 209}

$$-\frac{\text{ArcTan}\left(\sqrt{x}\right)}{x}-\text{ArcTan}\left(\sqrt{x}\right)-\frac{1}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/x^2,x]
```

```
[Out] -(1/Sqrt[x]) - ArcTan[Sqrt[x]] - ArcTan[Sqrt[x]]/x
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^2}\,dx &= -\frac{\tan^{-1}\left(\sqrt{x}\right)}{x}+\frac{1}{2}\int \frac{1}{x^{3/2}(1+x)}\,dx\\
&= -\frac{1}{\sqrt{x}}-\frac{\tan^{-1}\left(\sqrt{x}\right)}{x}-\frac{1}{2}\int \frac{1}{\sqrt{x}\,(1+x)}\,dx\\
&= -\frac{1}{\sqrt{x}}-\frac{\tan^{-1}\left(\sqrt{x}\right)}{x}-\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= -\frac{1}{\sqrt{x}}-\tan^{-1}\left(\sqrt{x}\right)-\frac{\tan^{-1}\left(\sqrt{x}\right)}{x}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 30, normalized size = 1.11

$$-\frac{\operatorname{ArcTan}\left(\sqrt{x}\right)}{x}-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-x\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/x^2,x]
```

```
[Out] -(ArcTan[Sqrt[x]]/x) - Hypergeometric2F1[-1/2, 1, 1/2, -x]/Sqrt[x]
```

Maple [A]
time = 0.01, size = 22, normalized size = 0.81

method	result	size
meijerg	$-\frac{1}{\sqrt{x}}-\frac{\arctan\left(\sqrt{x}\right)(1+x)}{x}$	19
derivativedivides	$-\arctan\left(\sqrt{x}\right)-\frac{\arctan\left(\sqrt{x}\right)}{x}-\frac{1}{\sqrt{x}}$	22

default	$-\arctan\left(\sqrt{x}\right)-\frac{\arctan\left(\sqrt{x}\right)}{x}-\frac{1}{\sqrt{x}}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
[Out] -arctan(x^(1/2))-arctan(x^(1/2))/x-1/x^(1/2)
```

Maxima [A]
time = 0.48, size = 21, normalized size = 0.78

$$-\frac{\arctan\left(\sqrt{x}\right)}{x}-\frac{1}{\sqrt{x}}-\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^2,x, algorithm="maxima")
[Out] -arctan(sqrt(x))/x - 1/sqrt(x) - arctan(sqrt(x))
```

Fricas [A]
time = 2.37, size = 17, normalized size = 0.63

$$-\frac{(x+1)\arctan\left(\sqrt{x}\right)+\sqrt{x}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^2,x, algorithm="fricas")
[Out] -((x + 1)*arctan(sqrt(x)) + sqrt(x))/x
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(22)=44$.
time = 0.57, size = 94, normalized size = 3.48

$$-\frac{x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}-\frac{2x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}-\frac{\sqrt{x}\operatorname{atan}\left(\sqrt{x}\right)}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}-\frac{x^2}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}-\frac{x}{x^{\frac{5}{2}}+x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/x**2,x)
[Out] -x**(5/2)*atan(sqrt(x))/(x**(5/2) + x**(3/2)) - 2*x**(3/2)*atan(sqrt(x))/(x
**(5/2) + x**(3/2)) - sqrt(x)*atan(sqrt(x))/(x**(5/2) + x**(3/2)) - x**2/(x
**(5/2) + x**(3/2)) - x/(x**(5/2) + x**(3/2))
```

Giac [A]
time = 0.44, size = 21, normalized size = 0.78

$$-\frac{\arctan\left(\sqrt{x}\right)}{x}-\frac{1}{\sqrt{x}}-\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^2,x, algorithm="giac")
[Out] -arctan(sqrt(x))/x - 1/sqrt(x) - arctan(sqrt(x))
```

Mupad [B]
time = 0.35, size = 21, normalized size = 0.78

$$-\mathrm{atan}\left(\sqrt{x}\right)-\frac{\mathrm{atan}\left(\sqrt{x}\right)}{x}-\frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x^(1/2))/x^2,x)
[Out] - atan(x^(1/2)) - atan(x^(1/2))/x - 1/x^(1/2)
```

3.159 $\int \frac{\text{ArcTan}(\sqrt{x})}{x^3} dx$

Optimal. Leaf size=42

$$-\frac{1}{6x^{3/2}}+\frac{1}{2\sqrt{x}}+\frac{\text{ArcTan}(\sqrt{x})}{2}-\frac{\text{ArcTan}(\sqrt{x})}{2x^2}$$

```
[Out] -1/6/x^(3/2)+1/2*arctan(x^(1/2))-1/2*arctan(x^(1/2))/x^2+1/2/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4946, 53, 65, 209}

$$-\frac{\text{ArcTan}(\sqrt{x})}{2x^2}+\frac{\text{ArcTan}(\sqrt{x})}{2}-\frac{1}{6x^{3/2}}+\frac{1}{2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/x^3,x]
```

```
[Out] -1/6*1/x^(3/2) + 1/(2*Sqrt[x]) + ArcTan[Sqrt[x]]/2 - ArcTan[Sqrt[x]]/(2*x^2
)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^3}\,dx &= -\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{4}\int \frac{1}{x^{5/2}(1+x)}\,dx\\
&= -\frac{1}{6x^{3/2}}-\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x^2}-\frac{1}{4}\int \frac{1}{x^{3/2}(1+x)}\,dx\\
&= -\frac{1}{6x^{3/2}}+\frac{1}{2\sqrt{x}}-\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{4}\int \frac{1}{\sqrt{x}\,(1+x)}\,dx\\
&= -\frac{1}{6x^{3/2}}+\frac{1}{2\sqrt{x}}-\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\
&= -\frac{1}{6x^{3/2}}+\frac{1}{2\sqrt{x}}+\frac{1}{2}\tan^{-1}\left(\sqrt{x}\right)-\frac{\tan^{-1}\left(\sqrt{x}\right)}{2x^2}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.01, size = 34, normalized size = 0.81

$$-\frac{\text{ArcTan}\left(\sqrt{x}\right)}{2x^2}-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-x\right)}{6x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/x^3,x]
[Out] -1/2*ArcTan[Sqrt[x]]/x^2 - Hypergeometric2F1[-3/2, 1, -1/2, -x]/(6*x^(3/2))
```

Maple [A]
time = 0.01, size = 27, normalized size = 0.64

method	result	size
derivativedivides	$-\frac{1}{6x^{\frac{3}{2}}}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2x^2}+\frac{1}{2\sqrt{x}}$	27

default	$-\frac{1}{6x^{\frac{3}{2}}}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2x^{2}}+\frac{1}{2\sqrt{x}}$	27
meijerg	$-\frac{1}{6x^{\frac{3}{2}}}+\frac{1}{2\sqrt{x}}-\frac{4\left(-\frac{3x^{2}}{8}+\frac{3}{8}\right)\arctan\left(\sqrt{x}\right)}{3x^{2}}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/x^(3/2)+1/2*arctan(x^(1/2))-1/2*arctan(x^(1/2))/x^2+1/2/x^(1/2)
```

Maxima [A]
time = 0.48, size = 26, normalized size = 0.62

$$\frac{3\,x-1}{6\,x^{\frac{3}{2}}}-\frac{\arctan\left(\sqrt{x}\right)}{2\,x^{2}}+\frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*x - 1)/x^(3/2) - 1/2*arctan(sqrt(x))/x^2 + 1/2*arctan(sqrt(x))
```

Fricas [A]
time = 3.21, size = 26, normalized size = 0.62

$$\frac{3\,(x^{2}-1)\arctan\left(\sqrt{x}\right)+(3\,x-1)\sqrt{x}}{6\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(3*(x^2 - 1)*arctan(sqrt(x)) + (3*x - 1)*sqrt(x))/x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(36) = 72$.
time = 1.19, size = 160, normalized size = 3.81

$$\frac{3x^{\frac{7}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}+\frac{3x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}-\frac{3x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}-\frac{3\sqrt{x}\operatorname{atan}\left(\sqrt{x}\right)}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}+\frac{3x^{3}}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}+\frac{2x^{2}}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}-\frac{x}{6x^{\frac{7}{2}}+6x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/x**3,x)
```

```
[Out] 3*x**(7/2)*atan(sqrt(x))/(6*x**(7/2) + 6*x**(5/2)) + 3*x**(5/2)*atan(sqrt(x
))/(6*x**(7/2) + 6*x**(5/2)) - 3*x**(3/2)*atan(sqrt(x))/(6*x**(7/2) + 6*x**
(5/2)) - 3*sqrt(x)*atan(sqrt(x))/(6*x**(7/2) + 6*x**(5/2)) + 3*x**3/(6*x**(
```

```
7/2) + 6*x**(5/2)) + 2*x**2/(6*x**(7/2) + 6*x**(5/2)) - x/(6*x**(7/2) + 6*x
**(5/2))
```

Giac [A]
time = 0.44, size = 26, normalized size = 0.62

$$\frac{3\,x-1}{6\,x^{\frac{3}{2}}}-\frac{\arctan\left(\sqrt{x}\right)}{2\,x^2}+\frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*x - 1)/x^(3/2) - 1/2*arctan(sqrt(x))/x^2 + 1/2*arctan(sqrt(x))
```

Mupad [B]
time = 0.35, size = 24, normalized size = 0.57

$$\frac{\mathrm{atan}\left(\sqrt{x}\right)}{2}+\frac{x-\frac{1}{3}}{2\,x^{3/2}}-\frac{\mathrm{atan}\left(\sqrt{x}\right)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x^(1/2))/x^3,x)
```

```
[Out] atan(x^(1/2))/2 + (x - 1/3)/(2*x^(3/2)) - atan(x^(1/2))/(2*x^2)
```

3.160 $\int x^{3/2}\text{ArcTan}(\sqrt{x})\,dx$

Optimal. Leaf size=36

$$\frac{x}{5}-\frac{x^2}{10}+\frac{2}{5}x^{5/2}\text{ArcTan}(\sqrt{x})-\frac{1}{5}\log(1+x)$$

```
[Out] 1/5*x-1/10*x^2+2/5*x^(5/2)*arctan(x^(1/2))-1/5*ln(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {4946, 45}

$$\frac{2}{5}x^{5/2}\text{ArcTan}(\sqrt{x})-\frac{x^2}{10}+\frac{x}{5}-\frac{1}{5}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*ArcTan[Sqrt[x]],x]
```

```
[Out] x/5 - x^2/10 + (2*x^(5/2)*ArcTan[Sqrt[x]])/5 - Log[1 + x]/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^{3/2}\tan^{-1}\left(\sqrt{x}\right)\,dx &= \frac{2}{5}x^{5/2}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{5}\int\frac{x^2}{1+x}\,dx\\ &= \frac{2}{5}x^{5/2}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{5}\int\left(-1+x+\frac{1}{1+x}\right)dx\\ &= \frac{x}{5}-\frac{x^2}{10}+\frac{2}{5}x^{5/2}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{5}\log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 30, normalized size = 0.83

$$\frac{1}{10}\left(-((-2+x)x)+4x^{5/2}\text{ArcTan}\left(\sqrt{x}\right)-2\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*ArcTan[Sqrt[x]],x]
[Out] (-((-2 + x)*x) + 4*x^(5/2)*ArcTan[Sqrt[x]] - 2*Log[1 + x])/10
```

Maple [A]
time = 0.01, size = 25, normalized size = 0.69

method	result	size
derivativedivides	$\frac{x}{5}-\frac{x^2}{10}+\frac{2x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right)}{5}-\frac{\ln(1+x)}{5}$	25
default	$\frac{x}{5}-\frac{x^2}{10}+\frac{2x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right)}{5}-\frac{\ln(1+x)}{5}$	25
meijerg	$\frac{x(-3x+6)}{30}+\frac{2x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right)}{5}-\frac{\ln(1+x)}{5}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*arctan(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] 1/5*x-1/10*x^2+2/5*x^(5/2)*arctan(x^(1/2))-1/5*ln(1+x)
```

Maxima [A]
time = 0.27, size = 24, normalized size = 0.67

$$\frac{2}{5}x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{10}x^2+\frac{1}{5}x-\frac{1}{5}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctan(x^(1/2)),x, algorithm="maxima")
[Out] 2/5*x^(5/2)*arctan(sqrt(x)) - 1/10*x^2 + 1/5*x - 1/5*log(x + 1)
```

Fricas [A]
time = 2.53, size = 24, normalized size = 0.67

$$\frac{2}{5}x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{10}x^2+\frac{1}{5}x-\frac{1}{5}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctan(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/5*x^(5/2)*arctan(sqrt(x)) - 1/10*x^2 + 1/5*x - 1/5*log(x + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(29) = 58$.
time = 0.98, size = 85, normalized size = 2.36

$$\frac{4x^{\frac{7}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{10x+10}+\frac{4x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{10x+10}-\frac{x^3}{10x+10}+\frac{x^2}{10x+10}-\frac{2x\log\left(x+1\right)}{10x+10}-\frac{2\log\left(x+1\right)}{10x+10}-\frac{2}{10x+10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*atan(x**(1/2)),x)
```

```
[Out] 4*x**(7/2)*atan(sqrt(x))/(10*x + 10) + 4*x**(5/2)*atan(sqrt(x))/(10*x + 10)
 - x**3/(10*x + 10) + x**2/(10*x + 10) - 2*x*log(x + 1)/(10*x + 10) - 2*log
(x + 1)/(10*x + 10) - 2/(10*x + 10)
```

Giac [A]
time = 0.45, size = 24, normalized size = 0.67

$$\frac{2}{5}\,x^{\frac{5}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{10}\,x^2+\frac{1}{5}\,x-\frac{1}{5}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*arctan(x^(1/2)),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2)*arctan(sqrt(x)) - 1/10*x^2 + 1/5*x - 1/5*log(x + 1)
```

Mupad [B]
time = 0.35, size = 24, normalized size = 0.67

$$\frac{x}{5}-\frac{\ln\left(x+1\right)}{5}+\frac{2\,x^{5/2}\,\mathrm{atan}\left(\sqrt{x}\right)}{5}-\frac{x^2}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*atan(x^(1/2)),x)
```

```
[Out] x/5 - log(x + 1)/5 + (2*x^(5/2)*atan(x^(1/2)))/5 - x^2/10
```

3.161 $\int \sqrt{x}\,\mathrm{ArcTan}\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=29

$$-\frac{x}{3}+\frac{2}{3}x^{3/2}\mathrm{ArcTan}\left(\sqrt{x}\right)+\frac{1}{3}\log(1+x)$$

```
[Out] -1/3*x+2/3*x^(3/2)*arctan(x^(1/2))+1/3*ln(1+x)
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4946, 45}

$$\frac{2}{3}x^{3/2}\mathrm{ArcTan}\left(\sqrt{x}\right)-\frac{x}{3}+\frac{1}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*ArcTan[Sqrt[x]],x]
```

```
[Out] -1/3*x + (2*x^(3/2)*ArcTan[Sqrt[x]])/3 + Log[1 + x]/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\tan^{-1}\left(\sqrt{x}\right)\,dx &= \frac{2}{3}x^{3/2}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int \frac{x}{1+x}\,dx\\ &= \frac{2}{3}x^{3/2}\tan^{-1}\left(\sqrt{x}\right)-\frac{1}{3}\int \left(1+\frac{1}{-1-x}\right)\,dx\\ &= -\frac{x}{3}+\frac{2}{3}x^{3/2}\tan^{-1}\left(\sqrt{x}\right)+\frac{1}{3}\log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.86

$$\frac{1}{3}\left(-x+2x^{3/2}\text{ArcTan}\left(\sqrt{x}\right)+\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*ArcTan[Sqrt[x]],x]
[Out] (-x + 2*x^(3/2)*ArcTan[Sqrt[x]] + Log[1 + x])/3
```

Maple [A]
time = 0.01, size = 20, normalized size = 0.69

method	result	size
derivativedivides	$-\frac{x}{3}+\frac{2x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)}{3}+\frac{\ln(1+x)}{3}$	20
default	$-\frac{x}{3}+\frac{2x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)}{3}+\frac{\ln(1+x)}{3}$	20
meijerg	$-\frac{x}{3}+\frac{2x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)}{3}+\frac{\ln(1+x)}{3}$	20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*arctan(x^(1/2)),x,method=_RETURNVERBOSE)
[Out] -1/3*x+2/3*x^(3/2)*arctan(x^(1/2))+1/3*ln(1+x)
```

Maxima [A]
time = 0.26, size = 19, normalized size = 0.66

$$\frac{2}{3}\,x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{3}\,x+\frac{1}{3}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctan(x^(1/2)),x, algorithm="maxima")
[Out] 2/3*x^(3/2)*arctan(sqrt(x)) - 1/3*x + 1/3*log(x + 1)
```

Fricas [A]
time = 3.45, size = 19, normalized size = 0.66

$$\frac{2}{3}\,x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{3}\,x+\frac{1}{3}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctan(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*x^(3/2)*arctan(sqrt(x)) - 1/3*x + 1/3*log(x + 1)
```

Sympy [A]
time = 0.78, size = 24, normalized size = 0.83

$$\frac{2x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{3}-\frac{x}{3}+\frac{\log\left(x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*atan(x**(1/2)),x)
[Out] 2*x**(3/2)*atan(sqrt(x))/3 - x/3 + log(x + 1)/3
```

Giac [A]
time = 0.45, size = 19, normalized size = 0.66

$$\frac{2}{3}\,x^{\frac{3}{2}}\arctan\left(\sqrt{x}\right)-\frac{1}{3}\,x+\frac{1}{3}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*arctan(x^(1/2)),x, algorithm="giac")
[Out] 2/3*x^(3/2)*arctan(sqrt(x)) - 1/3*x + 1/3*log(x + 1)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\sqrt{x}\,\operatorname{atan}\left(\sqrt{x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*atan(x^(1/2)),x)
[Out] int(x^(1/2)*atan(x^(1/2)), x)
```

3.162 $\int \frac{\text{ArcTan}(\sqrt{x})}{\sqrt{x}} \, dx$

Optimal. Leaf size=20

$$2\sqrt{x}\,\text{ArcTan}(\sqrt{x}) - \log(1+x)$$

```
[Out] -ln(1+x)+2*x^(1/2)*arctan(x^(1/2))
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4946, 31}

$$2\sqrt{x}\,\text{ArcTan}(\sqrt{x}) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}} \, dx &= 2\sqrt{x}\,\tan^{-1}\left(\sqrt{x}\right) - \int \frac{1}{1+x} \, dx\\ &= 2\sqrt{x}\,\tan^{-1}\left(\sqrt{x}\right) - \log(1+x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$2\sqrt{x}\,\text{ArcTan}(\sqrt{x}) - \log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

Maple [A]
time = 0.02, size = 17, normalized size = 0.85

method	result	size
derivativedivides	$-\ln\left(1+x\right)+2\sqrt{x}\ \arctan\left(\sqrt{x}\right)$	17
default	$-\ln\left(1+x\right)+2\sqrt{x}\ \arctan\left(\sqrt{x}\right)$	17
meijerg	$-\ln\left(1+x\right)+2\sqrt{x}\ \arctan\left(\sqrt{x}\right)$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(1+x)+2*x^(1/2)*arctan(x^(1/2))
```

Maxima [A]
time = 0.26, size = 16, normalized size = 0.80

$$2\sqrt{x}\arctan\left(\sqrt{x}\right)-\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

Fricas [A]
time = 3.86, size = 16, normalized size = 0.80

$$2\sqrt{x}\arctan\left(\sqrt{x}\right)-\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

Sympy [A]
time = 0.12, size = 17, normalized size = 0.85

$$2\sqrt{x}\operatorname{atan}\left(\sqrt{x}\right)-\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*sqrt(x)*atan(sqrt(x)) - log(x + 1)
```

Giac [A]
time = 0.41, size = 16, normalized size = 0.80

$$2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

Mupad [B]
time = 0.35, size = 16, normalized size = 0.80

$$2\sqrt{x}\,\mathrm{atan}\left(\sqrt{x}\right) - \ln\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x^(1/2))/x^(1/2),x)
```

```
[Out] 2*x^(1/2)*atan(x^(1/2)) - log(x + 1)
```

3.163 $\int \frac{\text{ArcTan}\left(\sqrt{x}\right)}{x^{3/2}} \, dx$

Optimal. Leaf size=22

$$-\frac{2\text{ArcTan}\left(\sqrt{x}\right)}{\sqrt{x}}+\log(x)-\log(1+x)$$

```
[Out] ln(x)-ln(1+x)-2*arctan(x^(1/2))/x^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4946, 36, 29, 31}

$$-\frac{2\text{ArcTan}\left(\sqrt{x}\right)}{\sqrt{x}}+\log(x)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/x^(3/2),x]
```

```
[Out] (-2*ArcTan[Sqrt[x]])/Sqrt[x] + Log[x] - Log[1 + x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^{3/2}}\,dx &= -\frac{2\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}+\int \frac{1}{x(1+x)}\,dx\\
&= -\frac{2\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}+\int \frac{1}{x}\,dx-\int \frac{1}{1+x}\,dx\\
&= -\frac{2\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}+\log(x)-\log(1+x)
\end{aligned}
$$

Mathematica [**A**]
time = 0.01, size = 22, normalized size = 1.00

$$-\frac{2\mathrm{ArcTan}\left(\sqrt{x}\right)}{\sqrt{x}}+\log(x)-\log(1+x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/x^(3/2),x]
```

```
[Out] (-2*ArcTan[Sqrt[x]])/Sqrt[x] + Log[x] - Log[1 + x]
```

Maple [**A**]
time = 0.02, size = 19, normalized size = 0.86

method	result	size
derivativedivides	$\ln(x)-\ln(1+x)-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}$	19
default	$\ln(x)-\ln(1+x)-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}$	19
meijerg	$\ln(x)-\ln(1+x)-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/x^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-ln(1+x)-2*arctan(x^(1/2))/x^(1/2)
```

Maxima [**A**]
time = 0.26, size = 18, normalized size = 0.82

$$-\frac{2\arctan\left(\sqrt{x}\right)}{\sqrt{x}}-\log\left(x+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(3/2),x, algorithm="maxima")
[Out] -2*arctan(sqrt(x))/sqrt(x) - log(x + 1) + log(x)
```

Fricas [A]
time = 5.14, size = 26, normalized size = 1.18

$$-\frac{x \log(x+1) - x \log(x) + 2 \sqrt{x} \arctan\left(\sqrt{x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(3/2),x, algorithm="fricas")
[Out] -(x*log(x + 1) - x*log(x) + 2*sqrt(x)*arctan(sqrt(x)))/x
```

Sympy [A]
time = 0.38, size = 20, normalized size = 0.91

$$\log(x) - \log(x+1) - \frac{2 \operatorname{atan}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/x**(3/2),x)
[Out] log(x) - log(x + 1) - 2*atan(sqrt(x))/sqrt(x)
```

Giac [A]
time = 0.45, size = 18, normalized size = 0.82

$$-\frac{2 \arctan\left(\sqrt{x}\right)}{\sqrt{x}} - \log(x+1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(3/2),x, algorithm="giac")
[Out] -2*arctan(sqrt(x))/sqrt(x) - log(x + 1) + log(x)
```

Mupad [B]
time = 0.36, size = 22, normalized size = 1.00

$$2 \ln\left(\sqrt{x}\right) - \ln(x+1) - \frac{2 \operatorname{atan}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x^(1/2))/x^(3/2),x)
[Out] 2*log(x^(1/2)) - log(x + 1) - (2*atan(x^(1/2)))/x^(1/2)
```

3.164 $\int \frac{\mathbf{ArcTan}\left(\sqrt{x}\right)}{x^{5/2}}\,dx$

Optimal. Leaf size=37

$$-\frac{1}{3x}-\frac{2\text{ArcTan}\left(\sqrt{x}\right)}{3x^{3/2}}-\frac{\log(x)}{3}+\frac{1}{3}\log(1+x)$$

```
[Out] -1/3/x-2/3*arctan(x^(1/2))/x^(3/2)-1/3*ln(x)+1/3*ln(1+x)
```

Rubi [A]

time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4946, 46}

$$-\frac{2\text{ArcTan}\left(\sqrt{x}\right)}{3x^{3/2}}-\frac{1}{3x}-\frac{\log(x)}{3}+\frac{1}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/x^(5/2),x]
```

```
[Out] -1/3*1/x - (2*ArcTan[Sqrt[x]])/(3*x^(3/2)) - Log[x]/3 + Log[1 + x]/3
```

Rule 46

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m +
 n + 2, 0])
```

Rule 4946

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Dist[b*c*n*(p/(m +
1)), Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x], x
] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &&
 IntegerQ[m])) && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{x^{5/2}}\,dx &= -\frac{2\tan^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}+\frac{1}{3}\int \frac{1}{x^2(1+x)}\,dx\\
&= -\frac{2\tan^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}+\frac{1}{3}\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)dx\\
&= -\frac{1}{3x}-\frac{2\tan^{-1}\left(\sqrt{x}\right)}{3x^{3/2}}-\frac{\log(x)}{3}+\frac{1}{3}\log(1+x)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.84

$$\frac{1}{3}\left(-\frac{1}{x}-\frac{2\text{ArcTan}\left(\sqrt{x}\right)}{x^{3/2}}-\log(x)+\log(1+x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/x^(5/2),x]
[Out] (-x^(-1) - (2*ArcTan[Sqrt[x]])/x^(3/2) - Log[x] + Log[1 + x])/3
```

Maple [A]
time = 0.03, size = 26, normalized size = 0.70

method	result	size
derivativedivides	$-\frac{1}{3x}-\frac{2\arctan\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}-\frac{\ln(x)}{3}+\frac{\ln(1+x)}{3}$	26
default	$-\frac{1}{3x}-\frac{2\arctan\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}-\frac{\ln(x)}{3}+\frac{\ln(1+x)}{3}$	26
meijerg	$\frac{-10x+30}{45x}-\frac{2\arctan\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}+\frac{\ln(1+x)}{3}+\frac{2}{9}-\frac{\ln(x)}{3}-\frac{1}{x}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/x^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/3/x-2/3*arctan(x^(1/2))/x^(3/2)-1/3*ln(x)+1/3*ln(1+x)
```

Maxima [A]
time = 0.26, size = 25, normalized size = 0.68

$$-\frac{2\arctan\left(\sqrt{x}\right)}{3x^{\frac{3}{2}}}-\frac{1}{3x}+\frac{1}{3}\log\left(x+1\right)-\frac{1}{3}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(5/2),x, algorithm="maxima")
[Out] -2/3*arctan(sqrt(x))/x^(3/2) - 1/3/x + 1/3*log(x + 1) - 1/3*log(x)
```

Fricas [A]
time = 3.13, size = 33, normalized size = 0.89

$$\frac{x^2\log\left(x+1\right)-x^2\log\left(x\right)-2\sqrt{x}\arctan\left(\sqrt{x}\right)-x}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^2*log(x + 1) - x^2*log(x) - 2*sqrt(x)*arctan(sqrt(x)) - x)/x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(31) = 62$.
time = 1.18, size = 143, normalized size = 3.86

$$-\frac{2x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{x}\right)}{3x^3+3x^2}-\frac{2\sqrt{x}\operatorname{atan}\left(\sqrt{x}\right)}{3x^3+3x^2}-\frac{x^3\log\left(x\right)}{3x^3+3x^2}+\frac{x^3\log\left(x+1\right)}{3x^3+3x^2}-\frac{x^2\log\left(x\right)}{3x^3+3x^2}+\frac{x^2\log\left(x+1\right)}{3x^3+3x^2}-\frac{x^2}{3x^3+3x^2}-\frac{x}{3x^3+3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/x**(5/2),x)
```

```
[Out] -2*x**(3/2)*atan(sqrt(x))/(3*x**3 + 3*x**2) - 2*sqrt(x)*atan(sqrt(x))/(3*x*
*3 + 3*x**2) - x**3*log(x)/(3*x**3 + 3*x**2) + x**3*log(x + 1)/(3*x**3 + 3*
x**2) - x**2*log(x)/(3*x**3 + 3*x**2) + x**2*log(x + 1)/(3*x**3 + 3*x**2) -
 x**2/(3*x**3 + 3*x**2) - x/(3*x**3 + 3*x**2)
```

Giac [A]
time = 0.41, size = 28, normalized size = 0.76

$$\frac{x-1}{3\,x}-\frac{2\,\arctan\left(\sqrt{x}\right)}{3\,x^{\frac{3}{2}}}+\frac{1}{3}\,\log\left(x+1\right)-\frac{1}{3}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(x - 1)/x - 2/3*arctan(sqrt(x))/x^(3/2) + 1/3*log(x + 1) - 1/3*log(x)
```

Mupad [B]
time = 0.35, size = 27, normalized size = 0.73

$$\frac{\ln\left(x+1\right)}{3}-\frac{2\,\ln\left(\sqrt{x}\right)}{3}-\frac{2\,\mathrm{atan}\left(\sqrt{x}\right)}{3\,x^{3/2}}-\frac{1}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(x^(1/2))/x^(5/2),x)
```

```
[Out] log(x + 1)/3 - (2*log(x^(1/2)))/3 - (2*atan(x^(1/2)))/(3*x^(3/2)) - 1/(3*x)
```

3.165 $\int \frac{\mathbf{ArcTan}(ax^5)}{x}\,dx$

Optimal. Leaf size=33

$$\frac{1}{10}i\mathrm{PolyLog}\big(2,-iax^5\big)-\frac{1}{10}i\mathrm{PolyLog}\big(2,iax^5\big)$$

```
[Out] 1/10*I*polylog(2,-I*a*x^5)-1/10*I*polylog(2,I*a*x^5)
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4944, 4940, 2438}

$$\frac{1}{10}i\mathrm{Li}_2\big(-iax^5\big)-\frac{1}{10}i\mathrm{Li}_2\big(iax^5\big)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x^5]/x,x]
```

```
[Out] (I/10)*PolyLog[2, (-I)*a*x^5] - (I/10)*PolyLog[2, I*a*x^5]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}\left(ax^5\right)}{x}\,dx &= \frac{1}{5}\mathrm{Subst}\left(\int \frac{\tan^{-1}(ax)}{x}\,dx,x,x^5\right)\\
&= \frac{1}{10}i\mathrm{Subst}\left(\int \frac{\log(1-iax)}{x}\,dx,x,x^5\right)-\frac{1}{10}i\mathrm{Subst}\left(\int \frac{\log(1+iax)}{x}\,dx,x,x^5\right)\\
&= \frac{1}{10}i\mathrm{Li}_2\big(-iax^5\big)-\frac{1}{10}i\mathrm{Li}_2\big(iax^5\big)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$\frac{1}{10}i\text{PolyLog}(2,-iax^5)-\frac{1}{10}i\text{PolyLog}(2,iax^5)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x^5]/x,x]
```

```
[Out] (I/10)*PolyLog[2, (-I)*a*x^5] - (I/10)*PolyLog[2, I*a*x^5]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.06, size = 57, normalized size = 1.73

method	result
default	$\ln(x)\arctan(a\,x^5)-\frac{\sum_{\_R1=\text{RootOf}(a^2\_Z^{10}+1)}\frac{\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-x}{\_R1}\right)}{\_R1^5}}{2a}$
meijerg	$-\frac{ia\,x^5\,\text{polylog}\left(2,i\sqrt{a^2x^{10}}\right)}{10\sqrt{a^2x^{10}}}+\frac{ia\,x^5\,\text{polylog}\left(2,-i\sqrt{a^2x^{10}}\right)}{10\sqrt{a^2x^{10}}}$
risch	$\frac{i\ln(x)\ln(-ia\,x^5+1)}{2}-\frac{i\left(\sum_{\_R1=\text{RootOf}\left(a\_Z^5+\text{RootOf}(\_Z^2+1,index=1)\right)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)}{2}-\frac{i\ln(x)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x^5)/x,x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)*arctan(a*x^5)-1/2/a*sum(1/_R1^5*(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x)/
_R1)),_R1=RootOf(_Z^10*a^2+1))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(19) = 38.
time = 0.48, size = 50, normalized size = 1.52

$$-\frac{1}{20}\pi\log\left(a^2x^{10}+1\right)+\frac{1}{5}\arctan\left(ax^5\right)\log\left(ax^5\right)-\frac{1}{10}i\,\text{Li}_2\left(i\,ax^5+1\right)+\frac{1}{10}i\,\text{Li}_2\left(-i\,ax^5+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x^5)/x,x, algorithm="maxima")
```

```
[Out] -1/20*pi*log(a^2*x^10 + 1) + 1/5*arctan(a*x^5)*log(a*x^5) - 1/10*I*dilog(I*
a*x^5 + 1) + 1/10*I*dilog(-I*a*x^5 + 1)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x^5)/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(a*x^5)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}(ax^5)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x**5)/x,x)
```

```
[Out] Integral(atan(a*x**5)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x^5)/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x^5)/x, x)
```

Mupad [B]
time = 0.34, size = 25, normalized size = 0.76

$$\frac{\mathrm{polylog}(2,-a\,x^5\,1\mathrm{i})\;1\mathrm{i}}{10}-\frac{\mathrm{polylog}(2,a\,x^5\,1\mathrm{i})\;1\mathrm{i}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x^5)/x,x)
```

```
[Out] (polylog(2, -a*x^5*1i)*1i)/10 - (polylog(2, a*x^5*1i)*1i)/10
```

3.166 $\int \frac{\mathbf{ArcTan}(ax^n)}{x}\,dx$

Optimal. Leaf size=39

$$\frac{i\text{PolyLog}(2,-iax^n)}{2n}-\frac{i\text{PolyLog}(2,iax^n)}{2n}$$

```
[Out] 1/2*I*polylog(2,-I*a*x^n)/n-1/2*I*polylog(2,I*a*x^n)/n
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4944, 4940, 2438}

$$\frac{i\text{Li}_2(-iax^n)}{2n}-\frac{i\text{Li}_2(iax^n)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[a*x^n]/x,x]
```

```
[Out] ((I/2)*PolyLog[2, (-I)*a*x^n])/n - ((I/2)*PolyLog[2, I*a*x^n])/n
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4940

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[I*(b/2), Int[Log[1 - I*c*x]/x, x], x] - Dist[I*(b/2), Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1
/n, Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c, n
}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\tan^{-1}(ax^n)}{x}\,dx = \frac{\text{Subst}\left(\int \frac{\tan^{-1}(ax)}{x}\,dx, x, x^n\right)}{n}$$
$$= \frac{i\text{Subst}\left(\int \frac{\log(1-iax)}{x}\,dx, x, x^n\right)}{2n} - \frac{i\text{Subst}\left(\int \frac{\log(1+iax)}{x}\,dx, x, x^n\right)}{2n}$$
$$= \frac{i\text{Li}_2(-iax^n)}{2n} - \frac{i\text{Li}_2(iax^n)}{2n}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 0.82

$$\frac{i(\text{PolyLog}(2, -iax^n) - \text{PolyLog}(2, iax^n))}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[a*x^n]/x,x]
```

```
[Out] ((I/2)*(PolyLog[2, (-I)*a*x^n] - PolyLog[2, I*a*x^n]))/n
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(31) = 62.
time = 0.08, size = 83, normalized size = 2.13

method	result
meijerg	$\frac{-\frac{2ia\,x^n\,\text{polylog}\left(2, i\sqrt{x^{2n}a^2}\right)}{\sqrt{x^{2n}a^2}}+\frac{2ia\,x^n\,\text{polylog}\left(2, -i\sqrt{x^{2n}a^2}\right)}{\sqrt{x^{2n}a^2}}}{4n}$
derivativedivides	$\frac{\ln(a\,x^n)\arctan(a\,x^n)+\frac{i\ln(a\,x^n)\ln(1+ia\,x^n)}{2}-\frac{i\ln(a\,x^n)\ln(1-ia\,x^n)}{2}+\frac{i\,\text{dilog}(1+ia\,x^n)}{2}-\frac{i\,\text{dilog}(1-ia\,x^n)}{2}}{n}$
default	$\frac{\ln(a\,x^n)\arctan(a\,x^n)+\frac{i\ln(a\,x^n)\ln(1+ia\,x^n)}{2}-\frac{i\ln(a\,x^n)\ln(1-ia\,x^n)}{2}+\frac{i\,\text{dilog}(1+ia\,x^n)}{2}-\frac{i\,\text{dilog}(1-ia\,x^n)}{2}}{n}$
risch	$-\frac{i\ln(x)\ln(1+ia\,x^n)}{2}-\frac{i\,\text{dilog}(1-ia\,x^n)}{2n}+\frac{i\ln(-i(-a\,x^n+i))\ln(x)}{2}-\frac{i\ln(-i(-a\,x^n+i))\ln(-ia\,x^n)}{2n}-\frac{i\,\text{dilog}(-ia}{2n}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(a*x^n)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*(ln(a*x^n)*arctan(a*x^n)+1/2*I*ln(a*x^n)*ln(1+I*a*x^n)-1/2*I*ln(a*x^n)*
ln(1-I*a*x^n)+1/2*I*dilog(1+I*a*x^n)-1/2*I*dilog(1-I*a*x^n))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x^n)/x,x, algorithm="maxima")
```

```
[Out] -a*n*integrate(x^n*log(x)/(a^2*x*x^(2*n) + x), x) + arctan(a*x^n)*log(x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(25) = 50$.
time = 2.29, size = 63, normalized size = 1.62

$$\frac{2\,n\arctan\left(ax^{n}\right)\log\left(x\right)+i\,n\log\left(i\,ax^{n}+1\right)\log\left(x\right)-i\,n\log\left(-i\,ax^{n}+1\right)\log\left(x\right)-i\,\mathrm{Li}_{2}(i\,ax^{n})+i\,\mathrm{Li}_{2}(-i\,ax^{n})}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x^n)/x,x, algorithm="fricas")
```

```
[Out] 1/2*(2*n*arctan(a*x^n)*log(x) + I*n*log(I*a*x^n + 1)*log(x) - I*n*log(-I*a*
x^n + 1)*log(x) - I*dilog(I*a*x^n) + I*dilog(-I*a*x^n))/n
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{atan}\left(ax^{n}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(a*x**n)/x,x)
```

```
[Out] Integral(atan(a*x**n)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(a*x^n)/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(a*x^n)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\mathrm{atan}\left(a\,x^n\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(atan(a*x^n)/x,x)
```

```
[Out] int(atan(a*x^n)/x, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstar
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```